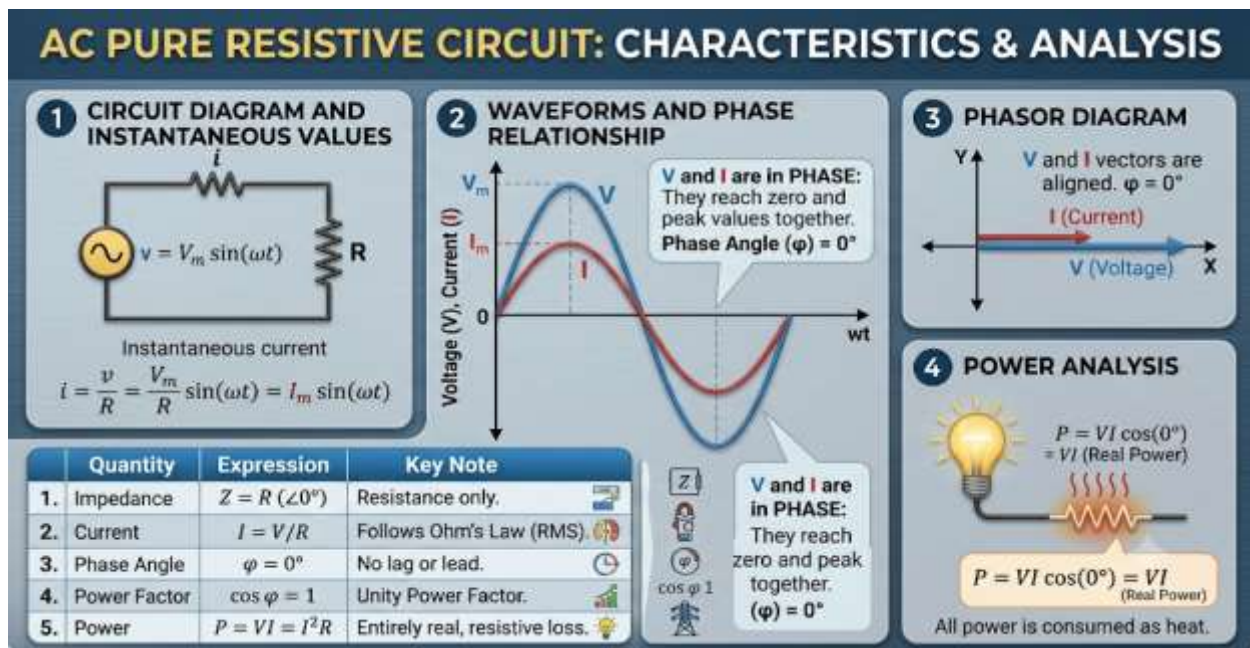




ANALYSIS OF SINGLE-PHASE AC CIRCUITS

When a sinusoidal AC source $v = V_m \sin(\omega t)$ is applied to a circuit containing resistance (R), inductance (L), and/or capacitance (C), the resulting current also varies sinusoidally, but its magnitude and phase relationship

with the voltage depend on which elements are present and how they are combined. Analysing an AC circuit means finding the impedance, current, phase angle, and power for a given combination of R, L, and C.

General approach for AC circuit analysis

- Identify the elements present (R, L, C) and whether they are in series.
- Calculate the reactance of each reactive element: $X_L = \omega L$, $X_C = 1/(\omega C)$.
- Combine resistance and net reactance to obtain the impedance Z (a complex quantity).
- Apply the AC form of Ohm's Law: $I = V/Z$ to find current magnitude and phase.
- Draw the phasor diagram and impedance triangle to visualise the phase relationship.
- Calculate active, reactive and apparent power, and power factor.

AC Circuit with Pure Resistance (R)

Consider a pure resistor R connected across a sinusoidal supply $v = V_m \sin(\omega t)$. By Ohm's Law, the instantaneous current is:

