

Electric flux density :

The flux due to the electric field 'E' can be calculated by using the definition of the flux.

$$\text{According to eqn } E = \frac{Q}{4\pi\epsilon_0 R^2} aR$$

the electric field intensity is dependent on the medium in which the charge is placed.

New vector D is defined by

$$D = \epsilon_0 E \quad \dots\dots\dots (1)$$

The electric flux Ψ in terms of D ,

$$\Psi = \int_s D \cdot ds \quad \dots\dots\dots (2)$$

$D \rightarrow$ electric flux density $\rightarrow C/m^2$

$x \rightarrow$ measured in coulomb's

Electric flux density is also called electric displacement

Example : For an infinite sheet of charge

$$E = \frac{\rho_s}{2\epsilon_0} a_n \quad \dots\dots\dots (3)$$

$$D = \epsilon_0 E = \frac{\epsilon_0 \rho_s}{2\epsilon_0} a_n$$

$$D = \frac{\rho_s}{2} a_n \quad \dots\dots\dots (4)$$

For a volume charge distribution

$$a = \int_v \rho_v dv$$

$$D = \int_v \frac{\rho_v av}{4\pi R^2} aR \quad \dots\dots\dots (5)$$

From eqn (4) & (5) 'D' is a function of charge & position only it is independent of medium

ELECTRIC POTENTIAL

The electric field intensity E is calculated from the electric scalar potential.

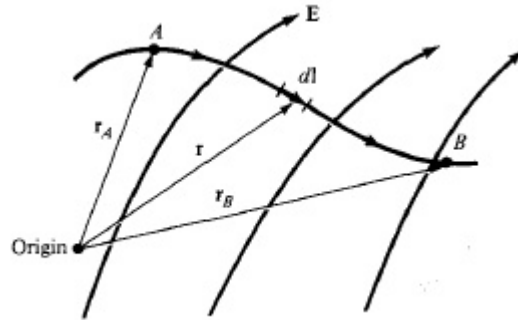


Fig 29. Displacement of point charge Q is an electrostatic field E

In fig 29 a point charge Q is move from A to point B in an electric field E .

From Coulomb's law : The force on Q is $F = Q.E$ the work done to displacing the charge by dl is

$$dw = -F dl = -QE dl \dots\dots\dots (1)$$

'-' \rightarrow work is being done by an external agent.

The potential energy required to move the charge Q from A to B is

$$W = -Q \int_A^B E dl \dots\dots\dots (2)$$

Work done by an external source for moving a charge a Q from A to B in an electric field E is potential difference. The potential difference between A & B is V_{AB}

$$V_{AB} = \frac{W}{Q} = - \int_A^B E dl \dots\dots\dots (3)$$

[work done by a unit charge]

Note :

- In V_{AB} $A \rightarrow$ Initial point, $B \rightarrow$ final point
- V_{AB} is negative, there is a loss in potential energy, V_{AB} is positive, there is a gain in potential energy.
- V_{AB} is independent of path taken
- V_{AB} is measured in joules per coulomb (or) volts (v)

In Fig 3 the electric field E due to a point charge Q located at the origin.

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \cdot a_r \dots\dots\dots (4)$$

Sub (4) in (3) r_B

$$\begin{aligned} (3) \Rightarrow V_{AB} &= - \int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \cdot a_r \cdot dr \ a_r \\ &= \frac{-Q}{4\pi\epsilon_0} \int_{r_A}^{r_B} r^{-2} a_r \ dr \\ &= \frac{-Q}{4\pi\epsilon_0} \left[\frac{r^{-1}}{-1} \right]_{r_A}^{r_B} \\ &= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right] \dots\dots\dots (5) \end{aligned}$$

$$V_{AB} = V_B - V_A \dots\dots\dots (6)$$

V_B & $V_A \rightarrow$ Absolute potentials at B & A respectively

To choose infinity as reference ie

$$V_A = 0 \qquad r_A \rightarrow \infty$$

Sub $r_B \rightarrow r$ in eqn (5)

$$V_{AB} = V = \frac{Q}{4\pi\epsilon_0 r} \dots\dots\dots (7)$$

In eqn (5) & (6) E points in the radial direction. Any contribution from a displacement in the θ & ϕ direction is

$$E \cdot dl = E \cos \alpha \ dl = E \cdot dr \dots\dots\dots (8)$$

$\alpha \rightarrow$ angle between E and dl

Conservative :

Vectors whose line integral does not depend on the path of integration are called conservative. (E is conservative)

