

DIGITAL IMAGE PROCESSING

Image Representation, Pixel Resolution, Sampling & Quantization

1. Image Representation & Pixel Resolution

1.1 Image Representation

Definition:

An image is represented as a two-dimensional function $f(x, y)$, where:

- $x \rightarrow$ horizontal coordinate
- $y \rightarrow$ vertical coordinate
- $f(x, y) \rightarrow$ intensity (brightness) at point (x, y)

When x, y and intensity are finite and discrete, the image is called a Digital Image.

Digital Image Matrix (Example)

$y \setminus x$	0	1	2	3
0	120	130	140	150
1	110	125	145	160
2	100	115	135	155
3	90	105	120	140

- Each number is called a Pixel.
- Pixel = Picture Element

Pixel

- Smallest element of a digital image.
- Stores intensity or colour information.
- Every image consists of millions of pixels.

1.2 Pixel Resolution

Definition:

Pixel resolution refers to the number of pixels used to represent an image.

Higher resolution $\rightarrow$ More pixels $\rightarrow$ Better image quality
Lower resolution $\rightarrow$ Fewer pixels $\rightarrow$ Poor image quality

Example:

Low resolution images show blocky, coarse squares, while high resolution images show smooth, finely detailed grids. More pixels produce smoother edges and greater detail.

Common Image Resolutions

Resolution	Pixels
256 × 256	65,536
512 × 512	262,144
1024 × 1024	1,048,576

Resolution Formula

Total Pixels = Width × Height

Example: $640 \times 480 = 640 \times 480 = 307,200$ pixels

Advantages of High Resolution

- ✓ Better clarity
- ✓ More details
- ✓ Suitable for medical and satellite images
- ✓ Better image processing results

Disadvantages of High Resolution

- ✗ Larger storage requirement
- ✗ More processing time
- ✗ Higher computational cost

Key Points (Exam Tips)

- Image = Matrix of Pixels
- Pixel = Smallest picture element
- Resolution = Number of pixels in an image
- Higher resolution → Better quality
- Lower resolution → Less detail

Memory Trick: More Pixels → Higher Resolution → Better Quality → More Memory

2. Sampling

2.1 Definition

Sampling is the process of converting a continuous image function $f(x, y)$ into a discrete image by measuring its intensity values at discrete intervals.

It is the first step in converting an analog image into a digital image.

2.2 Why Sampling is Needed?

- Computer can work only with discrete data.
- Sampling converts a continuous image into a matrix of pixels.

2.3 Process of Sampling

Continuous image $f(x, y)$ → Sampling (at intervals $\Delta x, \Delta y$) → Discrete image $f[m, n]$

$m, n \rightarrow$ integer coordinates
 $\Delta x, \Delta y \rightarrow$ sampling intervals (spacing between samples)

2.4 Illustration

A continuous image $f(x, y)$ is a smooth curve/surface over x and y . Sampling it at a grid of points $(m\Delta x, n\Delta y)$ produces the sampled image $f[m, n]$, shown as discrete dots on the grid.

At each grid point $(m\Delta x, n\Delta y)$, the intensity is measured. These values form the pixels.

2.5 Sampling Intervals

- $\Delta x \rightarrow$ distance between two samples along the x-axis.
- $\Delta y \rightarrow$ distance between two samples along the y-axis.

*If Δx and Δy are small $\rightarrow$ More samples $\rightarrow$ Better representation
 If Δx and Δy are large $\rightarrow$ Fewer samples $\rightarrow$ Less detail (loss of information)*

2.6 Effect of Sampling Interval

- Small sampling interval $\rightarrow$ High sampling density $\rightarrow$ Good quality
- Large sampling interval $\rightarrow$ Low sampling density $\rightarrow$ Poor quality (loss of details)

2.7 Key Points

- ✓ Sampling converts continuous image $f(x, y)$ into discrete image $f[m, n]$.
- ✓ It is done by measuring intensity at regular intervals.
- ✓ Output of sampling is a matrix of pixels.
- ✓ Choice of sampling interval decides the quality and amount of information.

Remember! More samples $\rightarrow$ More detail $\rightarrow$ Better quality

3. Quantization

3.1 Definition

Quantization is the process of mapping a large (continuous) set of intensity values to a small (discrete) set of levels. It reduces the number of possible intensity values.

Sampling → in space
Quantization → in intensity

3.2 Why Quantization is Needed?

- Computer can represent only a finite number of intensity levels.
- To reduce the amount of memory required.
- To simplify processing.

3.3 Process of Quantization

Continuous intensity values (0 to L-1) → Quantization (map to discrete levels) → Quantized image [k levels]

L – total no. of gray levels in the input image.
k – no. of gray levels after quantization ($k < L$).

3.4 Uniform Quantization

- The range [0, L-1] is divided into k intervals.
- Each interval is assigned a quantization level.
- Step size (Δ) = L / k

Example: If $L = 256$ and $k = 8$, $\Delta = 256/8 = 32$
Quantization levels: 0, 32, 64, 96, 128, 160, 192, 224

3.5 Example

(a) Original image (L = 16 levels: 0 to 15)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

(b) Quantized image (k = 4 levels: 0 to 3)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	1	1	1	1	2	2	2	2	3	3	3	3

- Step size $\Delta = L/k = 16/4 = 4$
- Intervals: [0-3], [4-7], [8-11], [12-15]

- Representative levels: 0, 1, 2, 3

3.6 Effect of Quantization

- Fewer levels (small k) → More quantization error (posterization) → Poor quality
- More levels (large k) → Less quantization error → Better quality, more memory

3.7 Key Points

- ✓ Quantization reduces the number of intensity levels.
- ✓ It is a mapping from L levels to k levels ($k < L$).
- ✓ Uniform quantization uses equal interval size ($\Delta = L/k$).
- ✓ Choice of k controls quality and storage.