24ME401 THERMAL ENGINEERING

For IVth Semester

B.E. Mechanical Engineering course

As per the revised regulation of

(2024 Regulation)

DEPARTMENT OF MECHANICAL ENGINEERING

ROHINI COLLEGE OF ENGINEERING AND TECHNOLOGY

UNIT – I

THERMODYNAMIC CYCLES

1. Derive air standard efficiency for an Otto cycle with the help of Pv diagram.

The Otto cycle, which was first proposed by a Frenchman, Beau de Rochas in 1862, was first used on an engine built by a German, Nicholas A. Otto, in 1876. The cycle is also called a constant volume or explosion cycle. This is the equivalent air cycle for reciprocating piston engines using spark ignition. Figures 1 and 2 show the P-V and T-s diagrams respectively

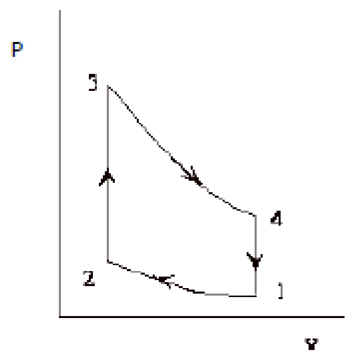


Fig.1: P-V Diagram of Otto Cycle.

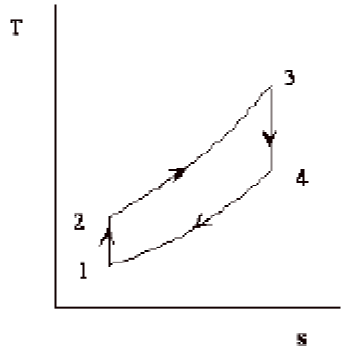


Fig.2: T-S Diagram of Otto Cycle.

At the start of the cycle, the cylinder contains a mass M of air at the pressure and volume indicated at point 1. The piston is at its lowest position. It moves upward and the gas is compressed isentropic ally to point 2. At this point, heat is added at constant volume which raises the pressure to point 3. The high pressure charge now expands isentropic ally, pushing the piston down on its expansion stroke to point 4 where the charge rejects heat at constant volume to the initial state, point 1.

The isothermal heat addition and rejection of the Carnot cycle are replaced by the constant volume processes which are, theoretically more plausible, although in practice, even these processes are not practicable.

The heat supplied, Qs, per unit mass of charge, is given by cv (T3 – T2) (1)

The heat rejected, Qr per unit mass of charge is given by

$$c_v(T_4 - T_1) \qquad (2)$$

and the thermal efficiency is given by

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1}{T_2}\left\{\frac{\left(\frac{T_4}{T_1} - 1\right)}{\left(\frac{T_3}{T_2} - 1\right)}\right\} \qquad (3)$$

$$\text{Now } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \left(\frac{V_3}{V_4}\right)^{\gamma - 1} = \frac{T_4}{T_3}$$

$$\text{And since } \frac{T_1}{T_2} = \frac{T_4}{T_3} \quad \textit{we have} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Hence, substituting in Eq. 3, we get, assuming that r is the compression ratio V1/V2

$$\eta_{th} = 1 - \frac{T_1}{T_2}$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

$$= 1 - \frac{1}{r^{\gamma - 1}} \qquad (4)$$

2. Derive mean effective pressure expression for an Otto cycle

It is seen that the air standard efficiency of the Otto cycle depends only on the compression ratio. However, the pressures and temperatures at the various points in the cycle and the net work done, all depend upon the initial pressure and temperature and the heat input from point 2 to point 3, besides the compression ratio.

A quantity of special interest in reciprocating engine analysis is the mean effective pressure. Mathematically, it is the net work done on the piston, W, divided by the piston displacement volume, $V_1 - V_2$. This quantity has the units of pressure. Physically, it is that constant pressure which, if exerted on the piston for the whole outward stroke, would yield work equal to the work of the cycle. It is given by

$$mep = \frac{W}{V_1 - V_2}$$

$$= \frac{\eta Q_{2-3}}{V_1 - V_2} \qquad (5)$$

where Q_{2-3} is the heat added from points 2 to 3.

Work done per kg of air

$$W = \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1} = mepV_s = P_m(V_1 - V_2)$$

$$mep = \frac{1}{(V_1 - V_2)}\left[\frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1}\right] \qquad (5A)$$

The pressure ratio P_3/P_2 is known as explosion ratio r_p

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\nu} = r^{\nu} \Rightarrow P_2 = P_1 r^{\nu},$$

$$P_3 = P_2 r_p = P_1 r^{\nu} r_p,$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\nu} = P_1 r^{\nu} r_p \left(\frac{V_2}{V_1}\right)^{\nu} = P_1 r_p$$

$$\frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = r$$

$$\therefore V_s = V_c(r - 1)$$

Substituting the above values in Eq 5A

$$mep = P_1 \frac{r(r_p - 1)(r^{\gamma - 1} - 1)}{(r - 1)(\gamma - 1)}$$

, Now

$$V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right)$$

$$= V_1\left(1 - \frac{1}{r}\right) \qquad (6)$$

Here r is the compression ratio, V_1/V_2

From the equation of state:

$$V_1 = M \frac{R_0}{m} \frac{T_1}{p_1} \qquad (7)$$

R_0 is the universal gas constant

Substituting for V_1 and for $V_1 - V_2$,

$$mep = \eta \frac{Q_{2-3} \frac{p_1 m}{M R_0 T_1}}{1 - \frac{1}{r}} \qquad (8)$$

The quantity Q2-3/M is the heat added between points 2 and 3 per unit mass of air (M is the mass of air and m is the molecular weight of air); and is denoted by Q'', thus

$$mep = \eta \frac{Q' \frac{p_1 m}{R_0 T_1}}{1 - \frac{1}{r}} \qquad (9)$$

We can non-dimensionalize the mep by dividing it by p1 so that we can obtain the following equation

$$\frac{mep}{p_1} = \eta \left[\frac{1}{1 - \frac{1}{r}} \right] \left[\frac{Q' m}{R_0 T_1} \right] \qquad (10)$$

Since $\frac{R_0}{m} = c_v(\gamma - 1)$, we can substitute it in Eq. 25 to get

$$\frac{mep}{p_1} = \eta \frac{Q'}{c_v T_1} \frac{1}{\left[1 - \frac{1}{r}\right][\gamma - 1]} \qquad (11)$$

3. In an air standard Otto cyle the pressure and temperature at the beginning of the cylcle is 42^0 C and 0.1MPa. The compression ratio and maximum temperature of the cycle are 8 and 1250^0 C respectively. Find (a) the temperature and pressure at the cardinal points of the cycle, (b) heat supplied per kg of air, (c) work done per kg of air (d) cycle efficiency and (e) the m.e.p. of the cylcle

Given data:

p_1=0.1MPa = 100kN/m^2

T_1=42^{0}C=315K

r=8

$$r = 8 \left(i.e \frac{V_1}{V_2} = \frac{V_4}{V_3} = 8 \right)$$

T_3=1250^{0}C=1523K

solution

Consider process 1-2 (adiabatic process)

