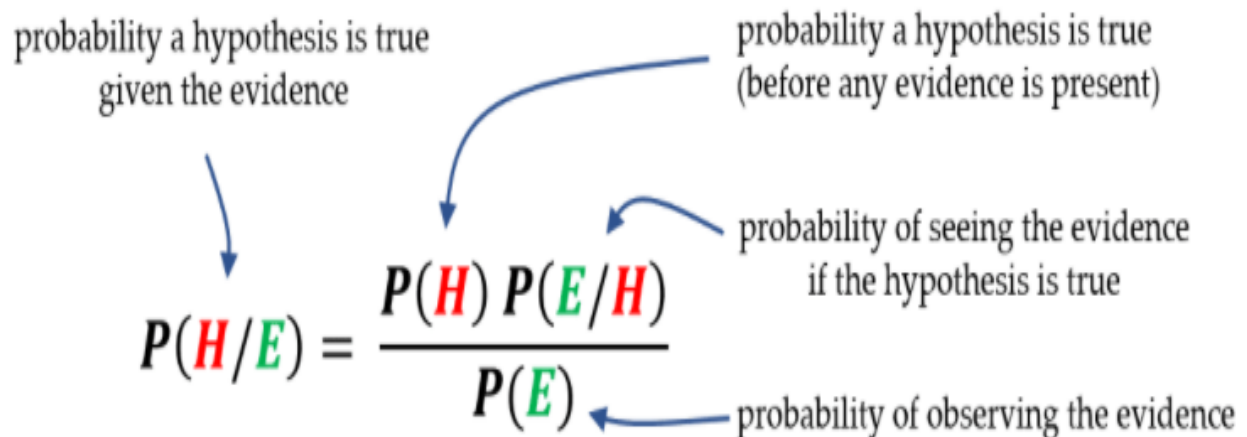


BAYESIAN CLASSIFICATION

- **Probabilistic learning:** Calculate explicit probabilities for hypothesis, among the most practical approaches to certain types of learning problems
- **Incremental:** Each training example can incrementally increase/decrease the probability that a hypothesis is correct. Prior knowledge can be combined with observed data.
- **Probabilistic prediction:** Predict multiple hypotheses, weighted by their probabilities
- **Standard:** Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

Given training data D, posteriori probability of a hypothesis h, $P(h|D)$ follows the Bayes theorem

The formula for Bayes' theorem is



The diagram shows the Bayes' theorem formula with arrows pointing from descriptive text to parts of the formula:

- $P(H/E)$ is labeled "probability a hypothesis is true given the evidence".
- $P(H)$ is labeled "probability a hypothesis is true (before any evidence is present)".
- $P(E/H)$ is labeled "probability of seeing the evidence if the hypothesis is true".
- $P(E)$ is labeled "probability of observing the evidence".

$$P(H/E) = \frac{P(H) P(E/H)}{P(E)}$$

MAP (maximum posteriori) hypothesis

$$\begin{aligned}
 h_{map} &= \arg \max_{h \in H} (P(h | D)) \\
 &= \arg \max_{h \in H} \left(\frac{P(D | h)P(h)}{P(D)} \right) \\
 &= \arg \max_{h \in H} (P(D | h)P(h))
 \end{aligned}$$

Practical difficulty: require initial knowledge of many probabilities, significant computational cost

Naïve Bayes Classifier (I)

- A simplified assumption: attributes are conditionally independent:
- Greatly reduces the computation cost, only count the class distribution.

Naïve Bayesian Classifier (II)

Given a training set, we can compute the probabilities

BAYESIAN CLASSIFICATION Algorithm

The classification problem may be formalized using a-posteriori probabilities:

- $P(C|X)$ = prob. that the sample tuple
- $X = \langle x_1, \dots, x_k \rangle$ is of class C .
- E.g. $P(\text{class} = N \mid \text{outlook} = \text{sunny}, \text{windy} = \text{true}, \dots)$
- Idea: assign to sample X the class label C such that $P(C|X)$ is maximal

Estimating a-posteriori probabilities

Bayes theorem:

$$P(C|X) = P(X|C) \cdot P(C) / P(X)$$

- $P(X)$ is constant for all classes
- $P(C)$ = relative freq of class C samples
- C such that $P(C|X)$ is maximum = C such that $P(X|C) \cdot P(C)$ is maximum
- Problem: computing $P(X|C)$ is unfeasible!

