

IDFT PROBLEMS

1. Find the IDFT of the sequence

$$X(k) = \{5, 0, 1-j, 0, 1, 0, 1+j, 0\} \quad \text{here } N=8$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi nk}{N}} \quad 0 \leq n \leq N-1$$

$$x(n) = \frac{1}{8} \sum_{k=0}^7 X(k) e^{j \frac{2\pi kn}{8}} \quad 0 \leq n \leq 7$$

• $n=0$

$$x(0) = \frac{1}{8} [X(0) + X(1) + X(2) + X(3) + X(4) + X(5) + X(6) + X(7)]$$

$$x(0) = \frac{1}{8} [5 + 0 + 1 - j + 0 + 1 + 0 + 1 + j + 0]$$

$$x(0) = \frac{1}{8} [8] \quad \boxed{x(0) = 1}$$

• $n=1,$

$$x(1) = \frac{1}{8} \sum_{k=0}^7 X(k) e^{j \frac{2\pi k(1)}{8}} = \frac{1}{8} \sum_{k=0}^7 X(k) e^{j \frac{\pi k}{4}}$$

$$x(1) = \frac{1}{8} \left[X(0) + X(1) e^{j \frac{\pi}{4}} + X(2) e^{j \frac{2\pi}{4}} + X(3) e^{j \frac{3\pi}{4}} + X(4) e^{j \frac{4\pi}{4}} \right. \\ \left. + X(5) e^{j \frac{5\pi}{4}} + X(6) e^{j \frac{6\pi}{4}} + X(7) e^{j \frac{7\pi}{4}} \right]$$

$$\boxed{x(1) = 0.75}$$

- Similarly $x(2) = 0.5$ $x(3) = 0.25$ $x(4) = 1$
 $x(5) = 0.75$ $x(6) = 0.5$ $x(7) = 0.25$
- **Final Answer:**
- $x(n) = \{1, 0.75, 0.5, 0.25, 1, 0.75, 0.5, 0.25\}$

Find the IDFT of $X(k) = \{4, 2, 0, 4\}$ directly.

Soln:

Given DFT is $X(k) = \{4, 2, 0, 4\}$. The IDFT of $X(k)$, i.e. $x(n)$ is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk} = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j(2\pi/N)nk}$$

i.e.
$$x(n) = \frac{1}{4} \sum_{k=0}^3 X(k) e^{j(\pi/2)nk}$$

$$\therefore x(0) = \frac{1}{4} \sum_{k=0}^3 X(k) e^0 = \frac{1}{4} [X(0) + X(1) + X(2) + X(3)]$$

$$= \frac{1}{4} [4 + 2 + 0 + 4] = 2.5$$

$$x(1) = \frac{1}{4} \sum_{k=0}^3 X(k) e^{j(\pi/2)k} = \frac{1}{4} [X(0) + X(1)e^{j(\pi/2)} + X(2)e^{j\pi} + X(3)e^{j(3\pi/2)}]$$

$$= \frac{1}{4} [4 + 2(0 + j) + 0 + 4(0 - j)] = 1 - j0.5$$

$$x(2) = \frac{1}{4} \sum_{k=0}^3 X(k) e^{j\pi k} = \frac{1}{4} [X(0) + X(1)e^{j\pi} + X(2)e^{j2\pi} + X(3)e^{j3\pi}]$$

$$= \frac{1}{4} [4 + 2(-1 + j0) + 0 + 4(-1 + j0)] = -0.5$$

$$x(3) = \frac{1}{4} \sum_{k=0}^3 X(k) e^{j(3\pi/2)k} = \frac{1}{4} [X(0) + X(1)e^{j(3\pi/2)} + X(2)e^{j3\pi} + X(3)e^{j(9\pi/2)}]$$

$$= \frac{1}{4} [4 + 2(0 - j) + 0 + 4(0 + j)] = 1 + j0.5$$

$$x_3(n) = \{2.5, 1 - j0.5, -0.5, 1 + j0.5\}$$

Find the IDFT of $X(k) = \{1, 0, 1, 0\}$.

Soln:

Given $X(k) = \{1, 0, 1, 0\}$

Let the IDFT of $X(k)$ be $x(n)$.

$$\therefore x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk} = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j(2\pi/N)nk}$$

$$x(0) = \frac{1}{4} \sum_{k=0}^3 X(k) e^0 = \frac{1}{4} [X(0) + X(1) + X(2) + X(3)] = \frac{1}{4} [1 + 0 + 1 + 0] = 0.5$$

$$\begin{aligned} x(1) &= \frac{1}{4} \sum_{k=0}^3 X(k) e^{j(\pi/2)k} = \frac{1}{4} [X(0) + X(1)e^{j(\pi/2)} + X(2)e^{j\pi} + X(3)e^{j(3\pi/2)}] \\ &= \frac{1}{4} [1 + 0 + e^{j\pi} + 0] = \frac{1}{4} [1 + 0 - 1 + 0] = 0 \end{aligned}$$

$$\begin{aligned} x(2) &= \frac{1}{4} \sum_{k=0}^3 X(k) e^{j\pi k} = \frac{1}{4} [X(0) + X(1)e^{j\pi} + X(2)e^{j2\pi} + X(3)e^{j3\pi}] \\ &= \frac{1}{4} [1 + 0 + e^{j2\pi} + 0] = \frac{1}{4} [1 + 0 + 1 + 0] = 0.5 \end{aligned}$$

$$\begin{aligned} x(3) &= \frac{1}{4} \sum_{k=0}^3 X(k) e^{j(3\pi/2)k} = \frac{1}{4} [X(0) + X(1)e^{j(3\pi/2)} + X(2)e^{j3\pi} + X(3)e^{j(9\pi/2)}] \\ &= \frac{1}{4} [1 + 0 + e^{j3\pi} + 0] = \frac{1}{4} [1 + 0 - 1 + 0] = 0 \end{aligned}$$

The IDFT of $X(k) = \{1, 0, 1, 0\}$ is $x(n) = \{0.5, 0, 0.5, 0\}$.