4.7 NOISY QUANTUM COMMUNICATION

Introduction

Quantum communication transfers quantum information using physical carriers such as photons, trapped ions or superconducting-qubit links. Unlike classical bits, a qubit can exist in a superposition $\alpha|0\rangle + \beta|1\rangle$. During transmission, interaction with the environment, imperfect devices, loss and thermal effects can change amplitudes or phases. These unwanted changes are called quantum noise.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1$$

A noisy channel is represented by a completely positive, trace-preserving (CPTP) map. In the operator-sum representation:

$$\mathcal{E}(\rho) = \Sigma_i E_i \rho E_i^\dagger, \quad \Sigma_i E_i^\dagger E_i = I$$

Sources of Noise in Quantum Communication

The major sources of noise are:

1. Environmental Noise

- Interaction between quantum particles and the surrounding environment.
- Causes unwanted changes in quantum states.
- Affects the accuracy of quantum information transmission.

2. Decoherence

- Loss of quantum coherence due to interaction with the environment.
- Destroys the phase relationship between quantum states.
- Reduces the reliability of quantum communication.

3. Photon Loss

- Photons may be absorbed or scattered during transmission.
- Particularly important in optical fibre communication.
- Reduces the amount of information received.

4. Thermal Noise

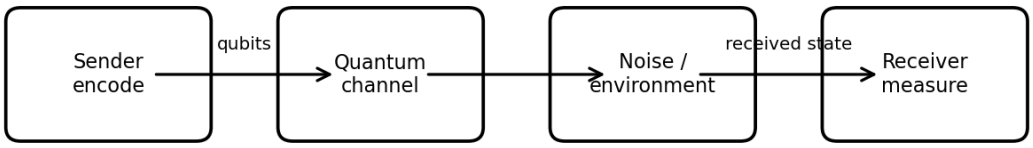
- Caused by temperature-related disturbances.
- Can affect the stability of quantum systems.
- Introduces errors during quantum operations.

5. Measurement Errors

- Occur when quantum measurements produce incorrect outcomes.
- Affect the reliability of the received information.

1.2 Quantum communication channel

Quantum communication channel



The sender prepares and encodes a quantum state. The state passes through a physical channel, where noise may act. The receiver applies suitable decoding and measurement operations. Classical communication may accompany the quantum link for basis reconciliation, syndrome exchange or key sifting.

1.3 Classification of quantum noise

Noise category	Description	Typical cause / effect
Bit-flip (X)	Changes $\|0\rangle \leftrightarrow \|1\rangle$	Bit-energy disturbance; wrong computational-basis value
Phase-flip (Z)	Changes relative phase; $\|1\rangle$ gains a minus sign	Phase instability; damages superposition coherence
Bit-phase flip (Y)	Combines bit and phase flip	Composite Pauli error
Depolarizing	Random Pauli disturbance; state moves towards maximally mixed state	Uncontrolled gate / channel errors
Amplitude damping	Excited state decays towards ground state	Energy loss, spontaneous emission
Phase damping	Suppresses off-diagonal coherence without population transfer	Environmental phase information leakage
Erasure / loss	Qubit is lost or flagged as absent	Photon absorption, scattering, detector inefficiency

1.4 Bit-flip channel

A bit-flip error applies the Pauli-X operator with probability p and does nothing with probability 1−p.

$$E_0 = \sqrt{(1-p)}\, I, \qquad E_1 = \sqrt{p}\, X, \qquad X = [[0,1],[1,0]]$$

$$\mathcal{E}_x(\rho) = (1-p)\rho + pX\rho X$$

For a basis state, $|0\rangle$ becomes $|1\rangle$ when the error occurs and $|1\rangle$ becomes $|0\rangle$. For a superposition, both basis amplitudes are exchanged.

1.5 Phase-flip channel

The phase-flip channel applies Pauli-Z with probability p. It preserves computational-basis populations but changes the relative phase of a superposition.

$$E_0 = \sqrt{(1-p)}\, I, \qquad E_1 = \sqrt{p}\, Z, \qquad Z = [[1,0],[0,-1]]$$

$$\mathcal{E}z(\rho) = (1-p)\rho + pZ\rho Z$$

For example, $\alpha|0\rangle+\beta|1\rangle$ becomes $\alpha|0\rangle-\beta|1\rangle$. This error is important because quantum interference depends on relative phase.

1.6 Depolarizing channel

A depolarizing channel replaces a qubit by the maximally mixed state with probability p, or equivalently applies a random Pauli error according to a chosen convention. One common single-qubit convention is:

$$\mathcal{E}dep(\rho) = (1-p)\rho + (p/3)(X\rho X + Y\rho Y + Z\rho Z)$$

The Bloch vector is shortened towards the origin. The output becomes less distinguishable from a maximally mixed state I/2 as noise increases. Some texts use a different parameter convention; state the convention used in an examination answer.

1.7 Amplitude-damping channel

Amplitude damping models energy relaxation from the excited state $|1\rangle$ to the ground state $|0\rangle$. The probability of decay is γ.

$$E_0 = [[1,0],[0,\sqrt{(1-\gamma)}]], \qquad E_1 = [[0,\sqrt{\gamma}],[0,0]]$$

$$\mathcal{E}AD(\rho) = E_0\rho E_0\dagger + E_1\rho E_1\dagger$$

Unlike a simple phase flip, amplitude damping changes the population of the excited state. It models spontaneous emission and energy loss in many physical systems.