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^{\gamma}$$

$$p_2 = \left(\frac{V_1}{V_2} \right) \times P_1 = (8)^{1.4} \times 100$$

$$P_2 = \mathbf{1837.9 kN/m^2}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \times T_1 = (8)^{1.4-1} \times 315$$

$$T_2 = \mathbf{723.68K}$$

consider process 2-3 (constant volume process);

$$\frac{p_3}{p_2} = \frac{T_3}{T_2}$$

$$p_3 = \frac{T_3}{T_2} \times p_2 = \frac{1523}{723.68} \times 1837.9$$

$$p_3 = \mathbf{3867.89kN/m^2}$$

Consider process 3 - 4 (adiabatic process);

$$\frac{p_4}{p_3} = \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$p_4 = \left(\frac{V_3}{V_4}\right) \times P_3 = \left(\frac{1}{8}\right)^{1.4} \times 3867.89$$

$$P_4 = \mathbf{210.44kN/m^2}$$

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T4 = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \times T_3 = \left(\frac{1}{8}\right)^{0.4} \times 1523$$

$$T_4 = \mathbf{662.9K}$$

Heat supplied, Q_s =$mC_v(T_3$-$T_2)$

=1 x 0.718 x (1523 – 723.68)

Q_s = 573.9 kJ/kg

Heat rejected Q_R= $mC_v(T_4$-$T_1)$ = 1 C0.718 X (662.9-315)

Q_R = 249.79 KJ/kg

Work done, W = Q_S – Q_R

W = 573.9 – 249.79

= 324.1 KjKG

Cycle efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$= 1 - \frac{1}{(8)^{1.4-1}} = 0.5647$$

$\boldsymbol{\eta} = \mathbf{56.47\%}$

More effective pressure, p_m

$$p_1 v_1 = mRT_1$$

$$v_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.287 \times 315}{100}$$

v1 = 0.9m³/kg

$$\frac{v_1}{v_2} = 8 \Rightarrow v_2 = \frac{0.9}{9} = 0.11 m^3/kg$$

$$p_m = \frac{W}{v_1 - v_2} = \frac{324.1}{0.9 - 0.11}$$

p_m = 409.2 kN/m²

Alternatively, we can use mean effective pressure formula,

$$p_m = p_1 r \left(\frac{k-1}{\gamma - 1}\right)\left[\frac{r^{\gamma - 1} - 1}{r - 1}\right]$$

$$k = \frac{p_3}{p_2} = \frac{3867.89}{1837.9} = 2.10$$

$$p_m = 100 \times 8 \times \left(\frac{2.1 - 1}{1.4 - 1}\right)\left(\frac{8^{1.4-1} - 1}{8 - 1}\right)$$

p_m = 407.75 Kn/m²

4. Air enters in an air standard Otto cycle at 1bar and 290k. The ratio of heat rejection and heat supplied is 0.4. The maximum Temperature of the cycle is 1500k. Find efficiency, compression ratio, and network and mean effective pressure.

Given data:

p_1 = 1 bar

T_1 = 290K

$$\frac{Q_R}{Q_S} = 0.4$$

T_3 = 1500K

Solution:

Efficiency of the cycle,

$$\eta = 1 - \frac{Q_R}{Q_S}$$

$= 1 - 0.4$

$\eta = 60\%$ Ans.

$$\eta = 1 - \frac{1}{(r)^{\gamma - 1}}$$

$$0.6 = 1 - \frac{1}{(r)^{1.4-1}} \Rightarrow r = \left(\frac{1}{0.4}\right)^{\frac{1}{0.4}}$$

r=9.88 Ans.

Compression ratio, r = 9.88

$$v_1 = \frac{RT_1}{p_1} = \frac{287 \times 290}{1 \times 10^5}$$

$v_1 = 0.8323\ m^3/kg$

$$\frac{v_1}{v_2} = 9.88$$

$$v_2 = \frac{0.8323}{9.88} = 0.0842 m^3 / kg$$

$$T_2 = T_1 \times (r)^{\gamma - 1}$$

$$= 290 \times (9.88)^{1.4-1} = 725K$$

Heat supply, $Q_s = C_v (T_3 - T_2)$

= 0.718 x (1500 – 725)

= 556.45 *kJ/kg*

Work done, $W = \eta \times Q_s = 0.6 \times 556.45$

= 333.87 *kJ/kg* **Ans.**

Mean effective pressure, p_m

$$p_m = \frac{W}{v_1 - v_2} = \frac{333.87}{0.8323 - 0.0842}$$

Pm = 446.29kN/m² ***Ans.***

5 **A six cylinder petrol engine has a compression ratio of 5:1. The clearance volume of each cylinder is 110CC. It operator on the four stroke constant volume cycle and the indicated efficiency ratio referred to air standard efficiency is 0.56. At the speed of 2400 rpm. It consumer 10kg of fuel per hour. The calorific value of fuel is 44000KJ/kg. Determine the average indicated mean effective pressure.**

Given data:

$r = 5$

$V_C = 110CC$

$\eta_{relative} = 0.56$

$N = 2400rpm$

$M_f = 10kg$

$= 10/3600$ kg/s

$C_v = 44000kJ/kg$

$Z = 6$

Solution:

Compression ratio:

$r = V_s + V_c/V_c \rightarrow 5 = V_s + 110/110 \rightarrow V_s = 440CC = 44x10^{-6}m^3$

Air standard efficiency:

$\eta = 1- 1 / (r^{\gamma-1}) = 47.47\%$ $(\gamma = 1.4)$

Relative efficiency:

$\eta_{relative} = \eta_{actual} / \eta_{air-standard} \rightarrow$

$0.56 = \eta_{actual}/47.47 \; \eta_{actual}$

$= 26.58\%$

Actual efficiency = work output/ head input

$0.2658 = W/ m_f C_V \rightarrow W = 0.2658$

x 10/3600x44000 W =

32.49kw.

The network output:

$W= P_m \times V_s \times N/60 \times Z \rightarrow 32.49\times10^3 = Pm \times 440 \times10^{-6} \times$

1200/60 x 6

Pm = 6.15 bar

6. The efficiency of an Otto cycle is 60 % and γ = 1.5, what is the compression ratio?

Given:

Cycle efficiency (η) = 0. 6

γ = 1. 5

Required: r

Solution:

Compression ratio (r) = V_1 / V_2

We know that, $\eta = 1 - 1/r^{\gamma - 1}$

$0.6 = 1 - 1/ r^{1.5 - 1}$

r = 6.25 --- Ans

7. An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearancevolume is 0.00263 m^3. The initial pressure and temperature are 1 bar and 50ºC. If the maximum pressure is limited to 25 bar, find (i) air standard efficiency of the cycle (ii) the mean effective pressure of the cycle.

