ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY

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DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

REGULATION 2021

UNIT V: DYNAMICS OF PARTICLES

Problem

An automobile travels 600m in 40s when it is accelerated at a constant rate of $0.6m/s^2$. Determine the initial and final velocity and the distance travelled for the first 12s.

<u>Given:</u>

Total travels distance=600m

Total time=40s

Acceleration a=$0.6m/s^2$

<u>To find</u>

Intitial and final velocity u &v

Distance travelled for the first 12s

<u>Soln</u>

Now

Distance travelled at 60 m

$$s = ut + \frac{1}{2}at2$$

$$600 = u \times 40 + \frac{1}{2} \times 0.6 \times (40)^{2}$$

Intial velocity.u=$3^{m}/_{s}$

Velocity v=u+at

$$V = 3 + 0.6 \times 40$$

Final velocity v=27 $^{m}/_{s}$

The distance travelled for the first 12s,1-2[1]

$a = 0.6\, m/m^2$ s=ut+ $\frac{1}{2}$at^2

$u = 3\,m/s$

$s = 3 \times 12 + \frac{1}{2} \times 0.6 \times (12)^2$

s=79.2 m

2. The motion of a p[article is defined by the relation $x = 3t^3 - 18t^2 + 26t + 8$

Where is the position expressed in metres and t is the time in seconds Determine (i)When the velocity is zero and (ii)The position and the total distance travelled when the acceleration becomes zero.

Given:

$$x = 3t^3 - 18t^2 + 26t + 8$$

x=position

t=seconds.

Soln:

Velocity=v= $\frac{dx}{dt}$

$$v = \frac{d}{dt}(3t^2 - 18t^2 + 26 \times 1 + 0$$

$$v = 9t^2 - 36t + 26$$

(ii)When velocity v= 0

0= $9t^2 - 36t + 26$

$9t^2 - 36t + 26$ =0

a=9 b= -36 c=26

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-36) \pm \sqrt{(-36)^2 - 4 \times 9 \times 26}}{2 \times 9}$$

$$t = \frac{-36 \pm 18.97}{18}$$

$t = \frac{36 \pm 18.97}{18}$ $t = \frac{36-18.97}{18}$

t= $3.094s$

t=0.946s

the velocity becomes zero t=0.946s and t=3.054s

Acceleration a= $\frac{dv}{dt}$

$$a= \frac{d}{dt}[9t^2 - 36t + 26]$$

$$a=9\times 2t-36\times 1 + 0$$

$$a=18t-36$$

Acceleration a= 0

$$0= 18t - 36$$

$$18t=36$$

$$t= \frac{36}{18}$$

$$t= 2s$$

Distance travelled from t=0 to t=2s

t= $2s$ $\quad x = 3t^3 - 18t^2 + 26 + 8$

$$x = 3 \times (2)^3 - 18 \times (2)^2 + 26 \times 2 + 8$$

$$x = 12\, m$$

t= $0s$ $\quad x = 3 \times (0)^3 - 18 \times (0)^2 + 26 \times 0 + 8$

$$x = 8\, m$$

When t= $0.946\, s$ ‘v’ becomes zero

$$x = 3(0.946)^3 - 18 \times (0.946)^2 + 26 \times 0.946 + 8$$

$$x = 19\, m$$

Total distance travelled= $(19 - 8) + (19 - 12)$

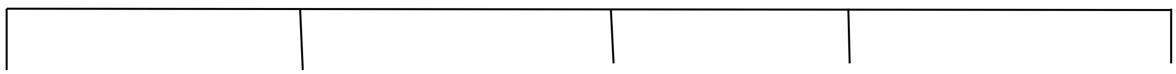
$$= 18\, m$$

3. A particle under constant deceleration is moving on a straight line and

covers a distance of 25 m in the first 3s and 40 m in nest 6s. Calculate the distance it covers in subsequent 2s and the total distance covered before it come to rest.

