INTEGRATOR

In an integrator circuit, the output voltage is the integration of the input voltage. The integrator circuit can be obtained without using active devices like op-amp, transistors etc. In such a case an integrator is called passive integrator. While an integrator using an active devices like op-amp is called active integrator. In this section, we will discuss the operation of active op-amp integrator circuit.

1. Ideal Active Op-amp Integrator

Consider the op-amp integrator circuit as shown in the Fig. 2.29.1.

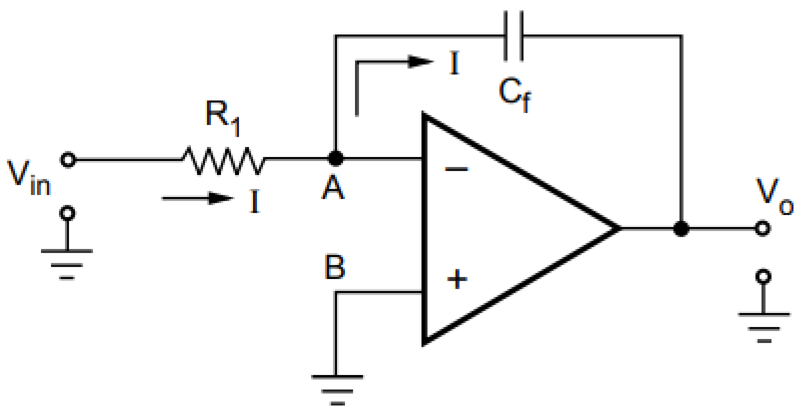

Fig. 2.29.1 Op-amp integrator

The node B is grounded. The node A is also at the ground potential from the concept of virtual ground.

$$V_A = 0 = V_B$$

As input current of op-amp is zero, the entire current I flowing through R_1, also flows through C_f, as shown in the Fig. 2.29.1.

From input side we can write, $I = \frac{V_{in} - V_A}{R_1} = \frac{V_{in}}{R_1}$ ... (2.29.1)

From output side we can write,

$$I = C_f \frac{d(V_A - V_o)}{dt} \quad \text{i.e.} \quad I = -C_f \frac{dV_o}{dt} \qquad \text{... (2.29.2)}$$

Equating the two equations (2.29.1) and (2.29.2),

$$\frac{V_{in}}{R_1} = -C_f \frac{dV_o}{dt} \qquad \text{... (2.29.3)}$$

Integrating both sides,

$$\int_0^t \frac{V_{in}}{R_1} dt = -C_f \int \frac{dV_o}{dt} dt \quad \text{i.e.} \quad \int_0^t \frac{V_{in}}{R_1} dt = -C_f V_o \qquad \text{... (2.29.4)}$$

$$\therefore \quad V_o = -\frac{1}{R_1 C_f} \int_0^t V_{in}\, dt + V_o(0) \qquad \text{... (2.29.5)}$$

where V_o (0) is the constant of integration, indicating the initial output Voltage.

The equation (2.29.5) shows that the output is - 1/ R_1 C_f times the integral of input and R_1 C_f is called time constant of the integrator.

The negative sign indicates that there is a phase shift of 180° between input and output. The main advantage of such an active integrator is the large time constant. By Miller's theorem the effective capacitance between input terminal A and the ground becomes C_f (1-A_v) where A_v is the gain of the op-amp which is very large. Due to such large effective capacitance, time constant is very large and thus a perfect integration results due to such circuit.

Sometimes a resistance R_{comp} = R_1 is connected to the non-inverting terminal to provide the bias compensation. This is shown in the Fig. 2.29.2.

