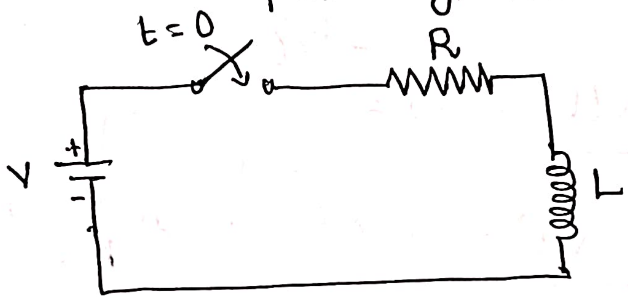
Forced Response of RL series circuit.

The circuit shows an RL circuit connected to a DC source of voltage V through a switch S. Assume switch is closed at time $t=0$ also assume the initial current flowing in the circuit is zero.



Apply KVL to ckt

$$V = V_R + V_L$$

$$V = iR + L \cdot \frac{di}{dt}$$

Taking Laplace transform on both side

$$\frac{V}{S} = R\, I(S) + L\left[S I(S) - i(0)\right]$$

$$L[1] = \frac{1}{S}$$

where $i(0)$ is the initial current

$$L\left(\frac{di}{dt}\right) = S\, I(S) - i(0)$$

let it be 0.

$$\therefore \frac{V}{S} = R I(S) + L S\, I(S)$$

$$\frac{V}{S} = I(S)\,(R + LS)$$

$$I(S) = \frac{V}{S(R+LS)}$$

By using partial Fraction method (15)

$$I(s) = \frac{V}{s(R+sL)} = \frac{A}{s} + \frac{B}{R+sL} \quad \text{——— (1)}$$

$$V = A(R+sL) + Bs$$

At $s=0$

$$V = AR$$

$$\boxed{A = \frac{V}{R}}$$

At $s = \frac{-R}{L}$

$$V = A\left(R - \frac{R}{L}\times L\right) + B\left(-\frac{R}{L}\right)$$

$$V = A(R-R) - B\cdot\frac{R}{L}$$

$$\boxed{\therefore B = -V\left(\frac{L}{R}\right)}$$

substituting the value of A and B in (1)

$$I(s) = \frac{V/R}{s} - \frac{V\cdot L/R}{R+sL}$$

$$= \frac{V}{Rs} - \frac{VL}{R(R+sL)}$$

$$= \frac{V}{R}\left(\frac{1}{s}\right) - \frac{V}{R}\left[\frac{1}{\frac{R}{L}+\frac{sL}{L}}\right]$$

$$= \frac{V}{R}\left(\frac{1}{s}\right) - \frac{V}{R}\left(\frac{1}{s+R/L}\right)$$

Taking inverse laplace transform on both sides

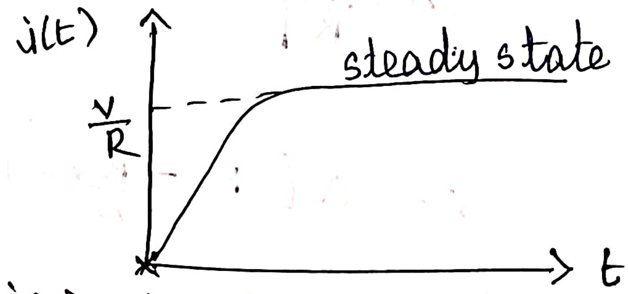
16

$$i(t) = \frac{V}{R} \cdot 1 - \frac{V}{R} e^{-(R/L)t}$$

$$i(t) = \frac{V}{R}\left[1 - e^{(-R/L)t}\right]$$

$$\boxed{i(t) = \frac{V}{R}\left[1 - e^{-t/\tau}\right]}$$ where time constant $\tau = L/R$

The above equation can be drawn as.



