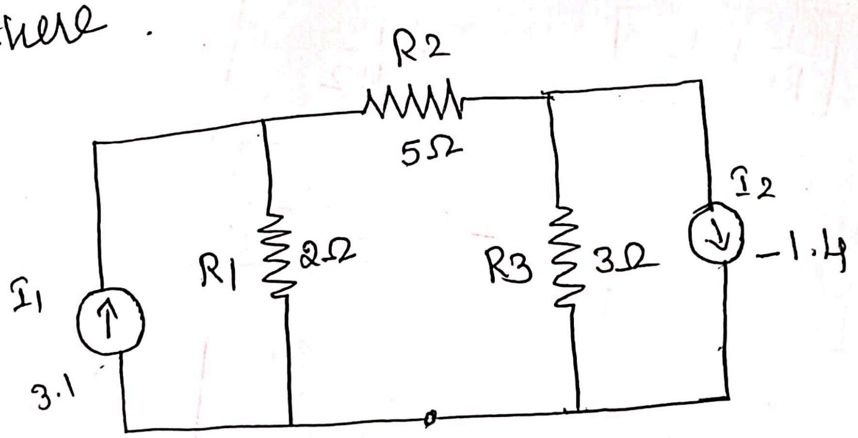
Node analysis (Branch current method)

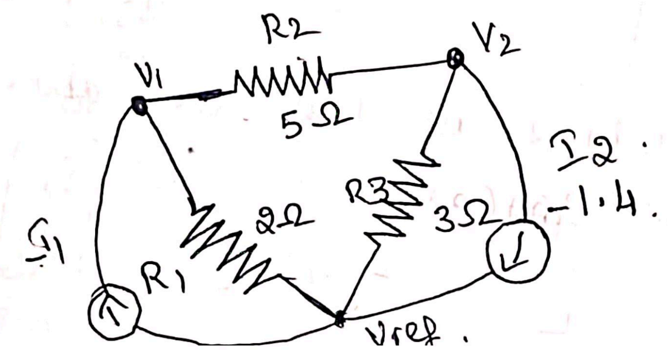
This method is used to determine voltage b/w nodes. It has the following steps.

1) Convert v/g source to current source
2) Identify the nodes & choose the reference node.
3) choose the reference node such that maximum number of components are connected.
4) Give number to other nodes and correspon-ding voltage V_1, V_2 ----
5) if n nodes are there (n-1) expressions will be there.

1)



The ckt can be re-drawn as.



(35) Apply KCL to node 1

$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$I_1 = \frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2}$$

$$I = V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2 \frac{1}{R_2} \quad \text{—— (1)}$$

Apply KCL at N_2.

$$I_2 + \frac{V_2}{R_3} + \frac{V_2 - V_1}{R_2} = 0.$$

$$\frac{V_2}{R_3} + \frac{V_2}{R_2} - \frac{V_1}{R_2} = -I_2.$$

$$-V_1\left[\frac{1}{R_2}\right] + V_2\left[\frac{1}{R_3} + \frac{1}{R_2}\right] = -I_2 \quad \text{—— (2)}$$

$$+V_1\left(\frac{1}{R_2}\right) - V_2\left[\frac{1}{R_3} + \frac{1}{R_2}\right] = (+) I_2$$

$$\boxed{\begin{bmatrix} \left[\frac{1}{R_1} + \frac{1}{R_2}\right] & -\frac{1}{R_2} \\ +\frac{1}{R_2} & -\left[\frac{1}{R_2} + \frac{1}{R_3}\right] \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{array}{c|l} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} & \begin{array}{l} 3.1 \\ -1.4 \end{array} \end{array}}$$