$$i = v/R = (V_m/R) \sin(\omega t) = I_m \sin(\omega t)$$

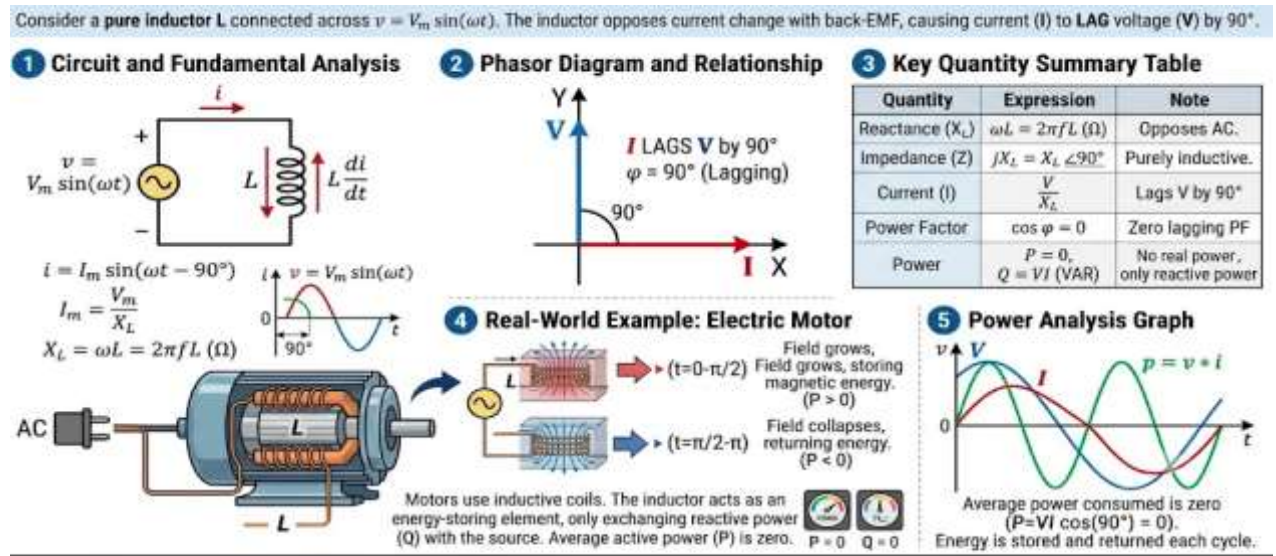
Since there is no energy-storing element, the current is in phase with the voltage — both reach their zero, positive peak, and negative peak at the same instant. The phase angle $\phi = 0^\circ$.

AC Circuit with Pure Inductance (L)

Consider a pure inductor L connected across $v = V_m \sin(\omega t)$. The inductor opposes any change in current by inducing a back-EMF equal to $L(di/dt)$, which must balance the applied voltage at every instant. Solving this relationship shows that the current lags the voltage by exactly 90° :

$$i = I_m \sin(\omega t - 90^\circ), \text{ where } I_m = V_m / X_L, \quad X_L = \omega L = 2\pi fL \text{ } (\Omega)$$

X_L , the inductive reactance, represents the opposition offered by the inductor to AC current; it increases with frequency and inductance, and is zero for DC ($f = 0$).



AC Circuit with Pure Capacitance (C)

Consider a pure capacitor C connected across $v = V_m \sin(\omega t)$. The current that flows is the charging current, $i = C(dv/dt)$, which is maximum when the voltage is changing fastest (near zero crossing) and zero when voltage is at its peak. This results in the current leading the voltage by 90° :

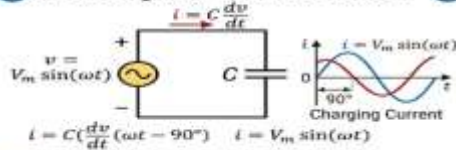
$$i = I_m \sin(\omega t + 90^\circ), \text{ where } I_m = V_m / X_C, \quad X_C = 1/(\omega C) = 1/(2\pi fC) \text{ } (\Omega)$$

X_C , the capacitive reactance, decreases with increasing frequency and capacitance, and becomes infinite for DC — confirming that a capacitor blocks DC in steady state.

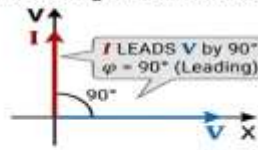
AC PURE CAPACITIVE CIRCUIT: CHARACTERISTICS & ANALYSIS

Consider a pure capacitive C connected across $v = V_m \sin(\omega t)$. The inductor opposes current change with back-EMF, causing current (i) to LAG voltage (V) by 90° .

1 Circuit Diagram & Current Generation



2 Phasor Diagram & Relationship

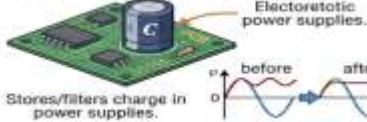


3 Key Quantity Summary Table

Quantity	Expression	Note
Reactance (X_C)	$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ (Ω)	Opposes AC. Infinite for DC (blocks DC).
Impedance (Z)	$Z = -jX_C = X_C \angle -90^\circ$	Purely capacitive opposition.
Current (i)	$i = \frac{v}{X_C}$	Leads V by 90°
Power Factor	$\cos \phi = 0$	Zero leading PF
Power	$P = 0$ $Q = VI$ (VAR)	No real power (P), only reactive power

4 Real-World Examples

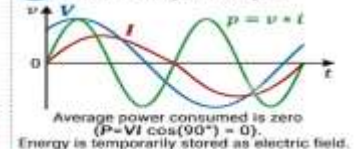
4a) Capacitive Power Supply Filter



4b) DC Blocking in Audio Signal



5 Power Analysis Graphs



AC Circuit with R and L in Series (R-L Circuit)

When a resistor and an inductor are connected in series across an AC supply, the same current flows through both, but the voltage across R ($V_R = IR$) is in phase with current, while the voltage across L ($V_L = IXL$) leads the current by 90° . The applied voltage is the phasor sum of these two.