when $t=0$ $i(0)=0$

Final value et can be found by substituting $t=\infty$

$$i(t)\Big|_{t=\infty} = \frac{V}{R}\left[1 - e^{-\infty}\right]$$

$$= \frac{V}{R} \qquad \left[\because e^{-\infty} = 0\right].$$

Note. At $t = \frac{L}{R}$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{\cancel{L} R}{R \cancel{L}}}\right]$$

$$= \frac{V}{R}\left[1 - e^{-1}\right]$$

$$= \frac{V}{R}\left[1 - 0.368\right]$$

$$i(t) = 0.632 \frac{V}{R}$$

$i(t)$ is 63.2% of steady state (or) final vlg

Voltage across inductor V_L

$$V_L = L \cdot \frac{di}{dt}$$

$$= L \cdot \frac{d}{dt} \frac{V}{R}(1 - e^{-t/\tau})$$

$$= \frac{VL}{R} \quad -e^{-t/\tau} \cdot \left(-\frac{1}{\tau}\right)$$

$$= \frac{V\cancel{L}}{\cancel{R}}\left(-\frac{\cancel{R}}{\cancel{L}}\right)\left(-e^{-t/\tau}\right)$$

$$\boxed{V_L = +V e^{-t/\tau}}$$

$$V_R = iR$$

$$= \frac{V}{R} \times R \left[1 - e^{-t/\tau}\right]$$

$$\boxed{V_R = V\left[1 - e^{-t/\tau}\right]}$$

Power across resistor is,

$$P_R = I^2 R$$

$$= \frac{V^2}{R^2}(1 - e^{-t/\tau})^2 \cdot \cancel{R}$$

$$\boxed{P_R = \frac{V^2}{R}(1 - e^{-t/\tau})^2}$$

Power across inductor,

$$P_L = I V_L$$

$$= \frac{V}{R}(1 - e^{-t/\tau}) \cdot V e^{-t/\tau}$$

$$\boxed{P_L = \frac{V^2}{R}\left(e^{-t/\tau} - e^{-2t/\tau}\right)}$$

(18)

$$E_L = \int_0^\infty P\,dt = \frac{V^2}{R}\int_0^\infty e^{-t/\tau} - e^{-2t/\tau}\,dt.$$

$$= \frac{V^2}{R}\left[\frac{e^{-t/\tau}}{-1/\tau} - \frac{e^{-2t/\tau}}{-2/\tau}\right]_0^\infty$$

$$= \frac{V^2}{R}\left[0 - \left[-\tau + \frac{\tau}{2}\right]\right]$$

$$= \frac{V^2}{R}\left[-\left[-\frac{L}{R} + \frac{L}{2R}\right]\right]$$

$$= \frac{V^2}{R}\left[-\frac{-2L+L}{2R}\right]$$

Energy $E_L = \frac{V^2}{R}\left(\frac{L}{2R}\right) = \left(\frac{V}{R}\right)^2 \cdot \frac{L}{2}.$

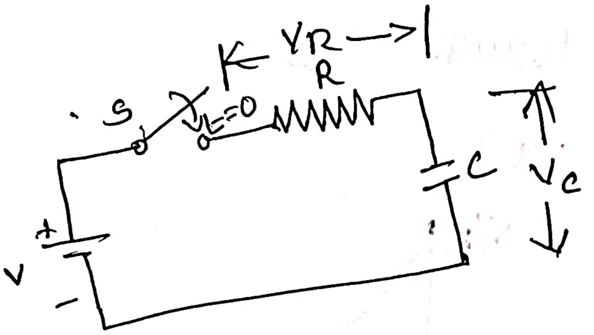
$$E_R = \int_0^\infty \frac{V^2}{R}\left(1 - e^{-t/\tau}\right)^2 dt = \infty$$

$$\eta = \frac{E_L}{E_L + E_R}$$

$$\boxed{\eta = 0}$$

Forced response of RC circuit

An RC circuit is connected to a DC supply voltage of 'V' volts through a switch. Assume the switch is closed at $t=0$. also assume initial charge and voltage in the capacitor is zero.



