DIGITAL IMAGE PROCESSING

Unit V – Image Restoration: Noise Models (Gonzalez & Woods)

Topic: Noise Models

1. Sources and Assumptions

Noise in digital images arises mainly during image acquisition (sensor limitations, poor illumination, sensor temperature) and/or during transmission (interference in the transmission channel, e.g. atmospheric disturbance in wireless transmission, or lightning/electrical-switching disturbance in analogue TV signals). Spatial-domain restoration filters are designed directly from a mathematical model of the noise, so understanding these models is the first step of any restoration procedure.

The principal noise descriptor used in restoration is the noise probability density function (PDF) — i.e. the statistical behaviour of the grey-level values of the noise component $\eta(x,y)$. Two simplifying assumptions are almost always made:

- The noise is independent of spatial coordinates (its statistics do not vary with position in the image).
- The noise is uncorrelated with the image itself (i.e., there is no correlation between pixel values of $f(x,y)$ and the noise values at those coordinates).

2. Gaussian Noise

Gaussian noise arises in an image mainly from electronic circuit noise and sensor noise caused by poor illumination and/or high temperature. Because of its tractability in both the spatial and frequency domains, it is the most commonly used noise model.

$$p(z) = 1 / [\, sqrt(2\pi) \cdot \sigma \,] \;\cdot\; e \,\hat{}\, [\, -(z-\mu)^2 / 2\sigma^2 \,]$$

where z represents grey level, μ is the mean (average) value of z, and σ is its standard deviation. The mean and variance of this PDF are μ and σ^2 respectively; about 70% of the values of z lie in the range $[(\mu-\sigma), (\mu+\sigma)]$, and about 95% lie in $[(\mu-2\sigma), (\mu+2\sigma)]$.

3. Rayleigh Noise

$$p(z) = (2/b)\cdot(z-a)\cdot e \,\hat{}\, [\, -(z-a)^2 / b \,] \quad for\ z \geq a \ ; \quad p(z) = 0\ for\ z < a$$

Mean: $\mu = a + \sqrt{(\pi b/4)}$.   Variance: $\sigma^2 = b(4-\pi)/4$. Unlike the Gaussian, the Rayleigh density is skewed to the right, which makes it a good approximation for the skewed histograms often seen in range imaging.

4. Erlang (Gamma) Noise

$$p(z) = [\, a\hat{}b \cdot z\hat{}(b-1) / (b-1)! \,] \cdot e \,\hat{}(-az) \quad for\ z \geq 0 \ ; \quad p(z) = 0\ for\ z < 0, \quad with\ a > 0\ and\ b\ a\ positive\ integer$$

Mean: $\mu = b/a$.   Variance: $\sigma^2 = b/a^2$. This density is frequently used to approximate the noise encountered in laser-based imaging systems.

5. Exponential Noise

$$p(z) = a \cdot e\,\hat{}\,(-az) \quad \text{for } z \geq 0 \;;\; p(z) = 0 \text{ for } z < 0, \quad \text{with } a > 0$$

Mean: $\mu = 1/a$. Variance: $\sigma^2 = 1/a^2$. This is a special case of the Erlang PDF with b = 1, and it too arises in laser imaging applications.

6. Uniform Noise

$$p(z) = 1/(b-a) \quad \text{for } a \leq z \leq b \;;\; p(z) = 0 \text{ otherwise}$$

Mean: $\mu = (a+b)/2$. Variance: $\sigma^2 = (b-a)^2/12$. Uniform noise is not usually descriptive of real physical noise sources, but it is very useful as a basis for numerical simulation and testing of restoration algorithms, since its parameters are easy to vary and its randomness easy to quantify.

7. Impulse (Salt-and-Pepper) Noise

$$p(z) = Pa \quad \text{for } z = a \;(\text{pepper}) \;;\; p(z) = Pb \quad \text{for } z = b \;(\text{salt}) \;;\; p(z) = 0 \quad \text{otherwise}$$

If b > a, grey level b appears as a bright (“salt”) dot and grey level a appears as a dark (“pepper”) dot in the image. If either Pa or Pb is zero, the noise is called unipolar. Impulse noise is generally caused by faulty or noisy switching during image acquisition or transmission and, unlike the other five models, corrupts only a fraction of the pixels while leaving the remaining pixels unaffected — it is typically several orders of magnitude larger in amplitude than the image signal, making affected pixels appear as sharp, isolated bright/dark dots.

8. Comparative Summary Table

Noise model	Typical source	Mean	Variance
Gaussian	Sensor / electronic circuit noise; poor illumination or high temperature	μ	σ^2
Rayleigh	Range imaging	$a + \sqrt{\pi b/4}$	$b(4-\pi)/4$
Erlang (Gamma)	Laser imaging	b/a	b/a^2
Exponential	Laser imaging	$1/a$	$1/a^2$
Uniform	Simulation / algorithm testing	$(a+b)/2$	$(b-a)^2/12$
Impulse (salt-and-pepper)	Faulty sensor / transmission-channel switching errors	— (bimodal, not described by μ alone)	—

10. Periodic Noise

Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition, for example from an unevenly running motor in the imaging system, or from mains-frequency interference in the sensor electronics. Unlike the noise models above, it is spatially dependent (not purely random) and appears as regularly repeating sinusoidal-like patterns superimposed on the image.

Its distinguishing feature is that periodic noise is most easily identified and removed in the frequency domain, where it produces concentrated, symmetric impulse-like bursts of energy at specific frequency locations — rather than being spread across the spectrum the way random noise is.

11. Estimating Noise Parameters from an Image

When the imaging system itself is not available for direct noise measurement, noise parameters can be estimated from the image data itself using the following procedure:

1. Select a small sub-image region Ss that is as flat / featureless as possible — i.e., one with the least amount of intensity variation from the underlying scene (for example, a patch of uniform background or sky).
2. Construct the grey-level histogram of the pixels within Ss; because the region is flat, this histogram is (ideally) the noise histogram itself, unaffected by image content.
3. Compute the sample mean and sample variance of the pixel values in Ss:

$$\bar{z} = (1/K)\,\Sigma\, zi \quad ; \quad \sigma^2 = (1/K)\,\Sigma\,(zi - \bar{z})^2 \qquad (K = \text{number of pixels in } Ss)$$

4. Compare the shape of the observed histogram with the standard PDF shapes (Sec. 9) to identify the closest-matching noise model, and use the computed mean and variance to solve for that model's parameters (e.g., for Gaussian noise, $\mu = \bar{z}$ and σ^2 as computed directly; for Uniform noise, solve a and b from $\mu = (a+b)/2$ and $\sigma^2 = (b-a)^2/12$).

This histogram-matching procedure is the standard practical method for identifying an unknown noise source before selecting an appropriate restoration filter.

12. Choosing a Restoration Filter Based on Noise Type

Noise identified	Best-suited spatial filter(s)
Gaussian	Arithmetic mean, Geometric mean filter
Salt-and-pepper (impulse)	Median filter, Contraharmonic mean filter (choose sign of Q appropriately)
Salt noise only (unipolar)	Minimum filter
Pepper noise only (unipolar)	Maximum filter
Gaussian or uniform (random, non-impulse)	Midpoint filter
Mixed (e.g., salt-and-pepper + Gaussian)	Alpha-trimmed mean filter
Periodic	Band-reject or notch filtering in the frequency domain