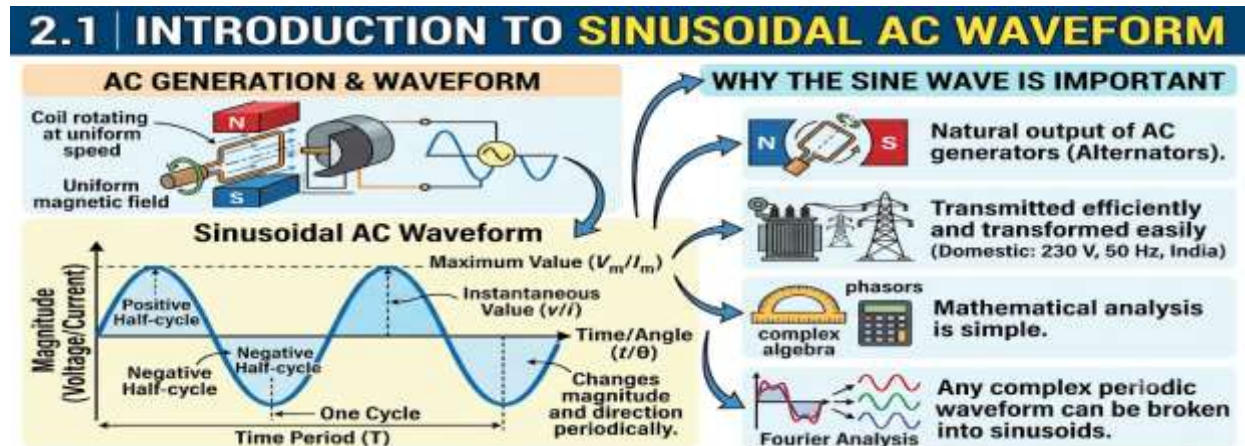


Ac circuits and magnetic circuits



2.1 Introduction to Sinusoidal AC Waveform

Alternating Current (AC) is a current that changes its magnitude and direction periodically with time. Unlike Direct Current (DC), which is constant, AC varies smoothly following a sine-wave pattern when generated by a rotating coil in a uniform magnetic field. This sinusoidal shape is the most important waveform in electrical engineering because it is the natural output of AC generators (alternators) and is the standard form of electric power supplied by utilities worldwide, including 230 V, 50 Hz domestic supply in India.

Why the sine wave is important

- It is the natural output of a coil rotating at uniform speed in a magnetic field.
- It can be transmitted efficiently and transformed easily using transformers.
- Mathematical analysis (phasors, complex algebra) becomes simple for sinusoidal quantities.
- Any complex periodic waveform can be broken into a sum of sinusoids (Fourier analysis).

2 Representation of an Alternating Quantity

Equation form

A sinusoidal alternating voltage or current can be expressed mathematically as a function of time:

$$v(t) = V_m \sin(\omega t) \quad i(t) = I_m \sin(\omega t \pm \phi)$$

where V_m and I_m are the maximum (peak) values, $\omega = 2\pi f$ is the angular frequency in rad/s, f is the frequency in Hz, t is time in seconds, and ϕ is the phase angle between voltage and current (zero for a purely resistive circuit).

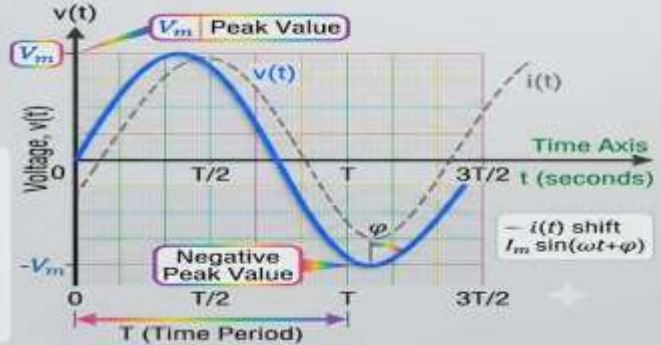
Section 2: Representation of an Alternating Quantity – Equation Form