Applying KVL to above circuit,

$$V = V_R + V_C$$

$$V = iR + \frac{1}{C}\int i\,dt$$

Taking Laplace transform on both sides.

$$\frac{V}{S} = R\,I(S) + \frac{1}{C}\cdot\frac{I(S)}{S}$$

$$= I(S)\left[R + \frac{1}{CS}\right]$$

$$I(S) = \frac{V}{S\left(R+\frac{1}{CS}\right)}$$

$$= \frac{V}{\left[RS + \frac{1}{C}\right]} = \frac{V}{\left[RS + \frac{R}{RC}\right]}$$

$$I(S) = \frac{V}{R\left[S + \frac{1}{RC}\right]}$$

20

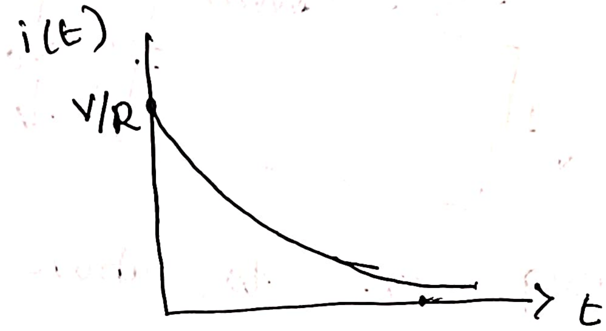
Taking inverse Laplace transform.

$$i(t) = \frac{V}{R} e^{-t/RC}$$

At $t = 0$ $i(t) = V/R$.

At $t = \infty$ $i(t) = 0$

The transient response of RC transient ckt can be drawn as



Voltage across resistor,

$$V_R = iR$$

$$= \frac{V}{\cancel{R}} \cdot e^{-t/RC} \times \cancel{R}$$

$$\boxed{V_R = V e^{-t/RC}}$$

Voltage across capacitor,

$$V_C = \frac{1}{C} \int_0^t i \, dt$$

$$= \frac{1}{C} \int_0^t \frac{V}{R} e^{-t/RC} \cdot dt$$

$$= \frac{V}{RC} \left[\frac{e^{-t/RC}}{-1/RC} \right]_0^t$$

$$= \frac{-V}{\cancel{RC}} \times \cancel{RC} \left[e^{-t/RC} - e^{0} \right]$$

$$= V \left[-e^{-t/RC} + 1 \right]$$

$$= V \left[1 - e^{-t/RC} \right]$$

$$\boxed{V_C = V \left[1 - e^{-t/\tau} \right]}$$

Time constant
$\tau = RC$

Power

$$P_C = VI = V \left[1 - e^{-t/\tau} \right] \frac{V}{R} e^{-t/RC}$$

$$= \frac{V^2}{R} \left[e^{-t/\tau} - e^{-2t/\tau} \right] \qquad [\because RC = \tau]$$

$$P_R = I^2 R$$

$$= \frac{V^2}{R^2} \left(e^{-t/\tau} \right)^2 \cancel{R}$$

$$\boxed{P_R = \frac{V^2}{R} e^{-2t/\tau}}$$

Energy

$$E_R = \int_0^\infty P_R \, dt = \frac{V^2}{R} \int_0^\infty e^{-2t/\tau} = \frac{V^2}{R} \left[\frac{e^{-2t/\tau}}{-2/\tau} \right]_0^\infty$$

$$= -\frac{V^2 \tau}{2R} \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{V^2 \tau}{2R} (-0 + 1) = \frac{V^2 RC}{2R}$$

(22)

$$\boxed{E_R = \frac{V^2 C}{2}}$$

$$E_C = \int_0^\infty P_C \, dt$$

$$= \frac{V^2}{R}\int_0^\infty \left(e^{-t/\tau} - e^{-2t/\tau}\right) dt$$

$$= \frac{V^2}{R}\left[\frac{e^{-t/\tau}}{-1/\tau} - \frac{e^{-2t/\tau}}{-2/\tau}\right]_0^\infty$$

$$= \frac{V^2}{R}\left[0 - \left(\frac{1}{-1/\tau} - \frac{1}{-2/\tau}\right)\right]$$

$$= \frac{V^2}{R}\left[\tau - \frac{\tau}{2}\right] = \frac{V^2}{R}\frac{\tau}{2} = \frac{V^2 RC}{2R}$$

$$\boxed{E_C = \frac{V^2 C}{2}}$$

Efficiency

$$\eta = \frac{E_C}{E_C + E_R}$$

$$= \frac{V^2 C/2}{V^2 C} = 1/2$$

$$\boxed{\eta = 50\%}$$