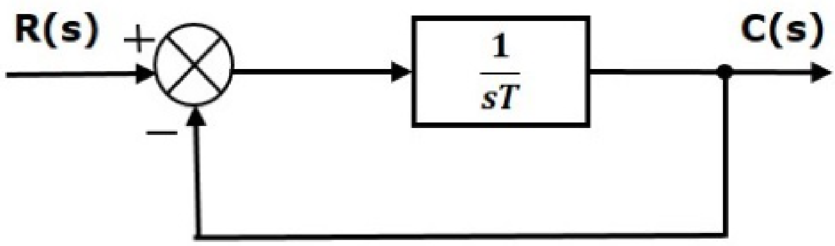
Time Domain Specifications of the First Order System

In this chapter, let us discuss the time response of the first order system. Consider the following block diagram of the closed loop control system. Here, an open loop transfer function, $\frac{1}{sT}$ is connected with a unity negative feedback.



We know that the transfer function of the closed loop control system has unity negative feedback as,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

Substitute, $G(s) = \frac{1}{sT}$ in the above equation.

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{1 + \frac{1}{sT}} = \frac{1}{sT+1}$$

The power of s is one in the denominator term. Hence, the above transfer function is of the first order and the system is said to be the **first order system**.

We can re-write the above equation as

$$C(s) = \left(\frac{1}{sT+1}\right) R(s)$$

Where,

- **C(s)** is the Laplace transform of the output signal c(t),
- **R(s)** is the Laplace transform of the input signal r(t), and
- **T** is the time constant.

Follow these steps to get the response (output) of the first order system in the time domain.

- Take the Laplace transform of the input signal $r(t)$.
- Consider the equation, $C(s) = \left(\frac{1}{sT+1}\right) R(s)$
- Substitute $R(s)$ value in the above equation.
- Do partial fractions of $C(s)$ if required.
- Apply inverse Laplace transform to $C(s)$.

TIME RESPONSE ANALYSIS

The knowledge of input signal is required to predict the response of a system. In most of the systems, the input signals are not known ahead of time and also it is difficult to express the input signals mathematically by simple equations. The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity and a constant acceleration. Hence test signals which resembles these characteristics are used as input signals to predict the performance of the system. The commonly use test input signals are impulse, step, ramp, acceleration and sinusoidal signals.

Standard Input Signals

1. Step signal
2. Unit step signal
3. Ramp signal
4. Unit ramp signal
5. Parabolic signal
6. Unit parabolic signal
7. Impulse signal
8. Sinusoidal signal

STEP SIGNAL

The step signal is a signal whose value changes from zero to A at t=0 and remains constant at A for t > 0. The step signal resembles an actual steady input to a system. A special case of step signal is unit step in which A is unity.

RAMP SIGNAL

The ramp signal is a signal whose value increases linearly with time from an initial value of zero at t=0. The ramp signal resembles a constant velocity input to the system. A special case of ramp signal is unit ramp signal in which the value of A is unity.

PARABOLIC SIGNAL

In parabolic signal, the instantaneous value varies as square of the time from an initial value of zero at t=0. The sketch of the signal with respect to time resembles a parabola. The parabolic signal resembles a constant acceleration input to the system. A special case of parabolic signal is unit parabolic signal in which A is unity.

IMPULSE SIGNAL

A signal of very large magnitude which is available for very short duration is called impulse signal. Ideal impulse signal is a signal with infinite magnitude and zero duration but with an area of A. The unit impulse signal is a special case, in which A is unity. Since perfect impulse cannot be achieved in practice, it is usually approximated by a pulse of small width but with area, A. Mathematically an impulse signal is the derivative of a step signal. Laplace transform of the impulse function is unity.

