

(χ^2) Chi-Square Test χ^2 test of Goodness of Fit

- χ^2 test is used to test whether differences between observed and expected frequencies are significant.
- χ^2 is used to test the independence of attributes
- The test statistic $\chi^2 = \sum \left[\frac{(O-E)^2}{E} \right]$
- Where O – Observed Frequency
- E – Expected Frequency
- If the data is given in a series of “n” numbers then degrees of freedom = $n - 1$.

Note:

- If the case of Binomial Distribution the degrees of freedom = $n - 1$
- Poisson distribution the degrees of freedom = $n - 2$
- Normal distribution the degrees of freedom = $n - 3$

1. The following table gives the number of aircraft accident that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	18	12	11	15	14	84

Solution:

The expected number of accidents on any day = $\frac{84}{6} = 14$

Let H_0 : The accidents occur uniformly over the week.

Observed Frequency	Expected Frequency	(O – E)	$\frac{(O - E)^2}{E}$
14	14	0	0
18	14	4	1.143

12	14	-2	0.286
11	14	-3	0.643
15	14	1	0.071
14	14	0	0

Now $\chi^2 = \sum \left[\frac{(O-E)^2}{E} \right] = 2.143$

Number of degrees of freedom $V = n - 1 = 7 - 1 = 6$

Critical value: The tabulated value of χ^2 at 5% for 6 d. f is 12.59

Conclusion:

Since $\chi^2 = 2.143 < 12.59$, then the null hypothesis H_0 is accepted.

i.e., we conclude that the accidents are uniformly distributed over the week

1. **4 coins were tossed 160 times and the following results were obtained.**

No. of heads : 0 1 2 3 4

Frequency : 19 50 52 30 9

Test the goodness of fit with the help of χ^2 on the assumption that the coins are unbiased

Solution:

Set the null hypothesis: H_0 : The coins are unbiased.

The probability of getting the success of heads is $p = \frac{1}{2}$

- And $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$
- When 4 coins are tossed, the probability of getting “r” heads is given by $P(X = r) = {}^nC_r p^r q^{n-r}$, $r = 0, 1, 2, \dots$
- The expected frequency of getting 0, 1, 2, 3, 4 heads are given by
- $P(X = 0) = 160 \times {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} = 10$
- $P(X = 1) = 160 \times {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} = 40$
- $P(X = 2) = 160 \times {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = 60$

- $P(X = 3) = 160 \times {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3} = 40$
- $P(X = 4) = 160 \times {}^4C_0 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{4-4} = 10$

Observed Frequency	Expected Frequency	(O - E)	$\frac{(o - E)^2}{E}$
19	10	-9	8.1
50	40	10	2.5
52	60	-8	1.067
30	40	-10	2.5
9	10	-1	0.1

Now $\chi^2 = \sum \left[\frac{(o-E)^2}{E} \right] = 14.267$

Number of degrees of freedom $V = n - 1 = 5 - 1 = 4$

Critical value: The tabulated value of χ^2 at 5% for 4 d. f is 9.488

Conclusion:

Since $\chi^2 = 14.267 > 9.488$, then the null hypothesis H_0 is rejected.

i.e., The coin are biased