Given:

Bore (d) = 0.25 m

Stroke (L) = 0.375 m

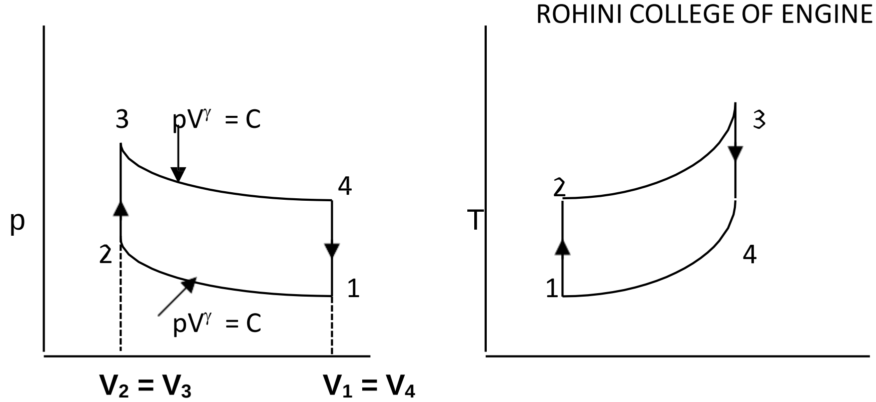
Clearance volume (V_2) = 0.00263

m^3Initial pressure (p_1) = 1 bar

Initial temperature (T_1) = 50ºC = 323 K

Maximum pressure (p_3) = 25 bar

***Required:* (i) η (ii) p_m**



Solution:

(i) Cycle efficiency (η) = $1 - 1/r^{\gamma - 1}$

r = compression ratio = V_1 / V_2

V_1 = stroke volume + clearance volume

$= (V_1 - V_2) + V_2$

$= (\pi/4) D^2 L + V_2$

$= (\pi/4) \times 0.25^2 + 0.375 + 0.00263$

$= 0.021038\ m^3$

$\therefore$ **r = 0. 021038 / 0. 00263 = 8**

$\therefore$ **$\eta = 1 - 1/8^{1.4-1} = 0.565$ --- Ans**

(ii) Mean effective pressure (p_m)

$$Pm = p_1 r \left[\frac{(r^{\gamma-1} - 1)(r_p - 1)}{(\gamma - 1)(r - 1)}\right]$$ → **for Otto cycle**

$r_p = p_3 / p_2$

$p_2 V_2^{\gamma} = p_1 V_1^{\gamma}$

$p_2 = (V_1 / V_2)^{\gamma}\ p_1 = (8)^{1.4} \times 1 = 18.38$ bar

$\therefore$ **$r_p = 25/18.38 = 1.36$**

$$p_m = 1 \times 8 \left[\frac{(8^{1.4-1} - 1)(1.36 - 1)}{(1.4 - 1)(8 - 1)}\right]$$

$p_m = 1.334$ bar Ans

8 Air enters an air standard Otto cycles at 100 kN/m^2 and 290 K. The ratio of heat rejection to heat

supplied is 0.4. The maximum temperature in the cycle is 1500 K.Find (a) efficiency, (b) network, (c) mep, & (d) compression ratio (r).

Given:

Initial pressure (p_1) = 100 kN/m^2 = 1

barInitial temperature (T_1) = 290 K

Heat rejection / Heat supplied = 0.4

Maximum temperature (T_3) = 1500 K

Required: **(a) η (b) W_{net} (c) mep (d)**

r*Solution:*

(a) Cycle efficiency (η) = $1 - 1 / I^{\gamma - 1}$

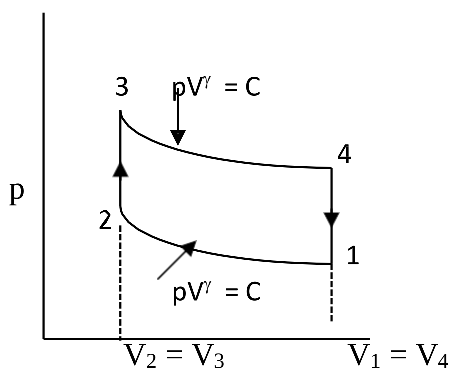
= (heat supplied – heat rejection) / heat supplied

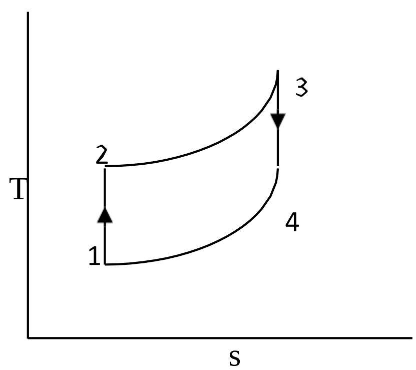
= 1 – (HR / HS) = 1 – 0.4 = 0.6---- Ans

(b) Network (W_{net}) = Heat supplied – Heat rejected

Heat supplied = m C_v ($T_3 - T_2$)

Take m = 1 kg & C_v = 0.717 kJ/kgK





<u>To find T_2</u>

$$T2 / T1 = (p2 / p1)^{(\gamma - 1)/\gamma} = (V1 / V2)^{(\gamma - 1)}$$

$$= r^{\gamma - 1}$$

From , $\eta = 1 - (1/ r^{\gamma - 1})$

i.e., $0.6 = 1 - (1 / r^{1.4-1})$

r = 9.88

$\therefore$ $T_2 = 290\,(9.88)^{1.4-1} = 725$ K

$\therefore$ **Heat supplied = 1 x 0.717 x (1500 – 725) = 555.675 kJ**

Heat rejected = 0.4 (555.675) = 222.27 kJ

$\therefore$ W_{net} **= 555.675 – 222.27 = 333.405 kJ ---- Ans**

(c) MEP

$$p_m = p_1\, r \left[\frac{(r^{\gamma-1} - 1)(r_p - 1)}{(\gamma - 1)(r - 1)}\right] \rightarrow \text{for Otto cycle}$$

$r_p = p_3 / p_2$

2 – 3 → Constant volume process

$p_3 / T_3 = p_2 / T_2$

$\therefore p_3 / p_2 = T_3 /$

T_2 **To find T_2**

$T_2 / T_1 = (V_1 / V_2)^{\gamma - 1}$

$T_2 = 290\,(9.88)^{1.4-1} = 724.94$ K

$Rp = p_3 / p_2 = 1500 / 724.94 = 2.069$

$$p_m = 1 \times 9.88 \left[\frac{(9.88^{1.4-1} - 1)(2.069 - 1)}{(1.4 - 1)(9.88 - 1)}\right]$$

= 4.459 bar ------ Ans

Module –II

Derive thermal efficiency expression for a Diesel cycle.

This cycle, proposed by a German engineer, Dr. Rudolph Diesel to describe the processes of his engine, is also called the constant pressure cycle. This is believed to be the equivalent air cycle for the reciprocating slow speed compression ignition engine. The P-V and T-s diagrams are shown in Figs 4 and 5 respectively.

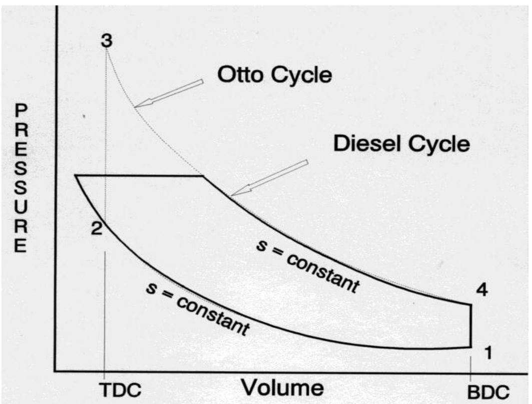


Fig.4: P-V Diagram of Diesel Cycle.

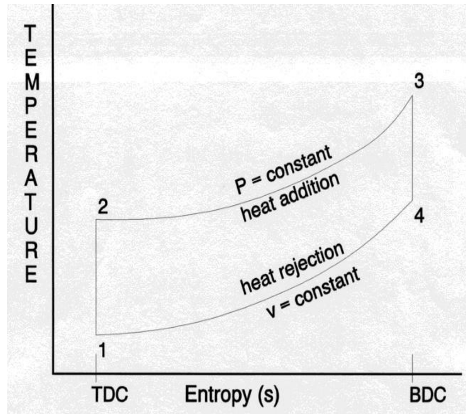


Fig.5: T-S Diagram of Diesel Cycle.