Fig 2.1: One cycle of a sinusoidal waveform

$$v(t) = V_m \sin(\omega t)$$

$$i(t) = I_m \sin(\omega t \pm \phi)$$

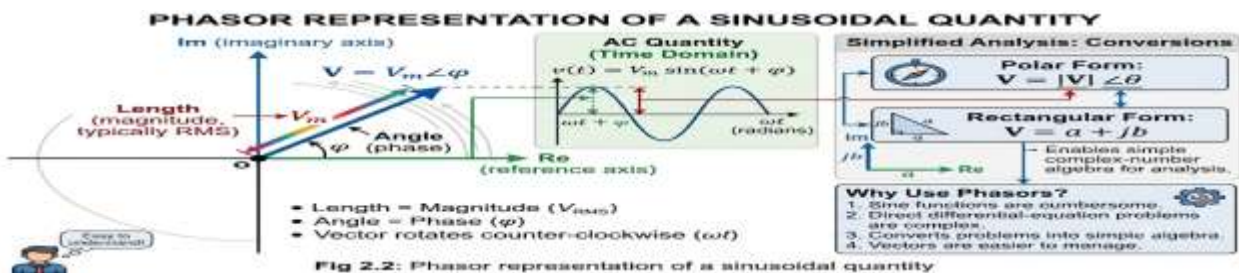
$v(t), i(t)$: Instantaneous voltage & current
 V_m, I_m : Maximum (peak) values
 ω : Angular frequency (rad/s)
 $\omega = 2\pi f$
 f : Frequency (Hz)
 t : Time (s)
 ϕ : Phase angle
 (zero for purely resistive circuit)



Phasor form

Since working with sine functions directly is cumbersome, alternating quantities are represented as phasors — rotating vectors whose length equals the magnitude (usually the RMS value) and whose angle equals the phase. This converts differential-equation problems into simple complex-number algebra.

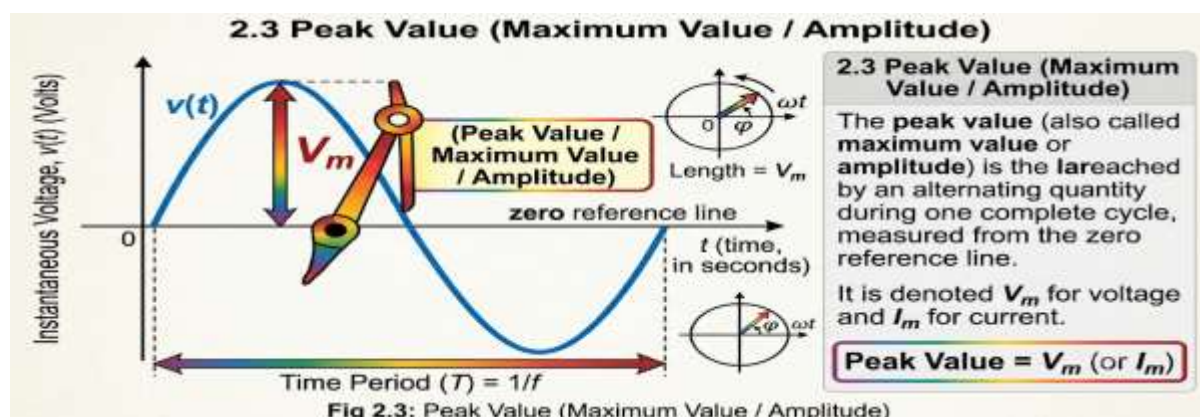
$$V = |V| \angle \theta \quad (\text{polar form}) \quad V = a + jb \quad (\text{rectangular form})$$



2.3 Peak Value (Maximum Value / Amplitude)

The peak value (also called maximum value or amplitude) is the largest value reached by an alternating quantity during one complete cycle, measured from the zero reference line. It is denoted V_m for voltage and I_m for current.

$$\text{Peak value} = V_m \quad (\text{or } I_m)$$



2.4 RMS (Root Mean Square) Value

The RMS value, also called the effective value, is the AC equivalent of a steady DC value: it is that constant DC voltage or current which would produce the same amount of heat in a resistor as the actual alternating quantity produces over one complete cycle. All AC voltmeters, ammeters, and equipment ratings (e.g. '230 V AC') refer to RMS values unless stated otherwise.

