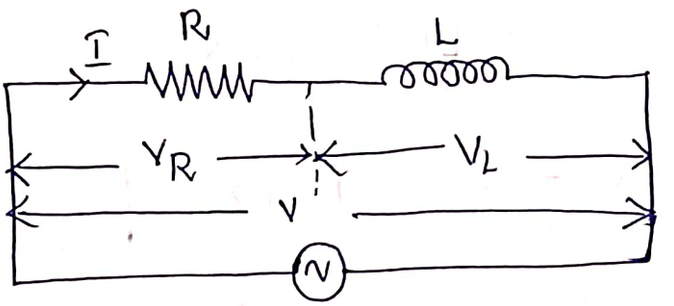
RL series circuit

A circuit that contains a pure resistance R ohm connected in series with a coil having pure inductance of L (Henry) is known as RL series circuit. Since the R and L are connected in series same amount of current flow through them.



V_R = Voltage across Resistor

V_L = Voltage across Inductor

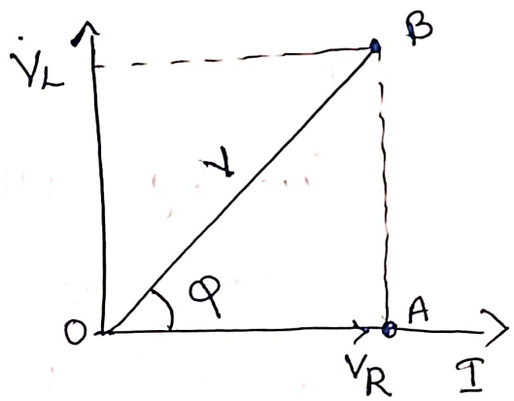
$V = V_R + V_L$.

Phasor diagram of RL series circuit

* current I is taken as reference
* The voltage drop across the resistor V_R is in-phase with the current.

The voltage drop across the inductive reactance V_L is 90^0 out-of-phase with I.

* The total voltage $V = V_R + V_L$



From triangle OAB

$$OB^2 = OA^2 + AB^2$$

$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$\boxed{V = I\sqrt{R^2 + X_L^2}}$$

$V = IZ$ where $Z = \sqrt{R^2 + X_L^2}$.

$|Z| = \frac{|V|}{|I|}$

From diagram

$$\tan\phi = \frac{AB}{OA} = \frac{V_L}{V_R} = \frac{IX_L}{IR}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

Let the instantaneous voltage $v = V_m \sin\omega t$

Let the instantaneous current $I = I_m \sin(\omega t - \phi)$

∴ Instantaneous power is.

$$P = VI$$

$$P = V_m \sin\omega t \; I_m \sin(\omega t - \varphi)$$

$$= V_m I_m \sin\omega t \sin(\omega t - \varphi)$$

$$= \frac{V_m I_m}{2} \cos(\omega t - \omega t + \varphi) - \cos(\omega t + \omega t + \varphi)$$

$$\boxed{P = \frac{V_m I_m}{2}\left[\cos\varphi - \cos(2\omega t - \varphi)\right]}$$

$C_{A+B} = C_A C_B - S_A S_B$

$C_{A-B} = C_A C_B + S_A S_B$

$C_{A+B} + C_{A-B} = 2 S_A S_B$

$S_A S_B = \frac{C_{A-B} - C_{A+B}}{2}$

Average power,

$$P = \text{Average of } \frac{V_m I_m}{2}\cos\varphi - \text{Avg}\,\frac{V_m I_m}{2}\cos(2\omega t - \varphi)$$

$$= \frac{V_m I_m}{2} \cdot \frac{1}{\pi}\int_0^{\pi} \cos\varphi - \cos(2\omega t - \varphi)\, d(\omega t)$$

$$= \frac{V_m I_m}{2\pi}\cos\varphi[\omega t]_0^{\pi} - \frac{V_m I_m}{2}\int_0^{\pi}\cos(2\omega t - \varphi)\, d\omega t$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi$$

Average power $= V_{rms} I_{rms} \cos\varphi$

active power $= VI\cos\varphi$

$$= V I \times \frac{R}{Z} = (IZ)\frac{IR}{Z}$$

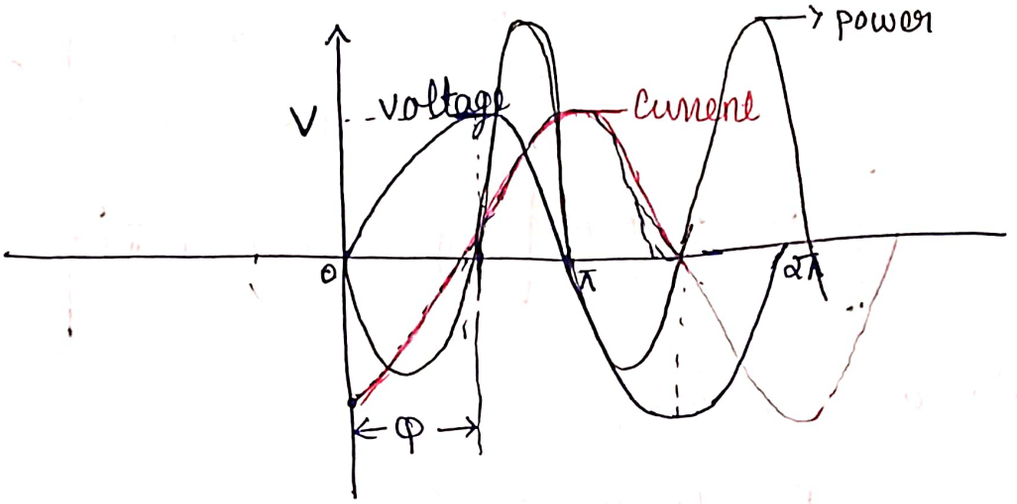
From dgm

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = R/Z.$$

$$P = I^2R$$

ie) the inductor doesnot consume any power in the ckt.

waveform and power curve of the RL series circuit



The current lags the voltage by φ. The power is negative between 0 and φ, and between 180 degree and (180 + φ), Power is positive during rest of the cycle.

Real power :- The power which is actually consumed or utilized in ac circuit is called real power (or) true power (or) active power. It is consumed by the resistive load in the circuit. It's unit is watts.

$$\text{Real power} = VI\cos\varphi \text{ watts}.$$

Reactive power :- A pure inductor and a pure capacitor do not consume any power.

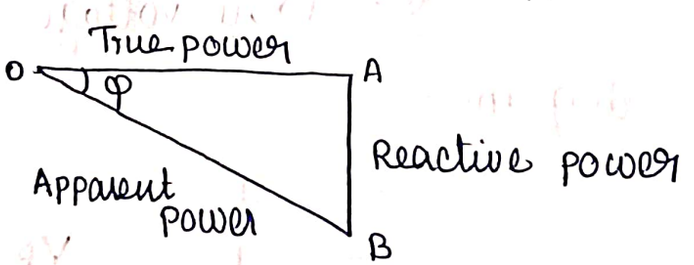
because what ever power is received from the source in a half-cycle will be returned back in the next half-cycle. Thus the power flow back and forth in both the direction in the circuit. This power is called reactive power. It is measured in Volt-Amps-Reactive (VAR). $P_r = VI \sin\varphi$

Apparent power:- The product of root mean square value of voltage and current is known as Apparent power. It is measured in kVA.

$= V_{rms} I_{rms}$

power triangle:-

power triangle is the representation of a right angle triangle showing the relation between active power, reactive power and apparent power.



Power factor:-

$\cos\varphi = \frac{\text{Real power in watts}}{\text{Apparent power in volt-Amp}}$