The cycle has processes which are the same as that of the Otto cycle except that the heat is added at constant pressure

The heat supplied, Qs is given by cp(T3 – T2) (22)

Whereas the heat rejected, Qr is given by cv (T4 – T1) (23) and

The thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma}\left\{\frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right)}\right\} \qquad (24)$$

From the T-s diagram, Fig. 5, the difference in enthalpy between points 2 and 3 is the same as that between 4 and 1, thus

$$\Delta s_{2-3} = \Delta s_{4-1}$$

$$\therefore c_v \ln\left(\frac{T_4}{T_1}\right) = c_p \ln\left(\frac{T_3}{T_2}\right)$$

$$\therefore \ln\left(\frac{T_4}{T_1}\right) = \gamma \ln\left(\frac{T_3}{T_2}\right)$$

$$\therefore \frac{T_4}{T_1} = \left(\frac{T_3}{T_2}\right)^{\gamma} \text{ and } \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma - 1} = \frac{1}{r^{\gamma - 1}}$$

Substituting in eq. 24, we get

$$\eta_{th} = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\left(\frac{T_3}{T_2}\right)^{\gamma} - 1}{\frac{T_3}{T_2} - 1}\right] \qquad (25)$$

$$\text{Now } \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c = cut-off\ ratio$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}\right] \qquad (26)$$

When Eq. 26 is compared with Eq. 8, it is seen that the expressions are similar except for the term in the parentheses for the Diesel cycle. It can be shown that this term is always greater than unity.

Now $r_c = \frac{V_3}{V_2} = \frac{V_3}{V_4} \Big/ \frac{V_2}{V_1} = \frac{r}{r_e}$ **where r is the compression ratio and re is the expansion ratio**

Thus, the thermal efficiency of the Diesel cycle can be written as

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\left(\frac{r}{r_e}\right)^{\gamma} - 1}{\gamma\left(\frac{r}{r_e} - 1\right)}\right] \qquad (27)$$

Let $r_e = r - \Delta$ since r is greater than r_e. Here, Δ is a small quantity. We therefore have

$$\frac{r}{r_e} = \frac{r}{r - \Delta} = \frac{r}{r\left(1 - \frac{\Delta}{r}\right)} = \left(1 - \frac{\Delta}{r}\right)^{-1}$$

We can expand the last term binomially so that

$$\left(1 - \frac{\Delta}{r}\right)^{-1} = 1 + \frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \cdots$$

$$\text{Also } \left(\frac{r}{r_e}\right)^{\gamma} = \frac{r^{\gamma}}{(r - \Delta)^{\gamma}} = \frac{r^{\gamma}}{r^{\gamma}\left(1 - \frac{\Delta}{r}\right)^{\gamma}} = \left(1 - \frac{\Delta}{r}\right)^{-\gamma}$$

We can expand the last term binomially so that

$$\left(1 - \frac{\Delta}{r}\right)^{-\gamma} = 1 + \gamma\frac{\Delta}{r} + \frac{\gamma(\gamma+1)}{2!}\frac{\Delta^2}{r^2} + \frac{\gamma(\gamma+1)(\gamma+2)}{3!}\frac{\Delta^3}{r^3} + \cdots$$

Substituting in Eq. 27, we get

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\frac{\Delta}{r} + \frac{(\gamma+1)}{2!}\frac{\Delta^2}{r^2} + \frac{(\gamma+1)(\gamma+2)}{3!}\frac{\Delta^3}{r^3} + \cdots}{\frac{\Delta}{r} + \frac{\Delta^2}{r^2} + \frac{\Delta^3}{r^3} + \cdots}\right] \qquad (28)$$

6. Derive the mean effective pressure expression for a diesel cycle

$$mep = \frac{1}{V_s}\left[P_2(V_3 - V_2) + \frac{P_3V_3 - P_4V_4}{\nu - 1} - \frac{P_2V_2 - P_1V_1}{\nu - 1}\right] \qquad (29)$$

The pressure ratio P3/P2 is known as explosion ratio rp

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\nu} = r^{\nu} \Rightarrow P_2 = P_1 r^{\nu},$$

$$P_3 = P_2 = P_1 r^{\nu}$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\nu} = P_1 r^{\nu}\left(\frac{V_2}{V_1}\right)^{\nu} = P_1 r_c^{\ \nu}$$

$$V_4 = V_1, V_2 = V_c,$$

$$\frac{V_1}{V_2} = \frac{V_c + V_s}{V_c} = r$$

$$\therefore V_s = V_c(r - 1)$$

Substituting the above values in Eq 29 to get Eq (29A)

In terms of the cut-off ratio, we can obtain another expression for mep/p1 as follows

$$mep = P_1\frac{\gamma r^{\gamma}(r_c - 1) - r(r_c^{\gamma} - 1)}{(r-1)(\gamma-1)} \qquad (29A)$$

We can obtain a value of rc for a Diesel cycle in terms of Q" as follows:

$$r_c = \frac{Q'}{c_p T_1 r^{\gamma-1}} + 1 \qquad (30)$$

We can substitute the value of η from Eq. 38 in Eq. 26, reproduced below and obtain the value of mep/p1 for the Diesel cycle.

$$\frac{mep}{p_1} = \eta\frac{Q'}{c_v T_1}\frac{1}{\left[1 - \frac{1}{r}\right][\gamma - 1]}$$

For the Diesel cycle, the expression for mep/p3 is as follows:

$$\frac{mep}{p_3} = \frac{mep}{p_1}\left(\frac{1}{r^{\gamma}}\right) \qquad (31)$$

Modern high speed diesel engines do not follow the Diesel cycle. The process of heat addition is partly at constant volume and partly at constant pressure. This brings us to the dual cycle.

7. A diesel engine working on air standard cycle takes in air at 1bar and 25^{0}C. The specify volume of air at inlet is 0.8^3/kg. The compression ratio is 14 and heat is added at constant pressure is 840kJ/kg. Find cut – off ratio and air standard efficiency.

Given data:

$p_1 = 1bar$

$T_1 = 25^0\ C = 25 + 273 = 293\ K$

$v_1 = 0.8m^3/kg$

$r = 14$

$Q_s = 840kJ/kg$

Solution:

$$\frac{v_1}{v_2} = 14$$

$$v_2 = \frac{0.8}{14} = 0.05714\ m^3 / kg$$

$$T_2 = T_1 \times \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

$T_2 = 298 \times (14)^{1.4-1}$

$T_2 = 856.38K$

We know that, $Q_s = m \times C_v (T_3 - T_2)$

$$T_3 = \frac{Q_s}{C_v} + T_2 = \frac{840}{0.718} + 856.38$$

$T_3 = 2026.3K$

Consider process 2-3:

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

$$v_3 = \frac{v_2}{T_2} \times T_3 = \frac{0.05714}{856.38} \times 2026.3$$

$= 0.1352m^3/kg$

Cut-off ratio, $\rho = \frac{v_3}{v_2} = \frac{0.1352}{0.05714}$

= 2.37 **Ans.**

$\therefore$ **Air Standard efficiency,** $\eta = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left(\frac{\rho^{\gamma} - 1}{\rho - 1} \right)$

$$= 1 - \frac{1}{1.4(14)^{1.4-1}} \left[\frac{(2.37)^{1.4} - 1}{(2.37 - 1)} \right]$$

= 57.42 % **Ans.**

8. An engine working on an ideal air standard diesel cycle has the compression ratio 15 and heat transfer 1400kJ/kg. Find the pressure and temperature at the end of the each process if the inlet conditions are 280K and 1.1 bar. Find also the air standard efficiency and mean effective pressure.