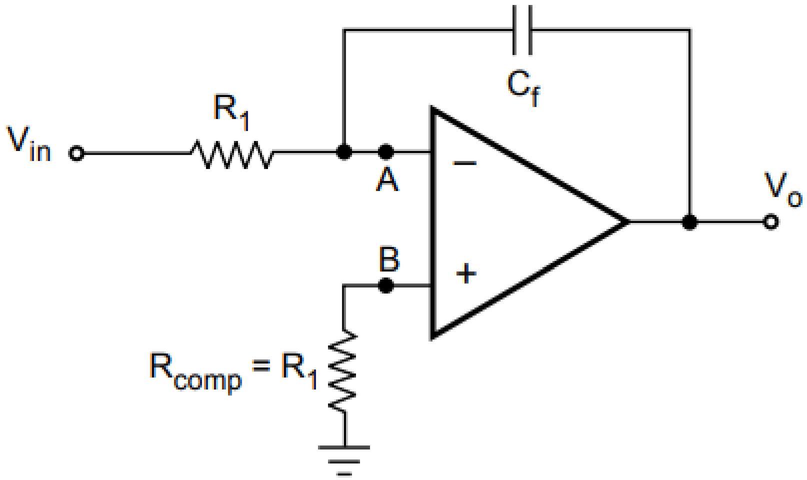

Fig. 2.29.2 Integrator with bias compensation

As the input current of op-amp is zero, the node B is still can be treated at ground potential in this circuit.

2. Input and Output Waveforms

Let us see the output waveforms, for various input signals. For simplicity of understanding, assume that the time constant $R_1C_f = 1$ and the initial voltage $V_o(0) = 0V$

i) Step input signal

Let the input waveform is of step type, with a magnitude of A units as shown in the Fig. 2.29.3

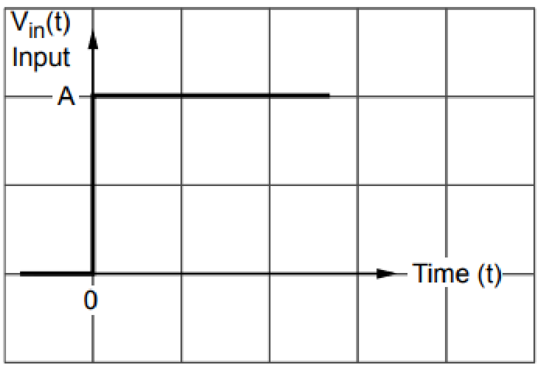

Fig. 2.29.3 Step input signal

Mathematically the step input can be expressed as,

$V_{in}(t) = A$ for $t \geq 0$

And $= 0$ for $t \leq 0$

From equation (2.29.5), with $R_1C_f = 1$ and $V_o(0) = 0$,

We can write,

$$\therefore \quad V_o(t) = -\int_0^t V_{in}(t)\,dt = -\int_0^t A\,dt$$

$$= -A\int_0^t dt = -A\,[t]_0^t$$

$$\therefore \quad \boxed{V_o(t) = -At} \qquad \ldots(2.29.7)$$

Thus output waveform is a straight line with a slope of -A where A is magnitude of the step input. The output waveform is shown in the Fig. 2.29.4.