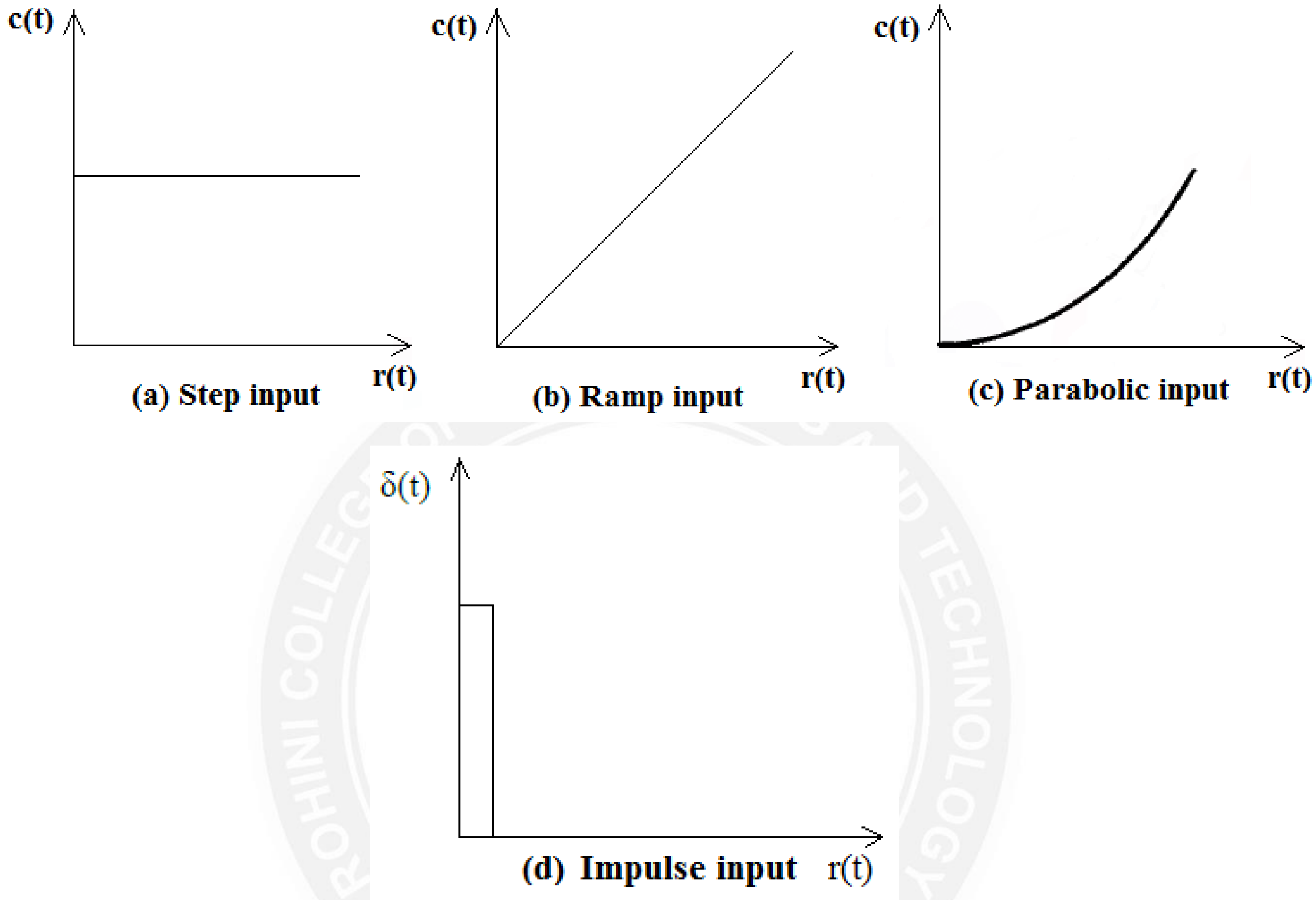
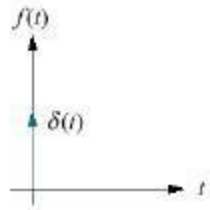
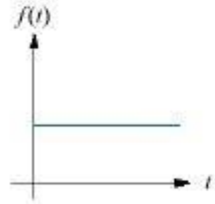
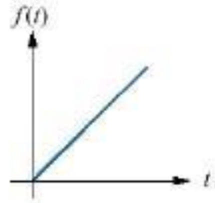
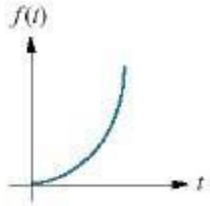
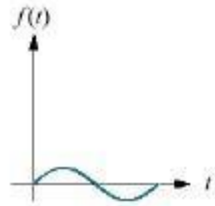


Figure 2.3.1 Standard test signals

[Source: "Control Systems Engineering" by I J Nagrath, M Gopal, Page: 196]

Input	r(t)	R(s)
Step input	A	A/s
Ramp input	At	A/s^2
Parabolic input	$At^2/2$	A/s^3
Impulse input	$\delta(t)$	1

Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)\,dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error