Given data:

r=15

Q_s = 1400 k J/kg

T_1 = 280 K

P_1=1.1bar = 110 k N/m^2

Solution :

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$T_2 = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \times T_1 = (15)^{1.4-1} \times 280$$

$$T_2 = 827.17 K$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma}{\gamma-1}} \times p_1 = \left(\frac{827.17}{280} \right)^{\frac{1.4}{1.4-1}} \times 110$$

$p_3 = p_2$ = 4874.39kN/m^2

Consider the process 2-3 (Constant pressure process);

$$Qs = m \times (T_3 - T_2) = 1400 \, kJ/kg$$

1400 = 1 $\times$ ***1.005*** $\times$ **(T_3 -827.17)**

T_3 = 2220.2K

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2220.2}{827.17} = 2.684$$

Cut-off ratio, $\rho = \frac{V_3}{V_2} = 2.684$

Consider the process 3-4 (adiabatic expansion);

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \times T_3 \qquad (\because V_4 = V_1)$$

$$= \left(\frac{V_3}{V_2} \times \frac{V_2}{V_1}\right)^{\gamma-1} \times T_3 = \left(\frac{2.684}{15}\right)^{1.4-1} \times 2220.2$$

T₄ **= 1115.5K** **Ans.**

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$p_4 = \left(\frac{T_4}{T_3}\right)^{\frac{\gamma}{\gamma-1}} \times p_3$$

$$= \left(\frac{1115.5}{2220.2}\right)^{\frac{1.4}{0.4}} \times 4874.39$$

$p_4 = 438.24 kN/m^3$ **Ans.**

Air standard efficiency,

$$\eta = 1\frac{1}{\gamma(r)^{\gamma-1}}\left(\frac{\rho^{\gamma}-1}{\rho-1}\right)$$

$$= 1 - \frac{1}{1.4(15)^{1.4-1}}\left(\frac{(2.684)^{1.4}-1}{2.684-1}\right)$$

= 57.16% **Ans**

Mean effective pressure,

$$P_m = \frac{p_1 r^{\gamma}\left[\gamma(\rho-1) - r^{1-\gamma}\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)}$$

$$= \frac{100 \times (15)^{1.4}\left[1.4(2.684-1) - (15)^{1-1.4}\left[(2.684)^{1.4}-1\right]\right]}{(1.4-1)(15-1)}$$

p$_m$ ***= 1066.33kN/m²*** **Ans.**

9.In an engine working on diesel cycle, inlet pressure and temperature are l bar and 17^0 C respectively. Pressure at the end of adiabatic compression is 35 bar. The ratio of expansion i.e., after constant pressure heat addition is 5. Calculate the hear addition, heat rejection the efficiency of the cycle, and mean effective pressure.

Assume =1.4, C_p = 1.004kJ/kgK and C_v = 0.717kJ/kgK.

Given data

Diesel cycle

p_1 = 1 bar = 100 kN/m^2

T_1 = 17^0C = 290K

p_2 =35bar = 3500 kN/m^2

$$\frac{V_4}{V_3} = \frac{V_1}{V_3} = 5$$

$\gamma = 1.4; C_P = 1.004$ kJ/kgK;

C_v=0.717 kJ/kgK

Solution

Consider process 1-2 ;

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = \left(\frac{3500}{100}\right)^{\frac{1.4-1}{1.4}} \times 290 = 800.87K$$

$$p_1V_1^{\gamma} = p_2V_2^{\gamma} \Rightarrow r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

Compression ratio, $r = \left(\frac{3500}{100}\right)^{\frac{1}{1.4}} = 12.67$

$$\frac{V_1}{V3} = \frac{V_1}{V_2} \times \frac{V_2}{V_3}$$

$$5 = 12.67 \times \frac{V_2}{V_3}$$

Cut – off ratio, $\rho = \frac{V_3}{V_2} = \frac{12.67}{5} = 2.534$

Consider process: Constant pressure process;

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \Rightarrow T_3 = \frac{V_3}{V_2} \times T_2 = 2.534 \times 800.87$$

T_3 = 2090.4K

Heat supplied during 2 – 3

$Q_s = C_p (T_3 - T_2)$

= 1.004(2029.4 – 800.87)

Q_s = 1233.45kJ/kg **Ans.**

Consider process 3-4: Adiabatic process;

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \times T_3 = \left(\frac{1}{5}\right)^{1.4-1} \times 2029.4$$

T_4 = 1066K

Heat rejection during constant volume process 4-1

$Q_R = C_V (T_4 - T_1)$ = 0.717(1066-290)

Q_R = 556.43kJ/kg **Ans.**

Efficiency, $\eta = 1 - \frac{Q_R}{Q_S} = 1 - \frac{556.43}{1233.45}$

= 54.88% **Ans.**

Mean effective pressure, p_m

$$P_m = \frac{p_1 r^{\gamma}\left[\gamma(\rho-1) - r^{1-\gamma}\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)}$$

$$= \frac{1 \times (12.67)^{1.4}\left[1.4(2.534-1) - (12.67)^{1-1.4}\left((2.534)^{1.4}-1\right)\right]}{(1.4-1)(12.67-1)}$$

P_m = 8.83bar **Ans.**

10. **In an air standard diesel cycle, the pressure and temperatures of air at the beginning of Cycle are 1bar and 40°C. The temperatures before and after the heat supplied are 400^0C and 1500^0C. Find the air standard efficiency and mean effective pressure of the cycle. What is the Power output if it makes 100cycles / min?**

Given data:

$p_1 = 1bar = 100kN/m^2$

$T_1 = 40^0C = 40 + 273 = 313K$

$T_2 = 400^0C = 673K$

$T_3 = 1500^0C = 1773K$

Solution:

Consider the process 1-2 (Isentropic compression)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

Compression ratio, $r = \frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} = \left(\frac{673}{313}\right)^{\frac{1}{1.4-1}}$

$= 6.779$

Consider the process 2-3 (Constant pressure heating)

$$\frac{v_2}{T_2} = \frac{v_3}{T_3}$$

Cut off ratio, $\rho = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{1773}{673} = 2.634$

Efficiency, $\eta = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left(\frac{\rho^\gamma - 1}{\rho - 1}\right)$

$$= 1 - \frac{1}{1.4(6.779)^{1.4-1}}\left(\frac{2.634^{1.4} - 1}{2.634 - 1}\right)$$

= 0.4142 = 41.42 % **Ans.**

Mean effective pressure,

$$\rho_m = \frac{p_1 r^\gamma\left(\gamma(\rho-1) - r^{1-\gamma}\left(\rho^\gamma - 1\right)\right)}{(\gamma-1)(r-1)}$$

$$\frac{100\times(6.779)^{1.4}\left(1.4(2.634-1)-(6.779)^{1-1.4}\left((2.634)^{1.4}-1\right)\right)}{(1.4-1)(6.779-1)}$$

$\rho_m = 597.77 kN/m^2$ **Ans.**

Heat supplied = m x C_p (T_3-T_2)

= 1 x 1.005 (1773 – 673)

Q_S = 1105.5*kJ/kg*

Work done $= \eta \times Q_S = 0.4142 \times 1105.5$

=457.89*kJ/kg* $\left[\because \eta = 1 - \frac{Q_R}{Q_S} = \frac{W}{Q_S}\right]$

Power = *W* x cycles/min = 457.89 x 100

= 45 x 10^{-3}*kJ/kg-min* = 763.16*kJ/kg-sec*

= *763.16kW/kg* **Ans.**

11) Find the air standard efficiency of a diesel cycle when the compression ratio andcut–off ratio are 15 & 1.84 respectively. Assume γ = 1.4.

