

5.3 DESIGN OF LAG, LEAD AND LAG LEAD COMPENSATOR USING BODE PLOTS

DESIGN PROCEDURE OF LAG COMPENSATOR USING BODE PLOTS

1. Determine the compensator gain K to meet the steady state error requirement.
2. Draw the Bode plots of $KG(s)$.
3. From the Bode plots, find the frequency ω_g at which the phase of $KG(s)$ is

$$\angle KG(\omega_g) = PM - 180^\circ + 5^\circ \sim 10^\circ$$

4. Calculate β to make ω_g the gain crossover frequency,

$$20 \log \beta = 20 \log K + 20 \log |G(j\omega_g)|$$

5. Choose T to be much greater than $1/\omega_g$, for example, $T=10/\omega_g$.
6. Verify the results using MATLAB.

DESIGN PROCEDURE OF LEAD COMPENSATOR USING BODE PLOTS

1. Draw the Bode plot for the uncompensated system and obtain the current phase margin available.
2. Calculate the phase margin required to meet the damping coefficient or percent overshoot requirement. Don't forget to add some extra phase margin to compensate for imperfections in the controller design (approximately 10 degrees of phase is good).

$$PM = \tan^{-1} \left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} \right)$$

$$\zeta \cong \frac{PM}{100}$$

3. Calculate the value of alpha from the following equation. Use the phase margin obtained in Step (4) as the maximum phase value:

$$\alpha = \frac{1 - \sin \phi_{max}}{1 + \sin \phi_{max}}$$

4. Calculate the gain corresponding to the maximum phase frequency using the equation below. We are going to look for the new phase margin frequency that we want to design for by looking for places where this gain is present on the Bode plot.

$$|G(j\omega_{max})| = \frac{1}{\sqrt{\beta}}$$

5. Find the new maximum phase margin frequency by looking for the point where the uncompensated system's magnitude curve is the negative value of the gain calculated in Step (4).
6. Select the break frequencies, T and βT using the maximum frequency equation given below:

$$\omega_{max} = \frac{1}{T\sqrt{\beta}}$$

7. Reset the system gain to adjust for the compensator's gain.
8. Check that the bandwidth still meets design requirements. Simulate the system and repeat the design as necessary.

DESIGN PROCEDURE OF LAG-LEAD COMPENSATOR USING BODE PLOTS

The lag-lead compensator is the analog to the PID controller. The lag-lead compensator can meet multiple design requirements: the lag component reduces high frequency gain, stabilizes the system and meets steady state requirements, while the lead component is used to meet transient response design requirements.

The general equation for this kind of compensator is given below:

$$G_{lag-lead}(s) = G_{lead}(s)G_{lag}(s) = \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\gamma T_2}} \right), \gamma > 1$$

- (1) Calculate the required bandwidth to meet the transient performance requirement (usually expressed in terms of the settling time, rise time or peak time). Use the equation provided above.
- (2) Set the DC gain of the uncompensated system to meet the steady state requirements (this requires use of the Final Value Theorem).
- (3) Draw the Bode plot for the uncompensated system and obtain the current phase margin available.
- (4) Calculate the phase margin required to meet the damping coefficient or percent overshoot requirement. Don't forget to add some extra phase margin to compensate

for imperfections in the controller design (approximately 10 degrees of phase is good).

- (5) Select a new phase margin frequency that is slightly less than the bandwidth.
- (6) At this new phase margin frequency, calculate the phase lead required to obtain the phase margin from Step (4). Add some additional phase to adjust for the lag compensator's effects, if you have not already done so in Step (4).
- (7) Design the lag compensator. Choose the higher breakpoint frequency as the phase margin frequency divided by 10. A plot of the interaction between beta and the phase margin is used to select beta, but for our purposes, I think that spacing the pole and zero of the lag compensator apart by a factor of 10 is sufficient for our design purposes.
- (8) We can calculate gamma as the inverse of beta, so now we are ready to design the lead compensator. Use the equation given below to find T.
- (9) Check that the bandwidth still meets design requirements. Simulate the system and repeat the design as necessary.