Quantum noise diagrams

Bit-flip channel (X)

$|0\rangle \rightarrow |1\rangle$

$|1\rangle \rightarrow |0\rangle$

Phase-flip channel (Z)

$|0\rangle \rightarrow |0\rangle$

$|1\rangle \rightarrow -|1\rangle$

Depolarizing channel

State is replaced by a random

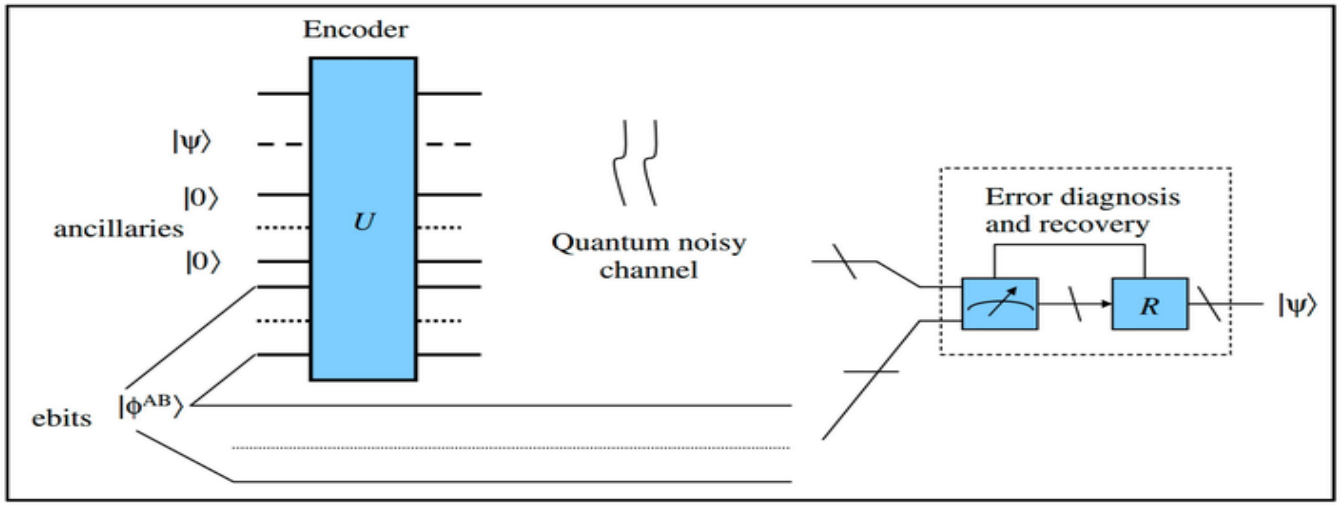
Pauli error with probability p

Amplitude-damping channel

$|1\rangle \rightarrow |0\rangle$ + emitted energy

$|0\rangle \rightarrow |0\rangle$

Working of Noisy Quantum Communication



The working process involves the following steps:

1. **Quantum State Preparation:** The sender prepares the required quantum state.

2. **Encoding:** The quantum information may be encoded using quantum error correction techniques.
3. **Transmission:** The encoded quantum state is transmitted through a communication channel.
4. **Noise Introduction:** The channel may introduce bit-flip, phase-flip or other errors.
5. **Error Detection:** The receiver checks for errors using suitable quantum error detection methods.
6. **Error Correction:** Appropriate correction operations are applied when possible.
7. **Decoding:** The original quantum information is recovered.

Comparison of major noise channels

Channel	**Operator / parameter**	**Main physical effect**	**Can affect populations?**
Bit flip	X; probability p	Exchanges $\|0\rangle$ and $\|1\rangle$	Yes
Phase flip	Z; probability p	Changes relative phase	No, in computational basis
Depolarizing	Random X, Y, Z; p	Drives state towards I/2	Generally yes
Amplitude damping	Kraus operators; γ	Excited state decays to ground state	Yes

Methods to reduce noise

- Quantum error-correcting codes add redundancy and recover logical information.
- Fault-tolerant gates prevent one physical fault from spreading uncontrollably.
- Dynamical decoupling uses pulse sequences to average selected environmental interactions.
- Better shielding, cryogenic control, low-loss optical fibre and improved detectors reduce physical noise.
- Quantum repeaters and entanglement purification can extend communication distance.

Applications

1. Quantum Key Distribution (QKD).
2. Secure quantum communication.
3. Quantum internet.
4. Distributed quantum computing.
5. Quantum teleportation.
6. Quantum networks.

1.9 Advantages

- Enables quantum information transmission.
- Supports secure quantum communication.
- Helps develop quantum networks.
- Encourages error-resistant quantum systems.

Thus Noisy quantum communication is governed by quantum channels that describe how physical disturbances transform density matrices. Bit-flip, phase-flip, depolarizing and amplitude-damping channels represent important error mechanisms. Understanding these models is essential for designing reliable quantum links and error-correction strategies.