Given:

Compression ratio (r) =

15 Cut – off ratio(ρ) = 1.84

γ = 1. 4

Required: η

***Solution*:**

$$\eta = 1 - \frac{1}{\gamma r^{\gamma-1}}\left[\frac{\rho^{\gamma-1}-1}{\rho-1}\right]$$

$$= 1 - \frac{1}{1.4(15)^{1.4-1}}\left[\frac{1.84^{1.4-1}-1}{1.84-1}\right]$$

η= 0.612---- Ans

12)In a diesel engine the pressure at the beginning of compression is 1 bar. Compression ratio is 14 : 1 and cut–off takes place at 10% of the stroke. Calculatethe air standard efficiency and ideal mep of the cycle (γ = 1. 4 for air).

Given:

Initial pressure (p_1) = 1 bar.Compression ratio (r) = 14.

Cut – off takes place at 10 % of stroke,i.e., $V_3 – V_2 = 0.1 (V_1 – V_2)$

$\gamma = 1.4$

Required **:** η & p_m

Solution:

$$\eta = 1 - \frac{1}{\gamma r^{\gamma-1}} \left[\frac{\rho^{\gamma-1} - 1}{\rho - 1}\right]$$

ρ = cut – off ratio = V_3 / V_2

r = V_1 / V_2 = 14

$\therefore V_1 = 14 V_2$

$\therefore V_3 - V_2 = 0.1 (14 V_2 - V_2)$

$V_3 = 2.3 V_2$.

$V_3 / V_2 = \rho = 2.3$

$$\therefore \quad \eta = 1 - \frac{1}{1.4 (14)^{1.4-1}} \left[\frac{2.3^{1.4-1} - 1}{2.3 - 1}\right]$$

= 0.577 ------ Ans

Mean effective pressure (p_m)

$$p_m = p_1 r^{\gamma} \left[\frac{\gamma (\rho - 1) - r^{1-\gamma} (\rho^{\gamma} - 1)}{(\gamma - 1) (r - 1)}\right]$$ **→ for Diesel cycle**

$$= 1 \times 14^{1.4} \left[\frac{1.4 (2.3 - 1) - 14^{1-1.4} (2.3^{1.4} - 1)}{(1.4 - 1) (14 - 1)}\right] = 8.133 \text{ bar}$$ **--------- Ans**

1.4 RANKINE CYCLE

The Rankine cycle is a model used to predict the performance of steam turbine systems. It was also used to study the performance of reciprocating steam engines. The Rankine cycle is an idealized thermodynamic cycle of a heat engine that converts heat into mechanical work while undergoing phase change. The heat is supplied externally to a closed loop, which usually uses water as the working fluid. It is named after William John Macquorn Rankine, a Scottish polymath and Glasgow University professor.

There are four processes in the Rankine cycle. These states are identified by numbers (in brown) in the above T–s diagram.

Process 1–2: The working fluid is pumped from low to high pressure. As the fluid is a liquid at this stage, the pump requires little input energy.

Process 2–3: The high-pressure liquid enters a boiler, where it is heated at constant pressure by an external heat source to become a dry saturated vapour. The input energy required can be easily calculated graphically, using an enthalpy–entropy chart (h–s chart, or Mollier diagram), or numerically, using steam tables.

Process 3–4: The dry saturated vapour expands through a turbine, generating power. This decreases the temperature and pressure of the vapour, and some condensation may occur. The output in this process can be easily calculated using the chart or tables noted above.

Process 4–1: The wet vapour then enters a condenser, where it is condensed at a constant pressure to become a saturated liquid.

In an ideal Rankine cycle the pump and turbine would be isentropic, i.e., the pump and turbine would generate no entropy and hence maximize the net work output. Processes 1–2 and 3–4 would be represented by vertical lines on the T–s diagram and more closely resemble that of the Carnot cycle. The Rankine cycle shown here prevents the vapor ending up in the superheat region after the expansion in the turbine, [1] which reduces the energy removed by the condensers.

The actual vapor power cycle differs from the ideal Rankine cycle because of

irreversibilities in the inherent components caused by fluid friction and heat loss to the surroundings; fluid friction causes pressure drops in the boiler, the condenser, and the piping between the components, and as a result the steam leaves the boiler at a lower pressure; heat loss reduces the net work output, thus heat addition to the steam in the boiler is required to maintain the same level of net work output.

$$\eta_{th} = \frac{W_T - W_P}{Q_{in}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} = 1 - \frac{(h_2 - h_3)}{(h_1 - h_4)}$$

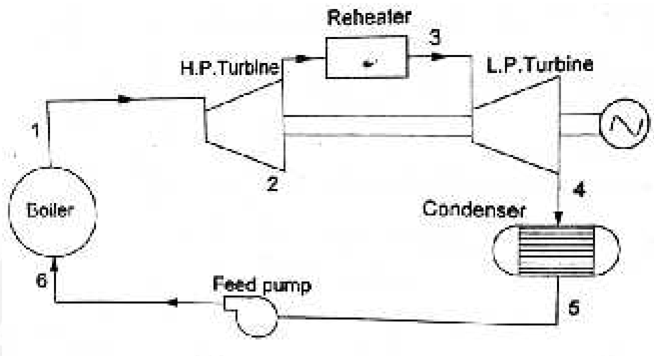
IMPROVISATIONS OF RANKINE CYCLE

Rankine cycle efficiency can be improved by using the following three methods.

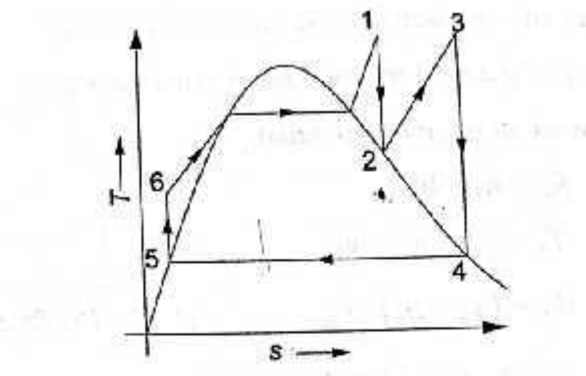
1. Reheating
2. Regeneration
3. Combined reheating and regeneration

Reheat Rankine Cycle

In the reheat cycle, the steam is extracted from a suitable point in the turbine and it is reheated with the help of flue gases in the boiler furnace.



