FIR & IIR FILTER REALIZATION – PARALLEL & CASCADE FORMS

FIR FILTERS:

The filters have a finite impulse response function which has finite length of time. The output y(n) of the filter is simply the convolution of impulse response and the input signal x(n) . In simply terms the co-efficient of a filter are the impulse response.

Properties of FIR filter:

1) Stability: These Filters are Stable i.e. Bounded Input Bounded Output (BIBO) because the poles located at the origin and are within the unit circle.

2) Feedback: There is no feedback for this kind of filters which in turn means rounding errors are not compounded by summed iterations, the error repeats in each of the iteration and by this implementation seems simple.

3) Linear Phase: This co-efficient sequence has to be made symmetric and the property seems to have an important role in phase – sensitivity applications.

Advantages of FIR Filter:

1. Simple to design, delay the input signal (Linear-phase) without altering the phase. Calculations are simple and need single instruction for looping.
2. Multi-rate Applications. Where you see interpolation (increase sampling rate) and decimation (sampling rate decreased) or both which in turn related to computational efficiency.
3. FIR Filters have desirable properties i.e. numeric properties. Finite-precision filters are implemented in DSP filters i.e. limited no. of bits.
4. Fractional arithmetic implementation can be seen by this type of the filters.

FIR FILTER REALIZATION

In general, the time domain representation of an N^{th} order FIR system is,

$$y(n) = \sum_{=0}^{N-1} b_m x(n-m) = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2) + \ldots\ldots + b_{N-1}\, x(n-(N-1))$$

and the z-domain representation of a FIR system is,

$$H(z) = \frac{Y(z)}{(\ )} = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots\ldots + b_{N-1} z^{-(N-1)}$$

The above two representation of FIR system can be viewed as a computational procedure to determine the output sequence y(n) from the input sequence x(n).

1.Direct form realization of FIR system

Consider the difference equation governing system a FIR system,

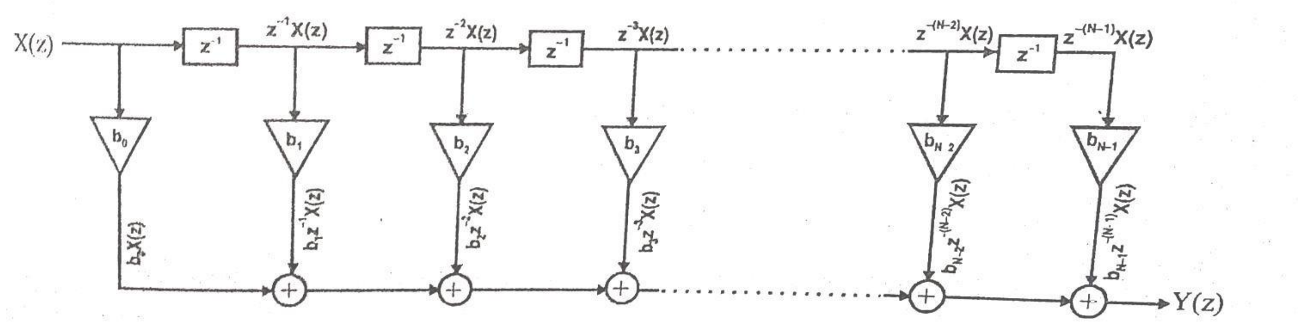
$$y(n) = \sum_{=0}^{N-1} b_m x(n-m)$$

$=b_0\ x(n) + b_1\ x(n-1) + b_2\ x(n-2) + \ldots\ldots+b_{N-1}\ x(n-(N-1))$

On taking Z transform of the above equation we get,

$$Y(z) = b_0\ X(z) + b_1\ z^{-1}\ X(z) + b_2 z^{-2}\ X(z)+ b_3 z^{-3}\ X(z)+\ldots+ b_{N-1}\ z^{-(N-1)}\ X(z)$$

The direct form structure provides a direct relation between time domain and z domain equations.



Direct form structure of FIR system

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-511]

From the direct form structure it is observed that the realization of an N^{th} order FIR discrete time system involves N number of multiplications and N-1 number of additions.Also the structure involves N-1 delays and so N-1 memory locations are required to store the delayed signals.

2.Cascade Form Realization of FIR system:

Consider the transfer function of a FIR system,

$$H(z) = \frac{Y(z)}{(\ )} = b_0 + b_1\ z^{-1} + b_2\ z^{-2} +\ldots\ldots+ b_{N-1}\ z^{-(N-1)}$$

The transfer function of FIR system is $(N-1)^{th}$ order polynomial in z.This polynomial can be factorized into first and second order factors and the transfer function can be expressed as a product of first and second order factors or sections as shown in figure.