<u>Given:</u>

A B C D E



S AB=25m S BC= $40\ m$

t= $3s$ t= $6s$ t= $2s$

(A-B) (UDRM)

S= ut+$\frac{1}{2}\ at^2$

S$_{\text{A-B}}$= $u_a\ t_{A-B} + a\ t_{AB^2}$

25$\pm u + 3 + \frac{1}{2}(a) \times (3)^2$

25= $3\ u + \frac{1}{2}\ a \times 9$

25=3 u+4.5 a

Both side÷by 3

$\frac{25}{3} = \frac{34}{3} + \frac{4.5}{3}$ a

8.33=u+4.5 a

u+1.5 $a = 8.33$ ----------- > (1)

A−C

s= $ut + \frac{1}{2}\ at^2$

65$\pm u \times 9 + \frac{1}{2}at^2$

s=65 t= $3 + 6 = 9$

65= $9u + 40.\,s\ u$

$\div 9$

$7.22 = u + 4.5\ a$

$u \pm 4.5\ a = 7.22$

$u = 7.22 - 4.5\,a$--------->(2)

sub (ii) in (i)

$7.22 - 4.5\,a + 1.5\,a = 8.33$

$-3a = 8.33 - 7.22$

$-3a = 1.108$

$a = 1.108/-3$

$a = -0.369\ m/s^2$-------------- >(3)

sub (iii) in (2)

$u = 7.22 - 4.5 \times (0.369)$

$u = 8.88\, m/s$

To find velocity at point c

$v = u + at$

$v_c = u_A + at_{A-C}$

$= 8.88 + (-0.369)(9)$

$v_c = 5.56\ m/s$

For the motion from C to D (UDRM)

$v_c = 5.56 m/s$ $\qquad t_{C-D} = 2s$ $\qquad a = -0.369\ m/s^2$

$s = ut + \frac{1}{2}\, at^2$

$S_{C-D} = u_c t_{C-D} + \frac{1}{2} a\ t_{CD}{}^2$

$= 5.56 \times 2 + \frac{1}{2} \times (-0.369) 2^2$

$S_{C-D} = 10.38$ m

Distance travelled in subsequent t=2s

s=10.38 m

For the motion from C-E (UDRM)

V_c=5.56 m/s a= $-0.369 m/s$ V_E= 0

We have

$v^2 - u^2 = 2as$

$v_E{}^2 - v_C{}^2 = 2as$

$0^2 - (5.56)^2 = 2 \times (-0.369) \times s_{CE}$

s_{CE}=41.8 m

Total distance travelled before it comes to res

$= S_{AB}\ S_{BC} + S_{CE}$

=25+40+41.8

Total distance=106.9 m

4. The position of a particle which moves along a straight line is defined as $s = t^3 - 6t^2 - 15t + 40$ where s is expressed in m and + is in sec. Determine the (a) time at which the velocity will be zero. (b) the position and distance travelled by the particle at that time (c) acceleration of the particle at that time (d) the distance travelled by the particle when t=4 to t=6

<u>Given:</u>

$s = t^3 - 6t^2 - 15t + 40$

<u>Soln:</u>

a) t=? Velocity v=0

v= $\frac{ds}{dt}$

v= $\frac{d}{dt}(t^3 - 6t^2 - 15t + 4)$

v=$3t^2 - 6 \times 2t - 15 \times 1 + 0$

v=$3t^2 - 12t - 15$

v=0

$3t^2 - 12t - 15 = 0$

t= $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a= 3 b= -12 c-15

t= $\frac{-b \pm \sqrt{(-12)^2 - 4 \times 3(-15)}}{2 \times 3}$

t= $\frac{12 \pm \sqrt{144 + 130}}{6}$

t= $\frac{12 \pm \sqrt{324}}{6}$

t= $\frac{12 \pm 18}{6}$

t= $\frac{12+18}{6} = \frac{30}{6}$

T=5 Sec

&

t= $\frac{12-18}{6} = \frac{-6}{6}$

t= -1 sec

t$\neq -1$

t= 5 sec

b) t=5 Sec & displacement s=?