$R_1 = 2,\ R_2 = 5$
$R_3 = 3$
$I_1 = 3.1 \quad I_2 = -1.4$

$$\Delta = \begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ \frac{1}{R_2} & -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{vmatrix} = \begin{vmatrix} \frac{1}{2} + \frac{1}{5} & -\frac{1}{5} \\ \frac{1}{5} & -\left(\frac{1}{5} + \frac{1}{3}\right) \end{vmatrix}$$

$$= \begin{vmatrix} 0.7 & -0.2 \\ 0.2 & -0.53 \end{vmatrix}$$

$$\Delta = -0.371 + 0.04$$

$$\Delta = \cancel{0.297}\ 0.331$$

If conductance are given then above matrix become

$$\begin{bmatrix} (G_1 + G_2) & -G_2 \\ G_2 & -(G_2 + G_3) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \begin{bmatrix} 3.1 & -\frac{1}{5} \\ -1.4 & -\left[\frac{1}{5} + \frac{1}{3}\right] \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 3.1 & -0.2 \\ -1.4 & -0.53 \end{bmatrix}}{0.297}$$

$$V_1 = \frac{-1.643 - 0.28}{\cancel{0.297}\ 0.331} = \frac{1.922}{0.33}$$

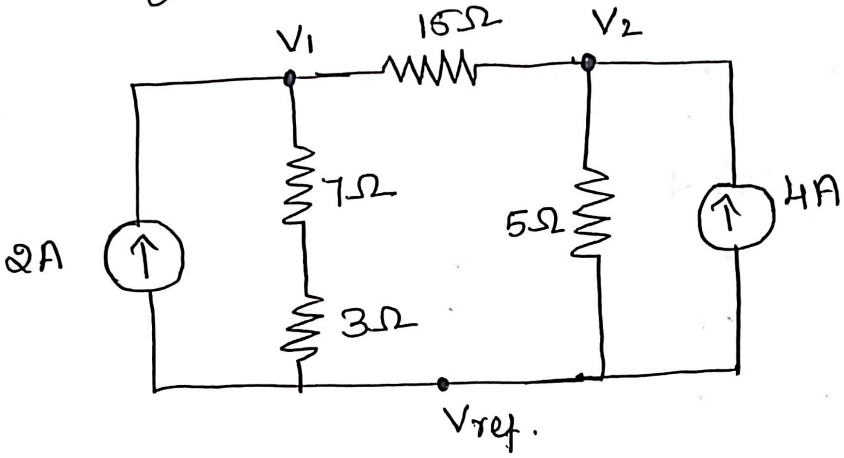
$V_1 = 5.8\,V$. (should be 5)

$$V_2 = \frac{\Delta V_2}{\Delta} = \frac{\begin{bmatrix} 0.7 & 3.1 \\ 0.2 & -1.4 \end{bmatrix}}{0.331} = \frac{0.98 - 0.62}{0.331}$$

$$= \frac{0.36}{0.331}$$

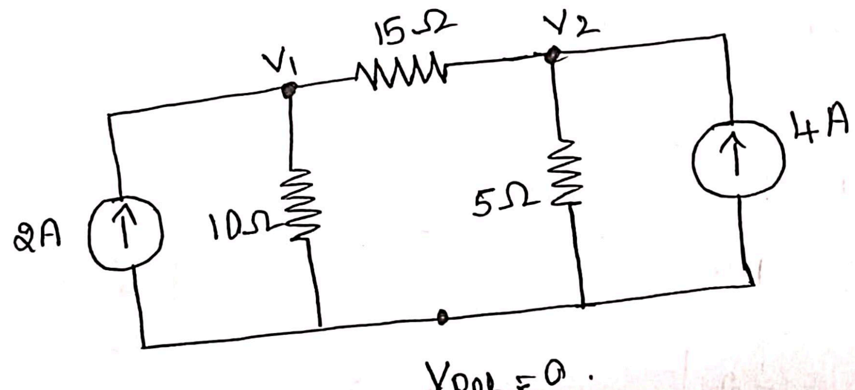
$\boxed{V_2 = 1.08}$ (should be 2)

2) Determine the current flowing left to right through the 15Ω resistor of fig 4.2a.



Soln

The circuit can be redrawn as.