Reheat Rankine cycle



T-s diagram for reheat Rankine cycle

Figure 1.1.1 Rankine cycle

[Source: "power plant Engineering" by Anup Goel ,Laxmikant D.jathar,Siddu :3]

The purpose of a reheating cycle is to remove the moisture carried by the steam at the final stages of the expansion process. In this variation, two turbines work in series. The first accepts vapor from the boiler at high pressure

After the vapor has passed through the first turbine, it re-enters the boiler and is reheated before passing through a second, lower- pressure, turbine. The reheat temperatures are very close or equal to the inlet temperatures, whereas the optimal reheat pressure needed is only one fourth of the original boiler pressure.

Among other advantages, this prevents the vapor from condensing during its expansion and thereby reducing the damage in the turbine blades, and improves the efficiency of the cycle, because more of the heat flow into the cycle occurs at higher temperature.

The reheat cycle was first introduced in the 1920s, but was not operational for long due to technical difficulties. In the 1940s, it was reintroduced with the increasing manufacture of high-pressure boilers, and eventually double reheating was introduced in the 1950s. The idea behind double reheating is to increase the average temperature.

It was observed that more than two stages of reheating are unnecessary, since the next stage increases the cycle efficiency only half as much as the preceding stage. Today, double reheating is commonly used in power plants that operate under supercritical pressure.

REGENERATIVE CYCLE

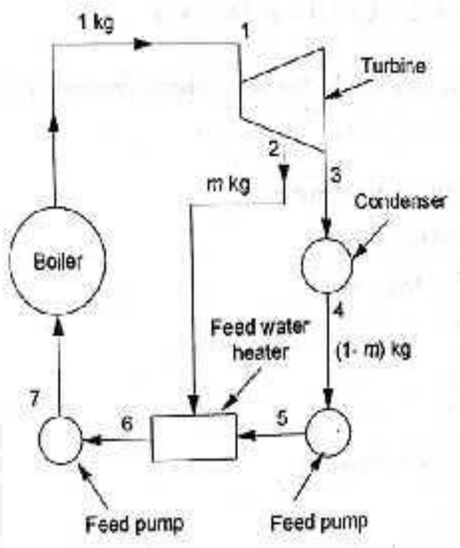


Figure 1.1.2 Regenerative cycle

[Source: "power plant Engineering" by Anup Goel ,Laxmikant D.jathar,Siddu :8]

SINGLE STAGE REGENERATIVE RANKINE CYCLE

The regenerative Rankine cycle is so named because after emerging from the condenser (possibly as a sub cooled liquid) the working fluid is heated by steam tapped from the hot portion of the cycle. On the diagram shown, the fluid at 2 is mixed with the fluid at 4 (both at the same pressure) to end up with the saturated liquid at 7. This is called "direct-contact heating". The Regenerative Rankine cycle (with minor variants) is commonly used in real power stations.

Another variation sends *bleed steam* from between turbine stages to feedwater heaters to preheat the water on its way from the condenser to the boiler. These heaters do not mix the input steam and condensate, function as an ordinary tubular heat exchanger, and are named "closed feedwater heaters".

Regeneration increases the cycle heat input temperature by eliminating the addition of heat from the boiler/fuel source at the relatively low feed water temperatures that would exist without regenerative feed water heating. This improves the efficiency of the cycle, as more of the heat flow into the cycle occurs at higher temperature.

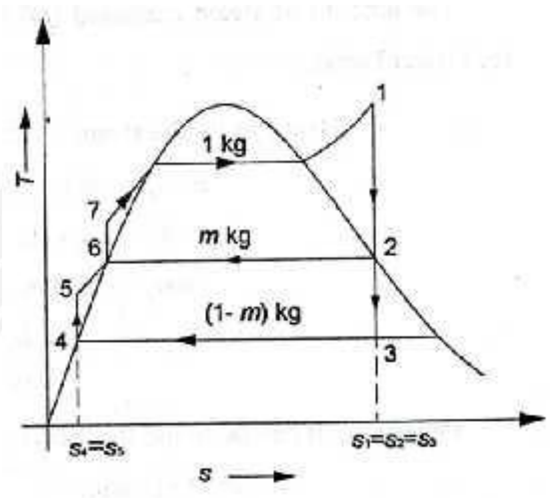


Figure 1.1.3 T-S Diagram of Regenerative cycle

[Source: "power plant Engineering" by Anup Goel ,Laxmikant D.jathar,Siddu :8]

Brayton cycle

Brayton cycle is a constant pressure cycle for a perfect gas. It is also called Joule cycle. The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas turbine plant would perform the processes that make up a Brayton cycle. The cycle is shown in the Fig. 1.8 (a) and it is represented on p-v and T-s diagrams as shown in Figs. 1.8 (b) and (c).

The various operations are as follows:

Operation 1-2. The air is compressed isentropic ally from the lower pressure p1 to the upper pressure p2, the temperature rising from T1 to T2. No heat flow occurs.

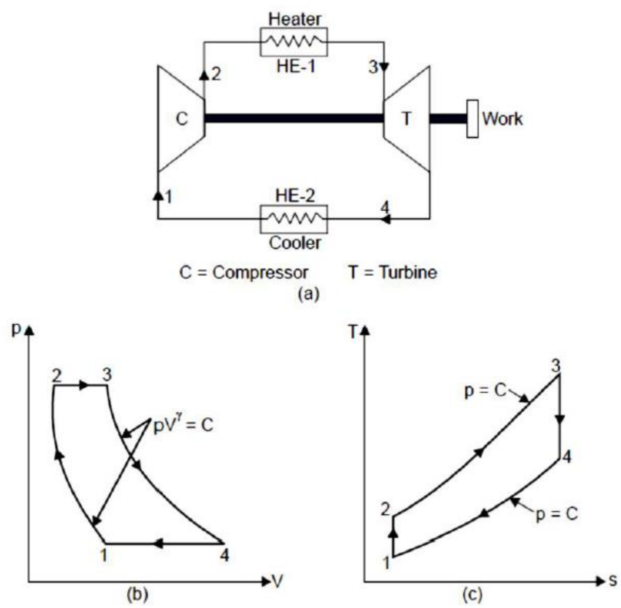
Operation 2-3. Heat flows into the system increasing the volume from V2 to V3 and temperature from T2 to T3 whilst the pressure remains constant at p2.

Heat received = mcp (T3 – T2).

Operation 3-4. The air is expanded isentropic ally from p2 to p1, the temperature falling from T3 to T4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V4 to V1 and the temperature from T4 to T1 whilst the pressure remains constant at p1.

Heat rejected = mcp (T4 – T1).



$$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat received}}$$
$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$
$$= \frac{mc_p (T_3 - T_2) - mc_p (T_4 - T_1)}{mc_p (T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

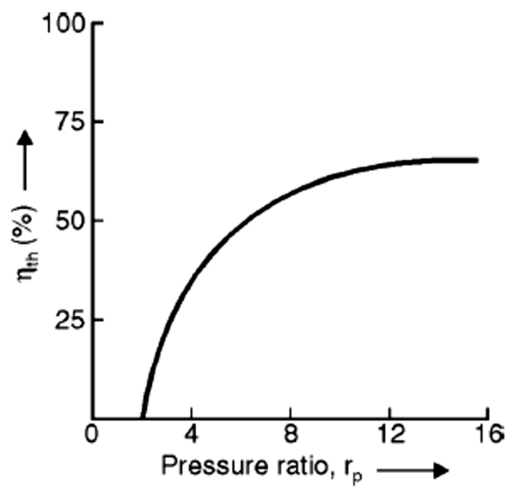
Now, from isentropic expansion,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 (r_p)^{\frac{\gamma-1}{\gamma}}, \text{ where } r_p = \text{pressure ratio.}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{or} \quad T_3 = T_4 (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \quad \eta_{\text{air-standard}} = 1 - \frac{T_4 - T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}} - T_1(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \quad ...(13.16)$$



The eqn. (13.16) shows that the efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.