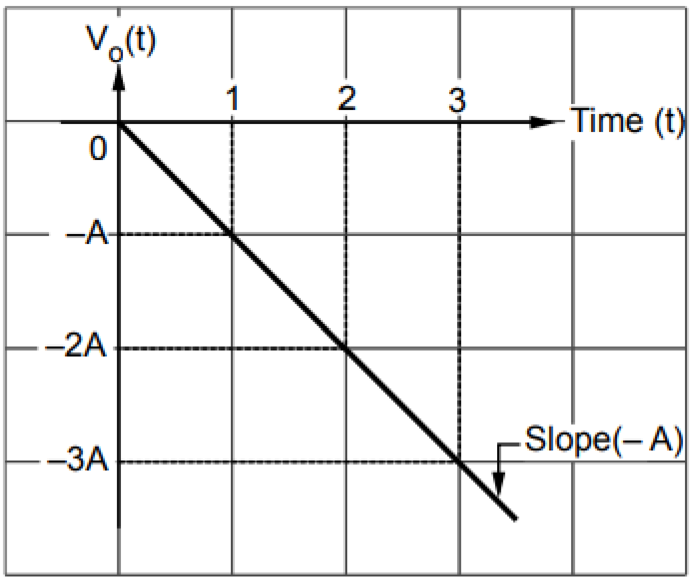

Fig. 2.29.4 Output waveform for step input

ii) Square wave input signal

Let the input waveform is a square wave as shown in the Fig. 2.29.5

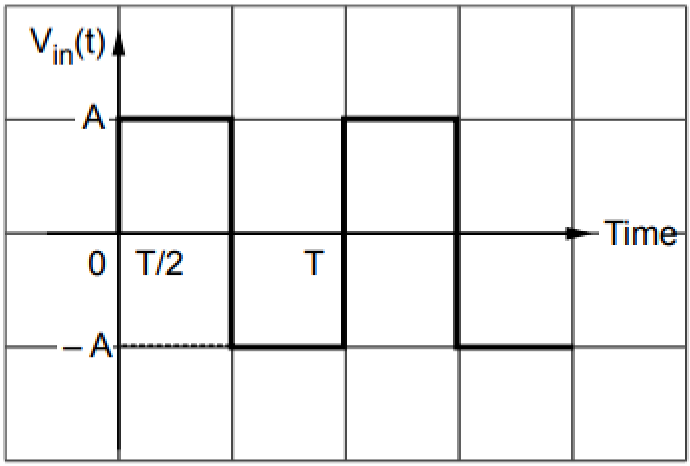

Fig. 2.29.5 Square wave input signal

It can be observed that the square wave is made up of steps i.e. a step of A between time period of 0 to T/2 while a step of - A units between a time period of T/2 to T and so on.

Mathematically it can be expressed as,

$V_{in}(t)= A, 0 < t < T/2$ ….. (2.29.8)

$= - A, T/2 < t < T$

This is the expression for the input signal for one period.

As discussed earlier, the output for step input is a straight line with a slope of -A. So for the period 0 to T/2 output will be straight line with slope - A. From t = T/2 till t = T, the slope of the straight line will become - (-A) i.e. + A.

So the output can be expressed Fig. 2.29.6 Output waveform for square wave input mathematically for one period as,

$V_o(t)= - A t \; 0 < t < T/2$

$= + A t \; T/2 < t < T$ ….. (2.29.9)

The output waveform is shown in the Fig. 2.29.6.

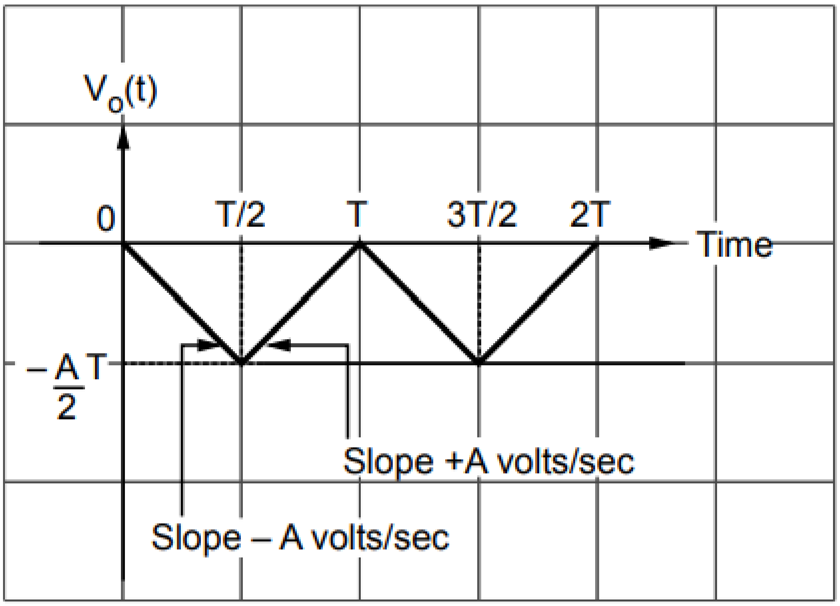

Fig. 2.29.6 Output waveform for square wave input

iii) Sine wave input signal

Let the input waveform is purely sinusoidal with a frequency of co rad/sec. Mathematically it can be expressed as,

$V_{in}(t) = V_m \text{ Sin } \omega t$ ... (2.29.10)

where V_m is the amplitude of the sine wave and T be the period of the waveform.