The potential at any point is the potential difference between that point and chosen point (reference point) at which the potential is zero

$$V = - \int_{\infty}^r E \cdot dl \quad \dots\dots\dots (9)$$

If the point charge Q is not located at the origin but at a point whose position vector is r' . the potential at $V_{(x,y,z)}$ (or) $V(r)$ at r becomes

$$V(r) = \frac{Q}{4\pi\epsilon_0|r-r'|} \quad \dots\dots\dots (10)$$

The electric field due to 'N' point charges located at points with position vectors $r_1, r_2 \dots r_n$. The potential at ' r ' is obtained by superposition principle.

$$V(r) = \frac{Q_1}{4\pi\epsilon_0|r-r_1|} + \frac{Q_2}{4\pi\epsilon_0|r-r_2|} + \dots + \frac{Q_n}{4\pi\epsilon_0|r-r_n|}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{Q_k}{|r-r_k|} \text{ (point charges) } \dots\dots\dots (11)$$

Continuous charge distributions

Line charge (potential at r)

$$Q = \rho_L dl$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_L \frac{\rho_L(r')}{|r-r'|} dl' \quad \dots\dots\dots (12)$$

Surface charge $Q = \rho_s ds$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_S \frac{\rho_s(r')}{|r-r'|} ds' \quad \dots\dots\dots (13)$$

Volume charge :

$$Q = \rho_v dv$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_v \frac{\rho_v(r')}{|r-r'|} dv' \quad \dots\dots\dots (14)$$

From eqn (7) to (14). The reference point has been chosen at infinity. If any other point is chosen as reference, equation. (10) \Rightarrow

$$V = \frac{Q}{4\pi\epsilon_0 r} + C \dots\dots\dots (15)$$

$C \rightarrow$ Constant, ie determined at the chosen point of reference

2. The potential v can be determined in two ways :

- ❖ Charge distribution
- ❖ Electric field 'E'

If the charge distribution is know we use one of the eqn (10) to (15)

IF 'E' is known

$$V = - \int E \cdot dl + C \dots\dots\dots (16)$$

$$V_{AB} = V_B - V_A = - \int_A^B E \cdot dl = \frac{W}{Q} \dots\dots\dots (17)$$

Relationship between E and V. Maxwell's equation

The potential difference between points A and B is independent of the path taken. (From previous section)

$$V_{BA} = -V_{AB}$$

$$\text{ie } V_{AB} + V_{BA} = \oint E \cdot dl = 0$$

$$\oint_L E \cdot dl = 0 \dots\dots\dots (1)$$

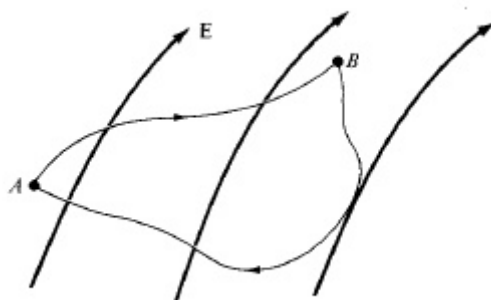


Fig30 : conservative nature of an electrostatic field

ie, the line integral of 'E' along a closed path is zero. No net work is done in moving a charge along a closed path in an electrostatic field.

Stokes theorem

$$\oint E \, dl = \oint_s (\nabla \times E) \, ds = 0$$

$$\nabla \times E = 0 \dots\dots\dots (2)$$

Any vector field satisfied eqn (1) or (2) is said to be conservative or irrotational.

(1) → Maxwell's second equation in integral form

(2) → Maxwell's second equation in point form (or) Differential form

The potential $V = - \int E \, dl$

$$dV = -E \cdot dl = -E_x \, dx - E_y \, dy - E_z \, dz \dots\dots\dots (3)$$

A total charge in $V(x, y, z)$ is the sum of partial charges with respect to x, y, z variables

$$dV = \frac{\partial v}{\partial x} \, dx + \frac{\partial v}{\partial y} \, dy + \frac{\partial v}{\partial z} \, dz \dots\dots\dots (4)$$

Compare (3) & (4)

$$E_x = - \frac{\partial v}{\partial x} \quad E_y = - \frac{\partial v}{\partial y} \quad E_z = - \frac{\partial v}{\partial z}$$

Thus $E = -\nabla V$

ie, Electric field intensity is the gradient of v . ' - ' sign indicates the direction of E is opposite to the direction in which 'v' increases.

$$\nabla \times \nabla V = 0$$