s= $t^3 - 6t^2 - 15t + 40$

s= $5t^3 - 6(5)^2 - 15 \times 5 + 40$

s= $-60\ m$

t= 0

s= $0^3 - 6 \times 0^2 - 15 \times 0 + 40$

s= $40\ m$

Distance travelled= $[s_t = 5] - [s_t = 0]$

$= -60 - 40 \qquad = -100\ m$

Distance travelled $= 100\ m$

3) when t=6 sec displacement 's'

s= $t^3 - 6t^2 - 15t + 40$

s= $6^3 - 6 \times 6^2 - 15 \times 6 + 40$

s= $4^3 - 6 \times 4^2 - 15 \times 4 + 40$

s= $-52\ m$

Distance travelled when t=4 to 5 sec

$= s_t = 5 - s_t = 4$

$= -60 - [-52]$

$= -60 + 52$

$= -8 = 8\ m$

Distance travelled when t=5 to 6

$= \mathrm{s_t} = 5 - \mathrm{s_t} = 5$

$= -50 - (-60)$

=10 m

Total distance travelled=8+10 $= 18\ m$

4) Acceleration a

a= $\frac{dv}{dt} = \frac{d}{dt}[3t^2 - 12t - 15]$

a=3$\times 2t - 12 \times 1$---(5)

a=6t−12

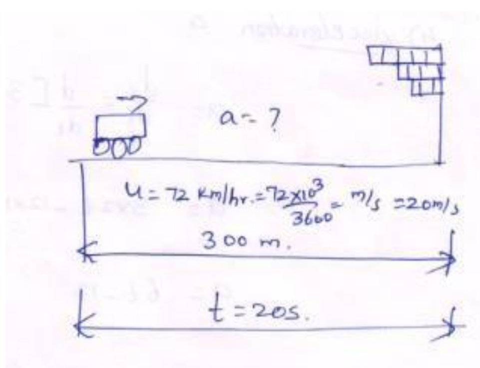
t=5 sec

a=6$\times 5 - 12$

a= $30 - 12$

a=18m/s^2

5) A driver of a car travelling at $72km/h$ Observes the traffic light 300 m ahead of him turning red. The traffic light is timed to remain red for 20 seconds before it turns without stopping to wait for its turn green, Determine (i) the required uniform acceleration of the car (ii) the speed with which the motorist crosses the traffic light.



<u>Soln:</u>

Displacement

s= $ut + \frac{1}{2}at^2$

300=20$\times 20 + \frac{1}{2} \times a \times 20^2$

a= $-0.5\ m/s^2$ (Retardation)

Final velocity

v=u+at

v= $20 + (-0.5) \times 20$

v= $10\,m/s$

v= $\frac{10\times3600}{1000} km/hr$

v= $36\ km/hr$

Problem:5

A particle starting from rest moves in a straight like and its acceleration is given by a=$50-36t^2 m/s^2$ Where t is in sec.Determine the velocity of the particle when it has travelled 52m.

