1.4 SYSTEM

A system is a process or device that receives an input signal, performs some operation on it, and produces an output signal.

CLASSIFICATION OF SYSTEM

Property	Type 1	Type 2	Description / Example
Linearity	Linear	Nonlinear	Obeys superposition / Does not.
Time Dependence	Time-Invariant	Time-Variant	System behaviour constant / changes over time.
Causality	Causal	Non-Causal	Output depends only on present/past / includes future.
Stability	Stable	Unstable	Bounded input → bounded output.
Memory	Static (Memoryless)	Dynamic (With Memory)	Depends only on current input / also on past or future.
Invertibility	Invertible	Non-Invertible	Input can / cannot be uniquely recovered.

CONTINUOUS TIME AND DISCRETE TIME SYSTEM

Continuous time system:

Continuous time system operates on a continuous time signal (input or excitation) and produces another continuous time signal (output or response) which is shown in Figure 1.4.1. The signal $x(t)$ is transformed by the system into signal $y(t)$, this transformation can be expressed as,

Response $y(t) = T\, x(t)$

where $x(t)$ is input signal, $y(t)$ is output signal, and T denotes transformation

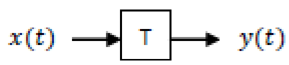

Figure 1.4.1 Representation of continuous time system

Discrete time system:

Discrete time system operates on a discrete time signal (input or excitation) and produces another discrete time signal (output or response) which is shown in Figure 1.4.2.

The signal $x(n)$ is transformed by the system into signal $y(n)$, this transformation can be expressed as,

$$\text{Response } y(n) = T\{x(n)\}$$

where x(n) is input signal, y(n) is output signal, and T denotes transformation

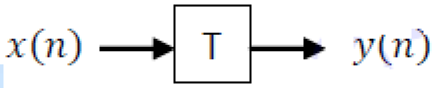

Figure 1.4.2 Representation of discrete time system

Linear system and Non-Linear system

Linear system:

A system is said to be linear if it obeys superposition theorem. Superposition theorem states that the response of a system to a weighted sum of the signals is equal to the corresponding weighted sum of responses to each of the individual input signals.

Condition for Linearity in continuous time systems:

$$T[ax_1(t) + bx_2(t)] = ay_1(t) + by_2(t)$$

where $y_1(t)$ *and* $y_2(t)$ *are the responses of* $x_1(t)$ *and* $x_2(t)$ *respectively*

Condition for Linearity in Discrete time systems:

$$T[ax_1(n) + bx_2(n)] = ay_1(n) + by_2(n)$$

where $y_1(n)$ *and* $y_2(n)$ *are the responses of* $x_1(n)$ *and* $x_2(n)$ *respectively*

Non Linear system:

A system is said to be Nonlinear if it does not obeys superposition theorem.

Condition for Non Linearity in continuous time systems:

$$i.e.,\, T[ax_1(t) + bx_2(t)] \neq ay_1(t) + by_2(t)$$

where $y_1(t)$ *and* $y_2(t)$ *are the responses of* $x_1(t)$ *and* $x_2(t)$ *respectively*

Condition for Non Linearity in Discrete time systems:

$$i.e.,\, T[ax_1(n) + bx_2(n)] \neq ay_1(n) + by_2(n)$$

where $y_1(n)$ *and* $y_2(n)$ *are the responses of* $x_1(n)$ *and* $x_2(n)$ *respectively*

Static (Memoryless) and Dynamic (Memory) system

Static system:

A system is said to be memoryless or static if the response of the system is due to present input alone.

Example in Continuous time domain: $y(t) = 2x(t)$

$$y(t) = x^2(t) + x(t)$$

Example in Discrete time domain: $y(n) = x(n)$

$$y(n) = x^2(n) + 3x(n)$$

Dynamic system:

A system is said to be memory or dynamic if the response of the system depends on factors other than present input also.