$5 \,|\, 15, 10$
$\;\; 3, 2$

KCL at Node 1

$$2A = \frac{V_1}{10} + \frac{V_1 - V_2}{15}$$

$$= \frac{V_1}{10} + \frac{V_1}{15} - \frac{V_2}{15}.$$

$$2A = V_1\left[\frac{1}{10} + \frac{1}{15}\right] - \frac{V_2}{15} \quad \text{——— (1).}$$

Rearranging

$$2 = \left[\frac{3+2}{30}\right]V_1 - \frac{2V_2}{30}$$

$$60 = 5V_1 - 2V_2. \quad \text{——— (2)}$$

KCL at node 2

$$4 = \frac{V_2 - V_1}{15} + \frac{V_2}{5} \quad \text{——— (3)}$$

$$4 = \frac{V_2 - V_1}{15} + \frac{3V_2}{15}$$

$$60 = V_2 - V_1 + 3V_2$$

$$60 = -V_1 + 4V_2 \quad \text{—— (2)}$$

$$5V_1 - 2V_2 = 60 \quad \text{—— (A)}$$

$$-V_1 + 4V_2 = 60 \quad \text{—— (B)}$$

$$\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{\begin{bmatrix} 60 & -2 \\ 60 & +4 \end{bmatrix}}{\begin{vmatrix} 5 & -2 \\ -1 & 4 \end{vmatrix}} = \frac{+240 + 120}{20 - 2} = \frac{360}{18} = 20V.$$

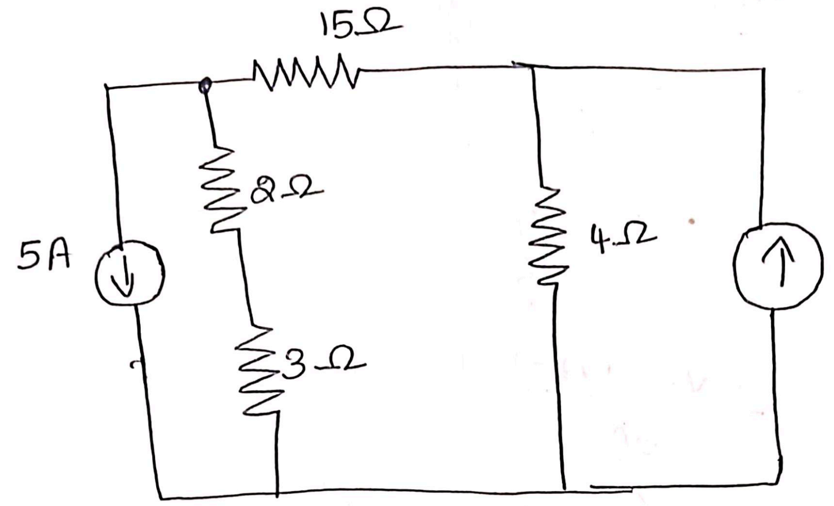
$$\boxed{V_1 = 20\,V}$$

$$V_2 = \frac{\Delta V_2}{\Delta} = \frac{\begin{bmatrix} 5 & 60 \\ -1 & 60 \end{bmatrix}}{\begin{bmatrix} 5 & -2 \\ -1 & 4 \end{bmatrix}} = \frac{300 + 60}{20 - 2} = \frac{360}{18} = 20V.$$

$$\boxed{V_2 = 20V}$$

ct through 15Ω is $\frac{V_1 - V_2}{15} = \frac{0}{15} = 0$, ∴ No current flow.