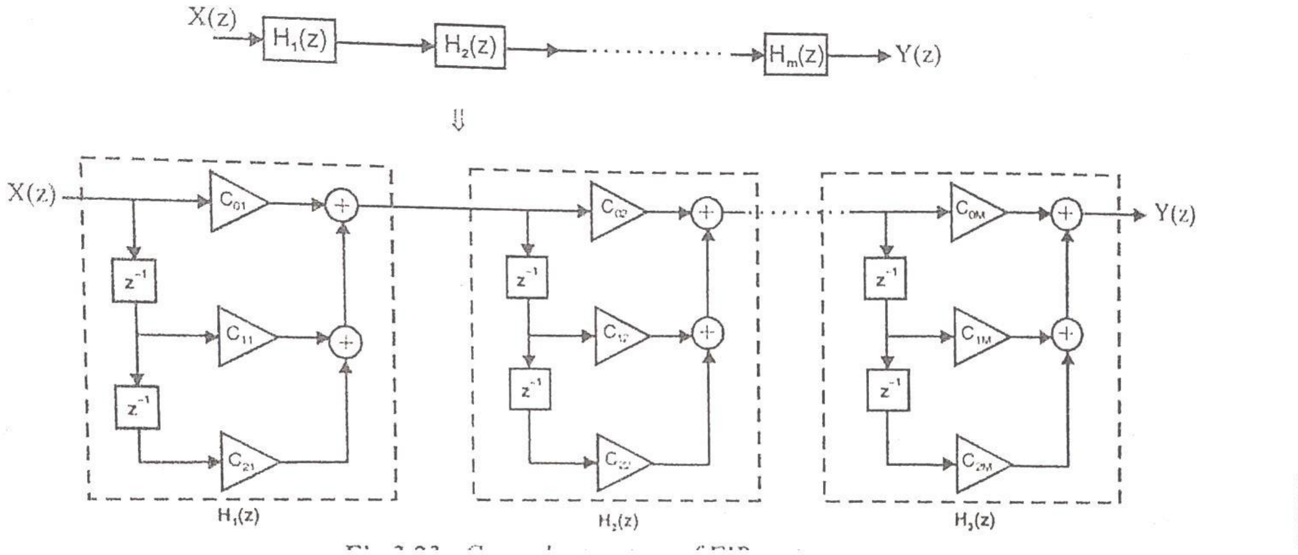
$$H(z)= \frac{Y(z)}{(\ )} = H_1(z) * H_2(z)*\ldots\ldots H_m(z)$$

$$=\prod_{1}^{=} \quad (z)$$

Where $H_i(z) = c_{0i} + c_{1i}\, z^{-1} + c_{2i}\, z^{-2}$

(or)

$H_i(z) = c_{0i} + c_{1i}\, z^{-1}$



Cascade structure of FIR system

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-505]

The individual second order or first order sections can be realized either in direct form structure or linear phase structure. The overall system is obtained by cascading the individual sections as shown in figure. The number of calculations depends on the realization of individual sections.

IIR FILTER REALIZATION

The infinite impulse response (IIR) filter is a recursive filter in that the output from the filter is computed by using the current and previous inputs and previous outputs. Because the filter uses previous values of the output, there is feedback of the output in the filter structure.

In general the time domain representation of an N^{th} order IIR system is,

$$y(n) = -\sum_{=1}^{N} a_m y(n-m) + \sum_{=0}^{M} b_m x(n-m)$$

and the z domain representation of an N^{th} order IIR system is,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots . + b_M z^{-M}}{1 + a_1 z^{-1} + \cdots . + a_N z^{-N}}$$

The above two representation of IIR system can be viewed as a computational procedure to determine the output sequence y(n) from the input sequence x(n).

1. Direct form-I structure of IIR system

Consider the difference equation governing an IIR system,

$$y(n) = -\sum_{=1}^{N} a_m y(n-m) + \sum_{=0}^{M} b_m x(n-m)$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) - \ldots - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \ldots + b_M x(n-M)$$

On taking Z transform of the above equation we get,

$$Y(z) = -a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - \ldots - a_N z^{-N} Y(z) + b_0 X(z) + b_1 z^{-1} X(z) + \ldots + b_M z^{-M} X(z)$$