AC R-L SERIES CIRCUIT ANALYSIS

1 CIRCUIT DIAGRAM

$v(t) = V_m \sin(\omega t)$

Winding has resistance (R) & inductance (L).

Current " i " flows through both.

Fluorescent tube choke coil

Single-phase induction motor

2 PHASOR & IMPEDANCE RELATIONSHIPS

(a) Voltage Phasor Diagram

$V_L = IX_L$

$V_R = IR$

V is the phasor sum:
 $V = \sqrt{V_R^2 + V_L^2}$. V leads I by ϕ ; current LAGS voltage.

(b) Impedance Triangle

$Z = \sqrt{R^2 + X_L^2}$

$X_L = \omega L$

$Z = R + jX_L = \sqrt{R^2 + X_L^2} \angle \phi$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

3 POWER & POWER FACTOR

$P = VI \cos \phi = I^2 R$

$Q = VI \sin \phi = I^2 X_L$

$S = VI$

Power Factor: $\cos \phi$ (Lagging)

Current LAGS voltage; lagging power factor (ϕ between 0° and 90°).

4 REAL-WORLD APPLICATION & COMPENSATION

R-L Circuit load, draws lagging current.

Single-phase Induction motor

Motor draws lagging current, reducing power factor.

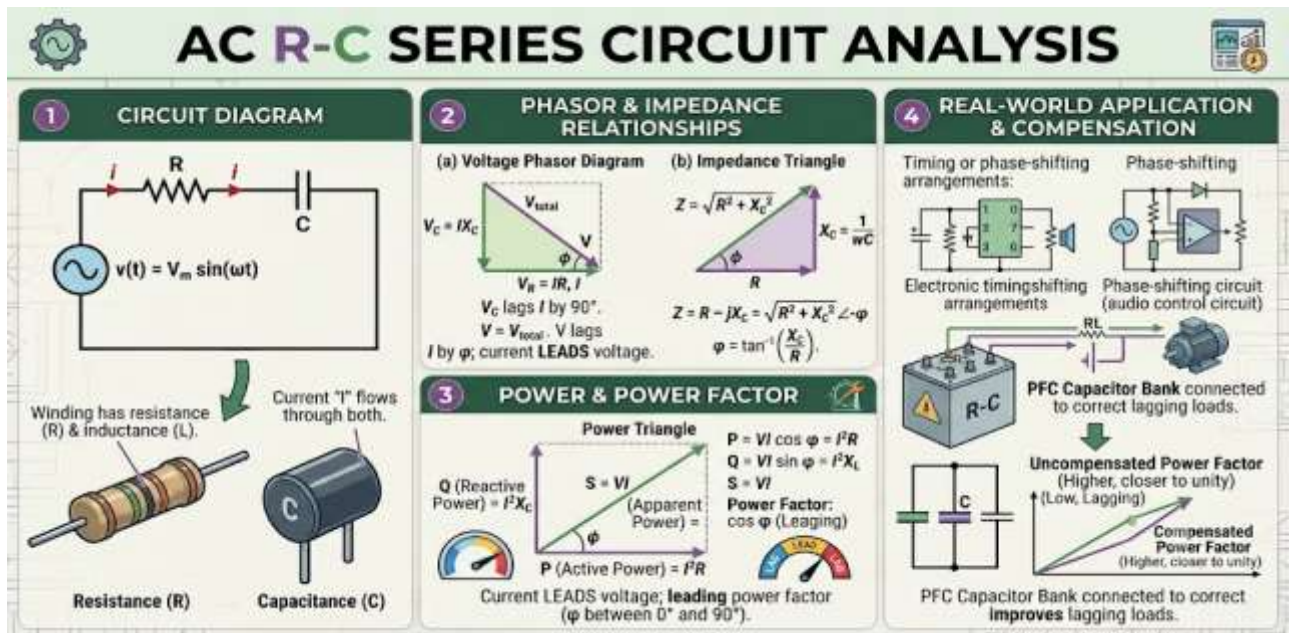
Uncompensated Power Factor (Lower, Lagging)

Compensated Power Factor (Higher, closer to unity)

Compensation with Parallel Capacitor (C) improves Power Factor.

AC Circuit with R and C in Series (R-C Circuit)

When a resistor and a capacitor are connected in series, the voltage across R ($V_R = IR$) is in phase with current, while the voltage across C ($V_C = IXC$) lags the current by 90° (equivalently, current leads V_C by 90°). The applied voltage is again the phasor sum.



AC Circuit with R, L and C in Series (R-L-C Circuit)

When R, L, and C are all connected in series, the same current I flows through all three elements. V_R is in phase with I , V_L leads I by 90° , and V_C lags I by 90° . Since V_L and V_C are exactly opposite in phase (180° apart), they partially or fully cancel, and the net reactive voltage is $(V_L - V_C)$.