Derivation outline (mid-ordinate / mean-of-squares method)

- Square the instantaneous values of the waveform over one cycle — this makes all values positive.
- Find the mean (average) of these squared values.
- Take the square root of that mean — hence 'Root of the Mean of the Squares'.

$$V_{\text{rms}} = \sqrt{\left[(1/T) \int_0^T v^2(t) dt \right]}$$

For a pure sine wave $v(t) = V_m \sin(\omega t)$, evaluating this integral gives the well-known result:

$$V_{\text{rms}} = V_m / \sqrt{2} \approx 0.707 V_m \quad I_{\text{rms}} = I_m / \sqrt{2} \approx 0.707 I_m$$

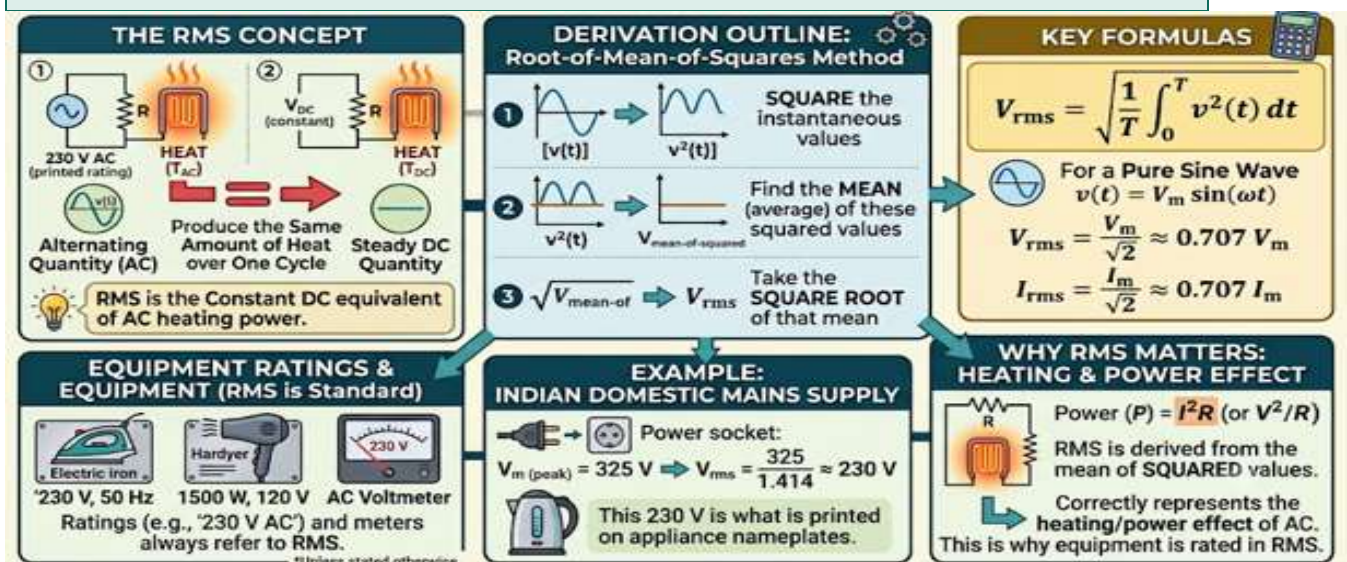
Example: Indian domestic mains supply

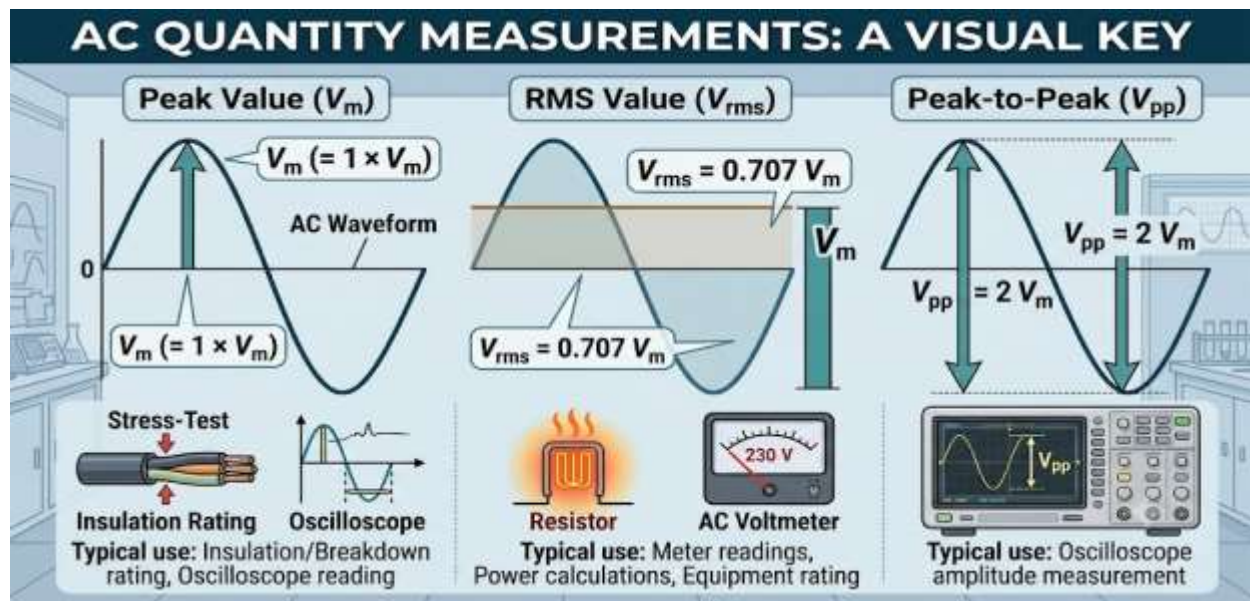
$$V_m = 325 \text{ V (peak)} \Rightarrow V_{\text{rms}} = 325 / 1.414 \approx 230 \text{ V}$$

This 230 V is what is printed on appliance nameplates.

Why RMS matters

