

#### 4.4. STATE REDUCTION/MINIMIZATION

The state reduction is used to avoid the redundant states in the sequential circuits. The reduction in redundant states reduces the number of required Flip-Flops and logic gates, reducing the cost of the final circuit.

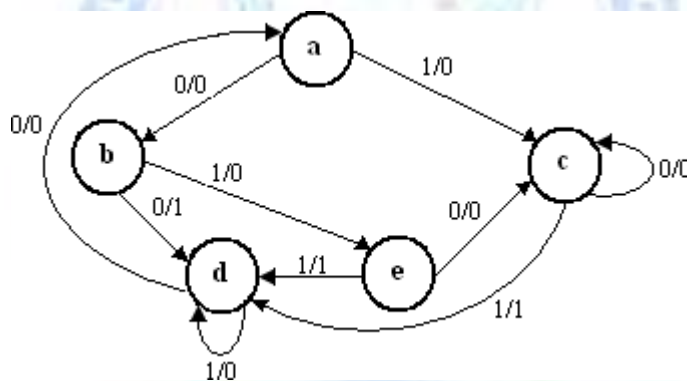
The two states are said to be redundant or equivalent, if every possible set of Inputs generate exactly same output and same next state. When two states are equivalent, one of them can be removed without altering the input-output relationship.

Since 'n' Flip-flops produced  $2^n$  state, are duction in the number of states may result in are Reduction in the number of Flip-Flops.

The need for state reduction or state minimization is explained with one example.

##### Examples:

1. Reduce the number of states in the following state diagram and draw the reduced state diagram



State diagram

**Step1:Determine the state table for given state diagram**

Presentstate	Nextstate		Output	
	X=0	X=1	X=0	X=1
a	b	c	0	0
b	d	e	1	0
c	c	d	0	1
d	a	d	0	0
e	c	d	0	1

### Step2:Find equivalent states

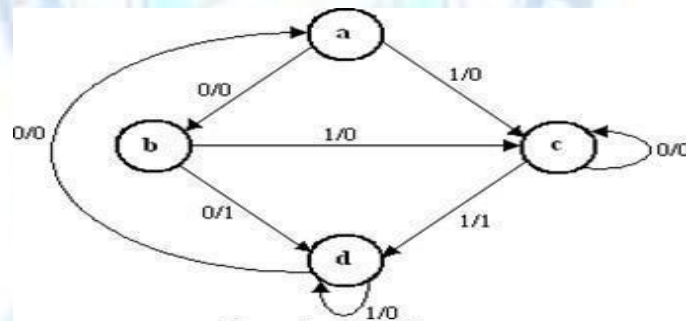
From the above state table can degenerate exactly same next state and same

Output for every possible set of inputs. The state **c** and **e** go to next states **c** and **d** and have out puts 0 and 1 for  $x=0$  and  $x=1$  respectively. Therefore state **e** can be removed and replaced by **c**. The final reduced state table is shown below.

Presentstate	Nextstate		Output	
	X=0	X=1	X=0	X=1
a	b	c	0	0
b	d	c	1	0
c	c	d	0	1
d	a	d	0	0

Reduced state table

### Step3:Draw state diagram



Reduced state diagram

2. Reduce the number of states in the following statetable and tabulate the reduced statetable.

Presentstate	Nextstate		Output	
	X=0	X=1	X=0	X=1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	g	f	0	1
g	a	f	0	1

**Soln:**

From the above state table **e** and **g** generate exactly same next state and same output for every possible set of inputs. The state **e** and **g** go to next states **a** and **f** and have outputs 0 and 1 for x=0 and x=1 respectively. Therefore state **g** can be removed and replaced by **e**.

The reduced state table-1 is shown below.

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
a	A	b	0	0
b	C	d	0	0
c	A	d	0	0
d	E	f	0	1
e	A	f	0	1
f	E	f	0	1

Now states d and f are equivalent. Both states go to the same next state(e,f)and have

same output(0,1).Therefore one state can be removed; **f** is replaced by **d**.

The final reduced state table-2 is shown below.