Work output during the cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle}$$
$$= mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)$$
$$= mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)$$
$$= mc_p\ T_3\ \left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

Work output during the cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle}$$
$$= mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)$$
$$= mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)$$
$$= mc_p\ T_3\ \left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

Since, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma - 1}{\gamma}} = \frac{T_2}{T_1}$$

Using the constant ‘z’ = $\frac{\gamma - 1}{\gamma}$,

we have, work output/cycle

$$W = K\left[T_3\left(1 - \frac{1}{r_p^{\,z}}\right) - T_1\,(r_p^{\,z} - 1)\right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3 \times \frac{z}{r_p(z+1)} - T_1\,z r_p^{\,(z-1)}\right] = 0 \text{ for a maximum}$$

$\therefore$ $$\frac{zT_3}{r_p^{\,(z+1)}} = T_1 z\,(r_p)^{(z-1)}$$

$\therefore$ $$r_p^{\,2z} = \frac{T_3}{T_1}$$

$\therefore$ $$r_p = (T_3/T_1)^{1/2z} \quad i.e., \quad r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma - 1)}} \qquad ...(13.17)$$

Thus, the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

13.10.3. Work Ratio

Work ratio is defined as the *ratio of net work output to the work done by the turbine.*

$$\therefore \quad \text{Work ratio} = \frac{W_T - W_C}{W_T}$$

$$\begin{bmatrix} \text{where,} & W_T = \text{Work obtained from this turbine,} \\ \text{and} & W_C = \text{Work supplied to the compressor.} \end{bmatrix}$$

$$= \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$= 1 - \frac{T_1}{T_3}\left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}\right] = 1 - \frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}} \qquad \text{...(13.18)}$$

1.In a Brayton cycle, the air enters the compressor at 1 bar and 25ºC. the pressure of air leaving the compressor is 3 bar and temperature at turbine inlet is 650ºC. determine per kgof air, i) cycle efficiency ii) heat supplied to air iii) work input iv) heat rejected in the cooler and v) temperature of air leaving the turbine.

Given data:

$P_1 = 1$ bar

$T_1 = 25^oC$

$T_3 = 650^oC$

$P_2 = 3$ bar

Solution:

Consider the process 1-2 adiabatic compression:

$T_2/T_1 = (P_2/P_1)^{y-1/y}$

$T_2 = (P_2/P_1)^{y-1/y}$ x T1

$T_2 = (3/1)^{1.4-1/1.4}$ x 298

3-4 adiabatic expansion:

$T_4/T_3 = (P_4/P_3)^{y-1/y}$

$T_4 = (P_4/P_3)^{y-1/y}$ x 923 = 674.3k

Air standard efficiency:

$\eta = 1- 1/(R_p)^{y-1/y} = 1- 1/(3)^{1.4-1/1.4} = 0.2694$

$= 26.94\%$

Heat supplied $Q_s = C_p (T_3 - T_2) = 1.005 (923 - 408) = 517.575$ KJ/kg

Heat rejected $Q_R = C_p (T_4 - T_1) = 1.008 (673.4 - 298) = 377.277$KJ/kg

Compressor work $W_C = C_p (T_2 - T_1) = 1.005 \times (408 - 298) = 110.55$Kj/kg

Similarly, for expander,

$W_e = C_p \times (T_3 - T_4) = 1.005 (923 - 6734)$

$W_e = 250.848 - 110.55 = 140.288$KJ/kg

Temperature of air leaving the turbine = 673.4K

2.In an air standard Brayton cycle, the air enters the compressor at 1 bar and 15ºC. The pressure leaving the compressor is 5 bar the maximum temperature in the cycle 900ºC. Find the following.

a) Compressor and expander work per kg of air. b) the cycle efficiency.

If an ideal regenerator is incorporated into the cycle, determine the percentage change in efficiency.

Given data:

$P_1 = P4 = 1 \text{ bar} = 100KN/m^2$

$T_1 = 15^oC = 288k$

$P_2 = P3 = 5 \text{ bar} = 500Kn/m^2$

$T_3 = 900^oC = 1173k$

Solution:

1-2 isentropic compression:

$T_2/T_1 = (p_2/P_1)^{y-1/y}$; $T_2 = (P_2/P_1)^{y-1/y} \times T_1 = 456k$Consider the process 3-4 isentropic expansion:

$T_4/T_3 = (P_4/P_3)^{y-1/y}$: $T_4 = (P_4/P_3)^{y-1/y} \times T_3 = 740.6k$

Work done by the compressor when it operates isentropic ally is given by

Compressor work $W_c = C_p (T_2 - T_1) = 1.005(456 - 288) = 168.756KJ$ for

expander $W_e = C_p (T_3 - T_4) = 1.005(1173 - 740.6) = 434.34KJ$

Air standard efficiency:

$\eta = 1- 1/(R_p)^{y-1/y} = 1- 1/(5)^{1.4-1/1.4} = 36.86\%$

When ideal regenerator is incorporated:

$T_3 = T_5 \times T_2 = T_6$

Heat supplied $Q_s = C_p (T_4 - T_3)$

Heat rejected $Q_R = C_p (T_6 - T_1)$

$T_1 = 288k$

$T_2 = T_6 = 456k$

$T_3 = T_5 = 740.6K$

$T_4 = 1173k$

$Q_s = 1.005 (1173 - 790.6)$
$= 434.56$ KJ/kg

$Q_R = 1.005 (456 - 288)$

$= 186.84$ KJ/kg

Efficiency:

$\eta = 1- Q_R/Q_s = 168.84/434.56 = 0.6114 = 61.14\%$

% change in efficiency: = 61.14 – 36.86/61.14 = 39.71%

3. A closed cycle ideal gas plant operates temperature limited of 800°C and 30°C and produces a power of 100Kw.The plant is designed such that there is no need for a regenerator. A fuel of calorific value 45000KJ/kg is used. Calculate the mass flow rate of air through the plant and the

rate of fuel combustion take C_p = 1 KJ/kgk and = 1.4

Given data:

T1 = 30ºC = 303k

T3 = 800ºC

P = 100KW,

Cp = 1 KJ /kgK KJ/kg,

y = 1.4

Solution:

For maximum net work done:

$T_4 = T_2 = \sqrt{T1 \, x T3} = \sqrt{1073 \text{ x } 303}$

= 570.2k

Net work done

$W_{net} = C_p [(T_3 - T_4) - (T_2 - T_1)$

= 235.6 KJ/kg

Total power development

$P = m_a \text{ x } W_{net}$ = 100/ 235.6

= 0.4244kg/sec

Heat supply to the system:

$m_f \text{ x } C_v = m_a \text{ x } C_p \text{ x } (T_3 - T_2)$

$m_f = m_a \text{ x } C_p \text{ x } (T_3 - T_2) / C_v$ 0.4244 x 1 (1073 – 570.2)/45.000

$= 4.742 \text{ x } 10^{-3}$ kg/s