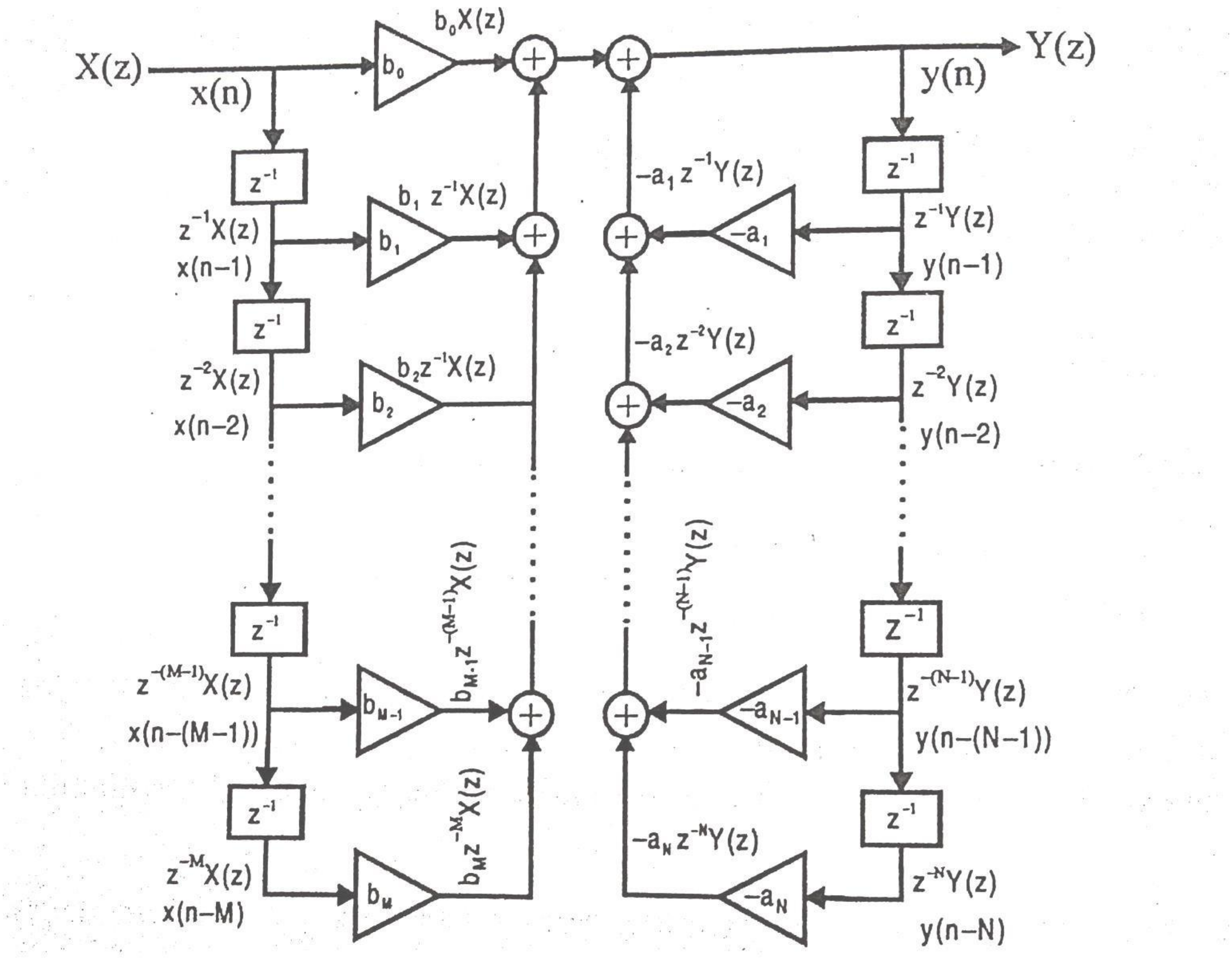


Fig.4.1.3.1. Direct form structure of IIR system

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-528]

2. Cascade form realization of IIR system

The transfer function H(z) can be expressed as a product of a number of second-order or first order sections as shown in equation

$$H(z) = \frac{Y(z)}{(\ )} = H_1(z) * H_2(z) * \ldots\ldots H_m(z)$$

$=\prod_{1}^{=} (z)$

Where , $H_i(z) = \frac{c0i + c1iz^{-1} + c_{2i}z^{-2}}{d_{0i} + d_{1i}z^{-1} + d_{2i}z^{-2}}$

(or)

$$H_i(z) = \frac{0 + 1 \quad -1}{d_{0i} + d_{1i}z^{-1}}$$

The individual second order or first order sections can be realized either in direct form-I or direct form –II structures. The overall system is obtained by cascading the individual sections as shown in figure.

Fig 4.1.3.2.Cascade structure of IIR system

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-528]

3. Parallel Form realization of IIR system

The transfer function H(z) of a discrete time system can be expressed as a sum of first and second order sections, using partial fraction expansion technique

$$H(z) = \frac{(\ )}{(\ )} = C + H_1(z) + H_2(z) + \ldots\ldots\ldots + H_m(z)$$

$=C+\sum_{1}^{=} Hi(z)$

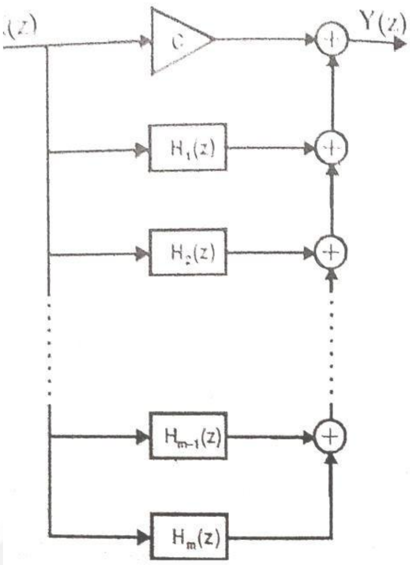
Where,

$$Hi(z) = \frac{c_{0i} + c_{1i}z^{-1}}{d_{0i} + d_{1i}z^{-1} + d_{2i}z^{-2}}$$

For second order section

$$Hi(z) = \frac{0}{d_{0i} + d_{1i}z^{-1}}$$

For first order section



Parallel structure of IIR system

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-529]

The individual first and second order sections can be realized either in direct form I or direct form-II structures.The overall system is obtained by connecting the individual sections in parallel.