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
a	a	b	0	0
b	c	d	0	0
c	a	<b>d</b>	0	0
d	e	<b>d</b>	0	1
e	a	<b>d</b>	0	1

**Reducedstatetable-2**

Thus 7 states are reduced into 5 states

2. Determine a minimal state table equivalent furnished below

Present state	Next state	
	X=0	X=1
1	1,0	1,0
2	1,1	6,1
3	4,0	5,0
4	1,1	7,0
5	2,0	3,0
6	4,0	5,0
7	2,0	3,0

**Soln:**

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
1	1	1	0	0
2	1	6	1	1
3	4	5	0	0
4	1	7	1	0
5	2	3	0	0
6	4	5	0	0
7	2	3	0	0

From the above state table, **5** and **7** generate exactly same next state and same output for every possible set of inputs. The state **5** and **7** go to next states **2** and **3** and have outputs 0 and 0 for x=0 and x=1 respectively. Therefore state **7** can be removed and replaced by **5**.

Similarly, 3 and 6 generate exactly same next state and same output for every possible set of inputs. The state 3 and 6 go to next states 4 and 5 and have outputs 0 and for  $x=0$  and  $x=1$  respectively. Therefore state 6 can be removed and replaced by 3. The final reduced state table is shown below.

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
1	1	1	0	0
2	1	3	1	1
3	4	5	0	0
4	1	5	1	0
5	2	3	0	0

Reduced state table

Thus 7 states are reduced into 5 state.



3. Minimize the following state table

Present state	Next state	
	X=0	X=1
A	D,0	C,1
B	E,1	A,1
C	H, 1	D,1
D	D,0	C,1
E	B,0	G, 1
F	H, 1	D,1
G	A,0	F,1
H	C,0	A,1
I	G, 1	H,1

**Soln:**

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
A	D	C	0	1
B	E	A	1	1
C	H	D	1	1
D	D	C	0	1
E	B	G	0	1
F	H	D	1	1
G	A	F	0	1
H	C	A	0	1
I	G	H	1	1

From the above state table, **A** and **D** generate exactly same next state and same output for every possible set of inputs. The state **A** and **D** go to next states **D** and **C** and have outputs 0 and 1 for  $x=0$  and  $x=1$  respectively. Therefore state **D** can be removed and replaced by **A**. Similarly, **C** and **F** generate exactly same next state and same output for every possible set of inputs. The state **C** and **F** go to next states **H** and **D** and have outputs 1 and 1 for  $x=0$  and  $x=1$  respectively. Therefore state **F** can be removed and replaced By **C**

Presentstate	Nextstate		Output	
	X=0	X=1	X=0	X=1
A	A	C	0	1
B	E	A	1	1
C	H	A	1	1
E	B	G	0	1
G	A	C	0	1
H	C	A	0	1
I	G	H	1	1

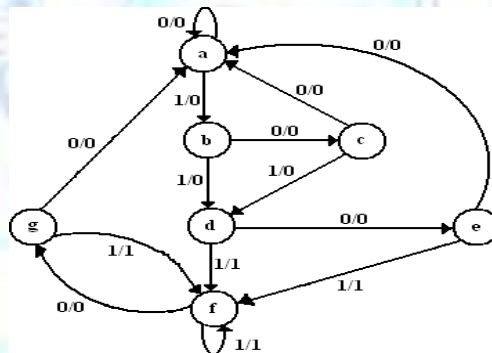
**Reducedstatetable-1**

From the above reduced state table-1, **A** and **G** generate exactly same next state and same output for every possible set of inputs. The state **A** and **G** go to next states **A** and **C** and have outputs 0 and 1 for  $x=0$  and  $x=1$  respectively. Therefore state **G** can be removed and replaced by **A**. The final reduced state table-2 is shown below.

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
A	A	C	0	1
B	E	A	1	1
C	H	A	1	1
E	B	A	0	1
H	C	A	0	1
I	A	H	1	1

Thus 9 states are reduced in to 6 states.

4. Reduce the following state diagram



SOLN:

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	g	f	0	1
g	a	f	0	1

State table

From the above state table **e** and **g** generate exactly same next state and same output for every possible set of inputs. The state **e** and **g** go to next states **a** and **f** and have outputs 0 and 1 for  $x=0$  and  $x=1$  respectively. Therefore state **g** can be removed and replaced by **e**. The reduced state table-1 is shown below.

**Reducedstatetable-1**

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	e	f	0	1



Now states d and f are equivalent. Both states go to the same next state(e,f)and have same output . The final reduced state table-2 is shown below.

Present state	Next state		Output	
	X=0	X=1	X=0	X=1
A	A	b	0	0
b	C	d	0	0
c	A	<b>d</b>	0	0
d	E	<b>d</b>	0	1
e	A	<b>d</b>	0	1

**Reduced state table-2**

Thus 7 states are reduced into 5 states.

The statediagramforthereducedstatetable-2is,

**Reduced state**