Given

a= $50 - 36t^2$

s= $52m$

To find

Velocity

Soln

Acceleration a= $\frac{dv}{dt}$

dv=a$\times$ dt

dv=a$\times$ dt

dv=($50-36t^2$)dt

$\int dv$=$\int(50 - 36t^2)dt$

$\int dv$=$\int(50 - 36t^2)dt$

$v = 50t - 36 \times \frac{t^3}{3}$

$v = 50t - 36 \times \frac{t^3}{3}$

$v = 50t - 12t^3 + c_1$

when $t = 0$ $v = 0$ $c_1 = 0$

$v = 50t - 12t^3$

$ds = v \times dt$

$ds = 50t - 12t^3 \times dt$

$ds = 50t - 12t^3 \times dt$

$\int ds = \int (50t - 12t^3)dt$

$s = \frac{50t^2}{2} - 12 \times \frac{t^4}{4} + c_2$

$s = 25t^2 - 3t^4 + c_2$

when $t=0$ $s = 0$ $c_2 = 0$

$s = 25t^2 - 3t^4$

Now s=52 m finding out t

$52 = 25 \times t - 3t^4$

$52 = 25t - 3t^4$

Put $t^2 = t$

$52 = 25\,t - 3t^2$

$3t^2 - 25t + 52 = 0$

a=3

b=-25

c=52

$t = \frac{-b \mp \sqrt{b^2-4ac}}{2a}$

t= $\frac{-(25) \mp \sqrt{(-25)^2 - 4 \times 3 \times 52}}{2 \times 3}$

t=2.0816sec & t=2sec

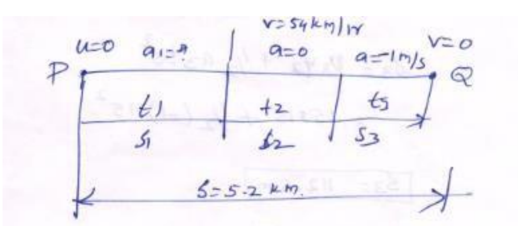
when t= 2sec   v=50$\times 2 - 12 \times 2^3$

v=2m/s

when t=200816 sec $v = 5050 \times 2.0816 - 12 \times (2.0810)^2$

$v = -4.163/m/s$

6. Two stations pand Q are 5.2km apart. A train stars from rest at the station P and accelerates uniformly to attain a speed of 54 km/hr in 30 sec. The speed is maintained until the brakes are applied. The train comes to rest at the station Q with uniform retaration of 1m/s^2. Determine the total time required to cover the distance b/w these two station



Consider Phase I

U=0

t_1=30 sec

v_1=15m/s

v_1=u+$a_1 t_1$$^{v=u+at}$

$15 = 0 + a_1 \times 30$

$a_1 = 0.5$ m/s^2

$s_1 = ut_1 + \frac{1}{2} a_1 t1^2$

$s_1 = 0 + \frac{1}{2} + a_1 t1^2$

$s_1 = 225$m

Consider Phase –III

$v_1 = 15$m/s

$a_3 = -1 m/s^2$

V=0

V=$u + at$

$0 = 15 - 1 \times t_3$

$0 = 15 - +3$

$t_3 = 15$ sec

$s_3 = u_3 v_3 + {}^1/_2\, a_3 t3^2$

$= 15 \times 15 + {}^1/_2\, (-1) 15^2$

$s_3 = 112.5m$

Consider Phase-II

$s_2 = s - [s_1 + s_3]$

$s_2 = 5200 - [225 + 112.5]$

$s_2 = 4862.5$m

$s_2 = ut + {}^1/_2\, at^2$ $\quad\quad a = 0$

s_2=ut

4862.5=15× t

t_2=$\frac{4862.5}{15}$

t_2=324.167sec

Total time=30+324.167+15

time=369.167 sec

multiply 2

120= $14\,u + 49\,a$

14 u+49 a=129=0 ---- (1)

÷=14

$u + 3.5a = 8.57$

u=8.57-3.5 a ---- (2)

Sub Eqn (2) in (1)

$u + a$=10 --------(1)

3.57$-3.5a + a = 10$

8.57$-2.5a = 10$

-2.5a= $10 - 8.57 = 1.43$

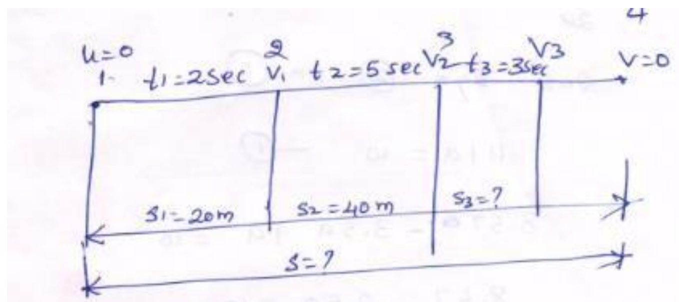
a=$\frac{1.43}{-2.5}$

a=$-0.572\ m/s^2$

u+$(-0.572) = 10$

u=10.572m/s

7. A particle under constant declaration is moving in a straight line and covers a distance of 20m in first 2seconds, and 40min the next 5sec. Calculate the distance it covers in the he subsequent 3sec and total distance travelled by the particle before it comes to rest.



