Faraday's law:

Michael Faraday proved that a magnetic field could produce a current.

According to Faraday's experiment, a static magnetic field cannot produce any current flow. But with a time varying field an electromotive force[e.m.f] induces which may drive a current in a closed path. This e.m.f is nothing but a voltage that induces from changing magnetic fields. Faraday discovered that the induced e.m.f in any closed circuit is equal to the time rate of change of magnetic flux linking with the closed circuit.

Faraday's Law can be stated as,

$e = -N\frac{d\varphi}{dt}$ volts...................... (1)

$N \rightarrow$ number of turns in the circuit

$e \rightarrow$ induced emf, $\lambda = N\Psi \rightarrow flux\ linkage$

For a single turn circuit , ie, N=1.

(1) $\Longrightarrow$ e=$-\frac{d\varphi}{dt}$ volts................. (2)

- $\rightarrow$ indicates the direction of induced e.m.f [induce e.m.f opposing the flux producing it]

The e.m.f produce a current which will produce a magnetic field which will oppose the original field.

In 1834, Henri Frederic Emile Lenz postulated the law. According to Lenz's law, the induced e.m.f acts to produce an opposing flux.

Let as consider Faraday's law. The induced e.m.f is a scalar quantity measured in volts. Thus the induced e.m.f is given by

e=$\oint E.dl$........................③

③→ indicates a voltage about a closed path such that if any part of the path is changed the e.m.f will also change.

The magnetic flux ϕ passing through a specified area is given by,

$$\phi = \int_s B.ds \text{ ....©}$$

where B= magnetic flux density

using eqn© ②⟹

$$e = -\frac{d}{dt}\int_s B.ds \text{ ....④}$$

from eqn ③&④

$$e = \oint E.dl = -\frac{d}{dt}\int_s B.ds \text{ ....⑤}$$

There are two conditions for the induced e.m.f.

1)The closed circuit in which e.m.f is induced is stationary and the magnetic flux is sinusoidally varying with time. In eqn ⑤ magnetic flux density is varying with time. We can use partial derivative to define relationship as 'B' may be changing with co-ordinates as well as time.

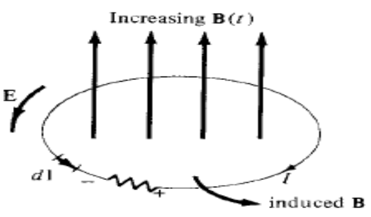

Fig: Induced emf due to a stationary loop in a time varying B field.

$$⑤ \Longrightarrow \quad \oint E.dl = -\int_s \frac{\partial B}{\partial t}.ds \text{.................⑥}$$

This is similar to transformer action and e.m.f is called transformer e.m.f using stokes theorem a line integral can be converted to surface integral.

$$\int_s (\nabla \times E)ds = \oint E.dl = -\int_s \frac{\partial B}{\partial t}.ds \text{.......................⑦}$$

In⑦ both surface integrals taken over identical surfaces,

$$\therefore (\nabla \times E)ds = -\frac{\partial B}{\partial t}.ds$$

Hence

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{..........................} ⑧$$

Equation ⑧ represents Maxwell's equation. if 'B' is not varying with time.

⑥⟹ $\oint E.dl = 0$, and

$$\nabla \times E = 0$$

The above results are same as in electrostatics.

2) Secondly magnetic field is stationary constant not varying with time. while the closed circuit is revolved to get the relative motion between them. This action is similar to generator action hence the induced e.m.f is called motional or generator e.m.f

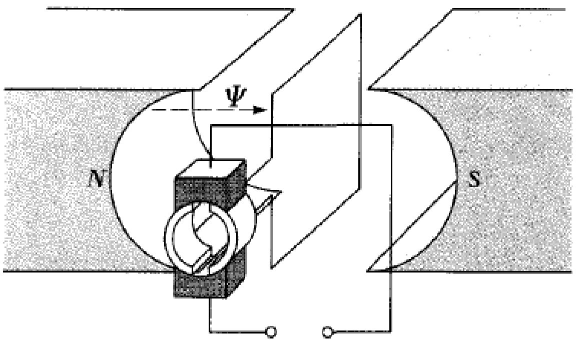

Fig: A direct-current machine

Consider a charge Q is moved in a magnetic field B at a velocity v. Then the force on a charge is given by

$$\overline{F} = Q\, v \times B \text{.........................} ⑨$$

The motional electric field intensity is defined as force per unit charge.

⑨⟹ $\overline{E}m = \frac{\overline{F}}{Q} = v \times B$ .... ⑩

Thus the induced e.m.f is given by

$$\oint Em\, dl = \oint (v \times B) dl \text{....................⑪}$$

Eqn ⑪ represents total e.m.f induced. When a conductor is moved in a uniform constant magnetic field.
If the magnetic flux density is varying with time, the induced e.m.f is the combination of transformer e.m.f and generator e.m.f

$$\boxed{\oint E.dl = -\int_s \frac{\partial B}{\partial t}.ds + \oint (v \times B).dl} \text{.....................⑫}$$