Example in Continuous time domain: $y(t) = 2x(t) + x(-t)$

$$y(t) = x^2(t) + x(2t)$$

Example in Discrete time domain: $y(n) = 2x(n) + x(-n)$

$$y(n) = x^2(1-n) + x(2n)$$

Time invariant (Shift invariant) and Time variant (Shift variant) system

Time invariant system:

A system is said to time invariant if the relationship between the input and output doesnot change with time.

In Continuous time domain: If $y(t) = T[x(t)]$

Then $T[x(t-t_0)] = y(t-t_0)$ should be satisfied for the system to be time invariant

In Discrete time domain: If $y(n) = T[x(n)]$

Then $T[x(n-n_0)] = y(n-n_0)$ should be satisfied for the system to be time invariant

Time variant system:

A system is said to time variant if the relationship between the input and output changeswith time.

Continuous time domain: If $y(t) = T[x(t)]$

Then $T[x(t-t_0)] \neq y(t-t_0)$ should be satisfied for the system to be time variant

In Discrete time domain: If $y(n) = T[x(n)]$

Then $T[x(n - n_0)] \neq y(n - n_0)$ should be satisfied for the system to be time variant

Causal and Non-Causal system

Causal system:

A system is said to be causal if the response of a system at any instant of time depends only on the present input, past input and past output but does not depends upon the future input and future output.

For continuous time systems,

Example: $y(t) = 3x(t) + x(t - 1)$

A system is said to be causal if impulse response $h(t)$ is zero for negative values of t i.e., $h(t) = 0 \; for \; t < 0$

For discrete time systems,

Example: $y(n) = 3x(n) + x(n - 1)$

A system is said to be causal if impulse response h(n) is zero for negative values of n i.e., $h(n) = 0 \; for \; n < 0$

Non-Causal system:

A system is said to be Non-causal if the response of a system at any instant of time depends on the future input and also on the present input, past input, past output.

For continuous time systems,

Example: $y(t) = x(t + 2) + x(t - 1)$

$y(t) = x(-t) + x(t + 4)$

A system is said to be non-causal if impulse response $h(t)$ is non-zero for negative values of t

i.e., $h(t) \neq 0 \; for \; t < 0$

For discrete time systems,

Example: $y(n) = x(n + 2) + x(n - 1)$

$$y(n) = x(-n) + x\,(n + 4)$$

A system is said to be non-causal if impulse response $h(n)$ is non-zero for negative values of n i.e., $h(n) \neq 0$ for $n < 0$

Stable and Unstable system

A system is said to be stable if and only if it satisfies the BIBO stability criterion.

BIBO stable condition for continuous time systems:

- Every bounded input yield bounded output.

$i.e.,$ if $0 < x(t) < \infty$ $then$ $0 < y(t) < \infty$ should be satisfied for the system to be stable

- Impulse response should be absolutely integrable

$$i.e., 0 < \int_{-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$$

If the BIBO stable condition is not satisfied, then the system is said to be unstable system

BIBO stable condition for Discrete time systems:

- Every bounded input yield bounded output.
- Impulse response should be absolutely summable

$$i.e., 0 < \sum_{k=-\infty}^{\infty} |\,h(k)\,| < \infty$$

If the BIBO stable condition is not satisfied, then the system is said to be unstable system.

Invertible System

Definition:

A system is **invertible** if there exists another system (called the **inverse system**) that can **recover the original input** from the output.

Mathematically:

If a system T gives $x(t) = T\{x(t)\}$ then it is invertible if there exists a system T^{-1} such that:

$$T^{-1}\{T\{x(t)\}\} = x(t)$$

Example 1 (Invertible):

y(t)=x(t)+2

Inverse:

x(t)=y(t)−2

You can uniquely recover the input x(t)x(t) from the output y(t), so this system is **invertible**.

Example 2 (Invertible):

y(t)=5x(t)

Inverse:

x(t)=y(t)/5

Non-Invertible System

Definition:

A system is **non-invertible** if it's **not possible to uniquely determine the input** from the output.

Example (Non-Invertible):

$$\boldsymbol{y(t) = x^2(t)}$$

If y(t)=4, the input x(t) could be +2or −2. Since there's **no unique solution**, the system is **non-invertible**.