Soln:

Phase (1)-2

The displacement s=$ut + {}^{1}/_{2}\, at^2$

t=2 sec s=20m

20=$u \times 2 + {}^{1}/_{2}\, at^2$

20=2u+$2a$

$\div$ 10 $u + a = 10$ --------------(1)

Phase 1-3

s$= u + {}^{1}/_{2}\, at^2$

60=$u \times + {}^{1}/_{2} \times a \times 7^2$

60=7$u + {}^{1}/_{2} \times 49a$

S$= 20 + 40 = 60$

t=2+5=7

Considered 3rd phase

$t=2+5+3=10$

$s_3=10.572\times 10-{}^{1}/_{2}\times 0.572\times 10^2$

$s_3=17.142$ m

$v_3=u+at$

$v_3=10.57-0.572\times 10$

$v_3=4.857$ m/s

Considered 4th phase

$u_4=4.857$ $v=0$ $a=-0.572\ m/s^2$

$V=u+at$

$0=4.857+(-0.572)\times t$

$t=8.5$ sec

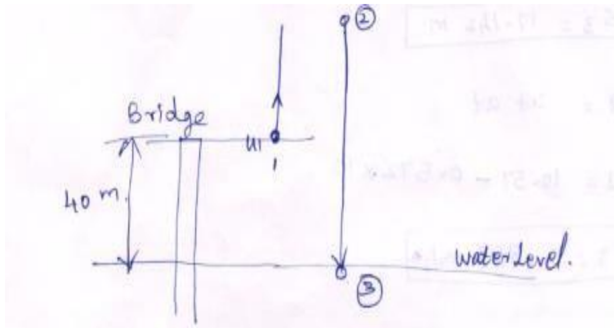
Total time $=2+5+3+8.5=18.5\ sec$

Total distance travel $=s=ut+\frac{1}{2}at^2$

$S=10.57\times 18.5-{}^{1}/_{2}\times 0.57\times 18.5^2$

$S=97.78$m

8. A stone is thrown vertically upwards at a point on a bridge located 40m above the water. If it strikes the water after 4sec, determine (i) the speed at which the stone was thrown up and (ii)The speed at which the stone strikes the water.



Soln:

For the a= $-9.81\, m/s^2$ $\quad v_2 = 0 \quad t_{1-2} = t \quad s_{1-2} = h$

v =u+at

0= $u - 9.81 \times t$

u= +9.81 t-------->(1)

Distance s=ut+ $\frac{1}{2} at^2$

$s_{1-2} = 9.81t \times t - \frac{1}{2} 9.81t^2$

$s_{1-2} = 9.81t^2 - 4.905t^2$

$s_{1-2} = 4.905t^2$ $\quad s_{1-2} = h$ ----->(2)

h=4.905 t^2

For motion 2 to 3

$s_{2-3} = h + 40 =$ $\quad v_2 = 0 \quad t_{2-3} = 4 - t$

a= $9.81\, m/s^2$

s= $ut + \frac{1}{2} at^2$

$s_{2-3} = u_2\, t_{2-3} + \frac{1}{2} at_{2-3}{}^2$

h+40 $= 0 + \frac{1}{2} \times 9.81 \times (4 - t)^2$

$h+40 = \frac{1}{2} 9.81 \times (4-t)^2 = 4.905[4-t]^2$

sub in (2)

$4.905t^2 + 40 = 4.905[16 + t^2 - 8t]$

$4.905t^2 + 40 = 78.48 - 4.905t^2 - 39.24t$

$40 = 78.48 - 39.24t$

$-39.24t = 40 - 78.48$

$-39.24t = -38.48$

$t = +0.98s$

$u = 9.81 \times t = 9.81 \times 0.98$

$u = 9.62 m/s$

$v_3 = v_2 + 9.81\,(4 - t)$

$v_3 = 0 + 9.81(4 - 0.98)$

$v_3 = 29.62\ m/s$