Power delivered to a resistor depends on the square of voltage/current ($P = I^2 R$). Because RMS is derived from the mean of the SQUARED values, it correctly represents the heating/power effect of AC — this is precisely why equipment is rated in RMS volts and amps, not peak or average values.



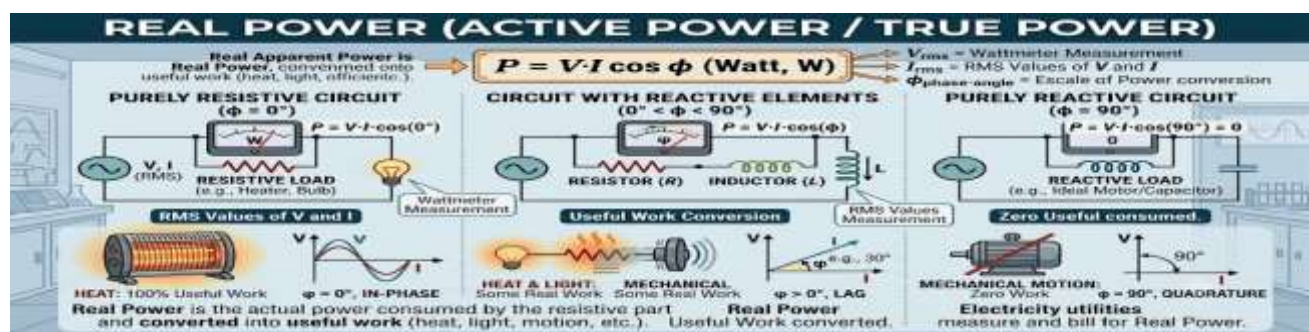


2.5 Real Power (Active Power)

Real power, also called active power or true power, is the power that is actually consumed by the resistive part of a circuit and converted into useful work — heat, light, mechanical motion, etc. It is the power measured by a wattmeter and billed by electricity utilities.

$$P = VI \cos \phi \text{ (Watt, W)}$$

where V and I are RMS values of voltage and current, and ϕ is the phase angle between them. In a purely resistive circuit ($\phi = 0^\circ$), all the apparent power is real power. In a circuit with only reactive elements ($\phi = 90^\circ$), real power is zero.