To find the output waveform, use the equation (2.29.5) with $R_1C_f = 1$ and $V_o(0) =$ 0 V.

$$\therefore \quad V_o(t) = -\int V_{in}\, dt = -\int V_m \sin \omega t\, dt = -V_m \left[\frac{1}{\omega}(-\cos \omega t)\right]$$

$$\therefore \quad \boxed{V_o(t) = -\frac{V_m}{\omega}(-\cos \omega t)} \qquad (2.29.11)$$

Thus it can be seen that the output of an integrator is a cosine waveform for a input. Due to inverting integrator, the output waveform is as shown in the Fig. 2.29.7.

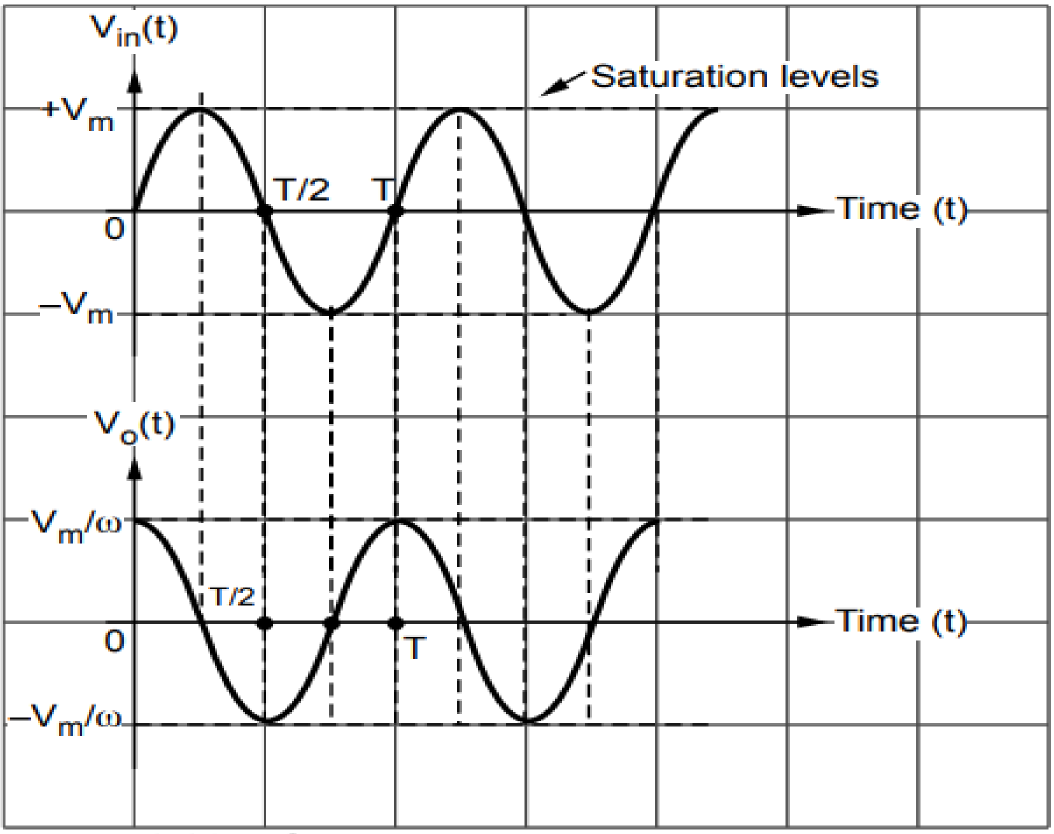

Fig. 2.29.7 Sine wave input and cosine output