2.6 Reactive Power

Reactive power is the power that oscillates back and forth between the source and the reactive components (inductors and capacitors) of a circuit. It performs no useful work but is essential for establishing the magnetic and electric fields needed for inductors, transformers, and motors to operate.

$$Q = VI \sin \phi \text{ (Volt-Ampere Reactive, VAR)}$$



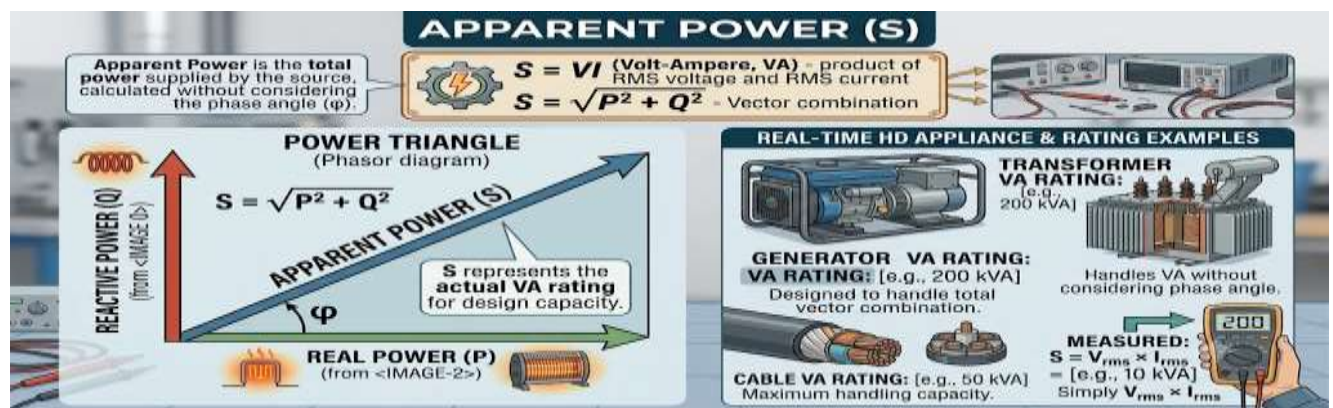
2.7 Apparent Power

Apparent power is the total power supplied by the source, obtained simply as the product of RMS voltage and RMS current, without considering the phase angle. It represents the actual VA (volt-ampere) rating that generators, transformers, and cables must be designed to handle.

$$S = VI \quad (\text{Volt-Ampere, VA})$$

Apparent power is the vector combination of real and reactive power:

$$S = \sqrt{P^2 + Q^2}$$



2.8 Power Triangle and Power Factor

The relationship between real power (P), reactive power (Q), and apparent power (S) is conveniently shown using a right-angled triangle called the power triangle, where P is the base (horizontal), Q is the height (vertical), and S is the hypotenuse.

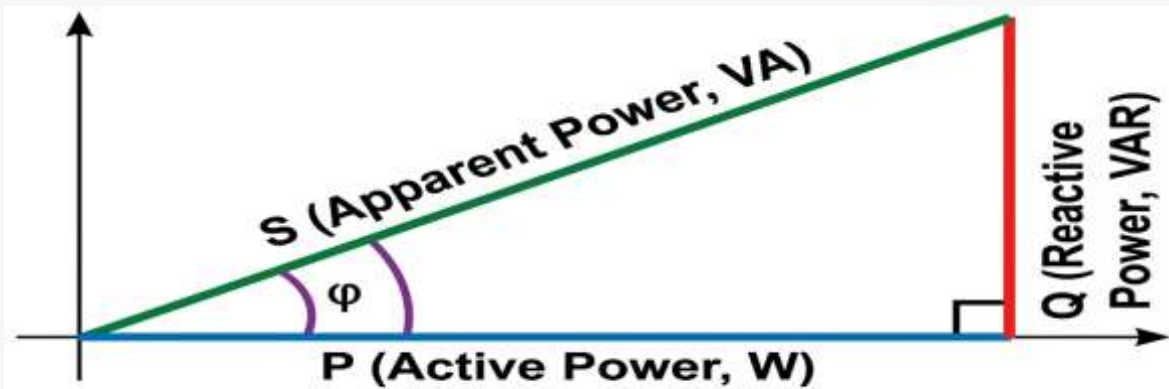


Fig. 2.3: Power Triangle Showing P, Q, S and ϕ

$$S^2 = P^2 + Q^2 \quad P = S \cos \phi \quad Q = S \sin \phi$$

Power Factor

Power factor is defined as the cosine of the phase angle ϕ between voltage and current. It represents the fraction of the apparent power that is actually converted into useful (real) work, and indicates how effectively electrical power is being utilised.

$$\text{Power Factor, } \cos \phi = P / S = \text{Real Power} / \text{Apparent Power}$$