3)



Ans

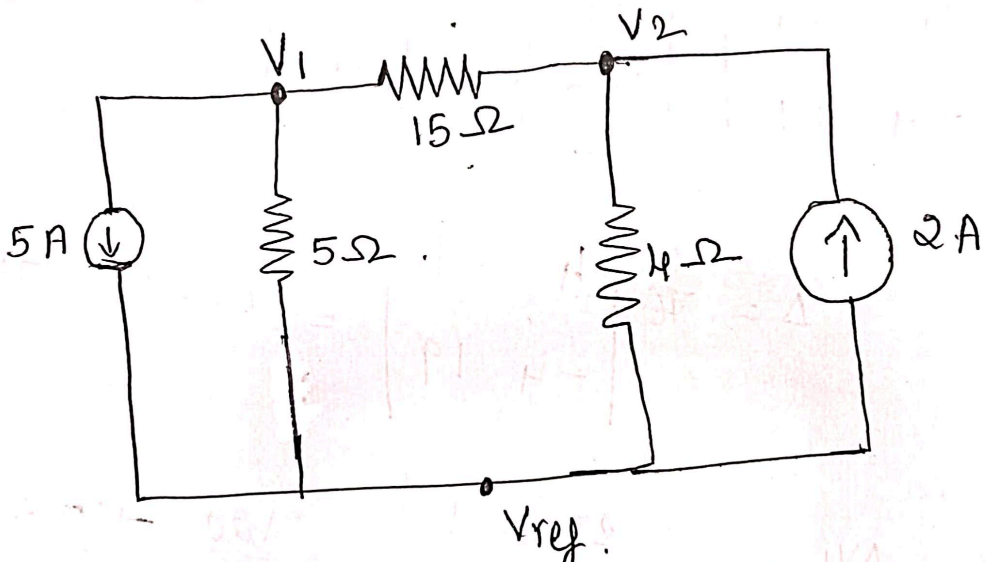
$V_1 = \frac{-145}{8} V$

$V_2 = \frac{5}{2} V.$

For the circuit shown determine V_1 and V_2.

Soln

The circuit can be re-drawn as.



* Mark the nodes,

* Apply KCL to node 1,

$$5 + \frac{V_1}{5} + \frac{V_1 - V_2}{15} = 0.$$

$$\frac{\cancel{75+} 3V_1 + V_1 - V_2}{15} = \cancel{0} -5$$

$$4V_1 - V_2 = -75 \quad \text{——} \quad (1)$$

(39) Apply KVL at N_2.

$$2 = \frac{V_2}{4} + \frac{V_2 - V_1}{15}$$

$$= \frac{V_2}{4} + \frac{V_2}{15} - \frac{V_1}{15}$$

$$2 = \frac{15V_2 + 4V_2 - 4V_1}{60}$$

$$19V_2 - 4V_1 = 120 \quad \text{--- (2)}$$

$$-4V_1 + 19V_2 = 120 \quad \text{--- (2)}.$$

$$\begin{bmatrix} 4 & -1 \\ -4 & 19 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -75 \\ 120 \end{bmatrix}$$

$$\Delta = \cancel{76} \begin{vmatrix} 4 & -1 \\ -4 & 19 \end{vmatrix} = 76 - 4 = 72.$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{\begin{vmatrix} -75 & -1 \\ 120 & 19 \end{vmatrix}}{72} = \frac{\cancel{120}}{\cancel{72}} = \frac{-1425 + 120}{72} = \frac{1305 \; \cancel{1545}}{72}.$$

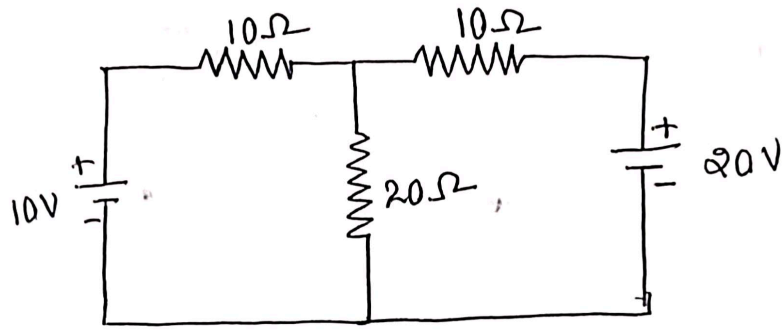
$$\boxed{V_1 = -18.125\ V}$$

$$V_2 = \frac{\Delta V_2}{\Delta}$$

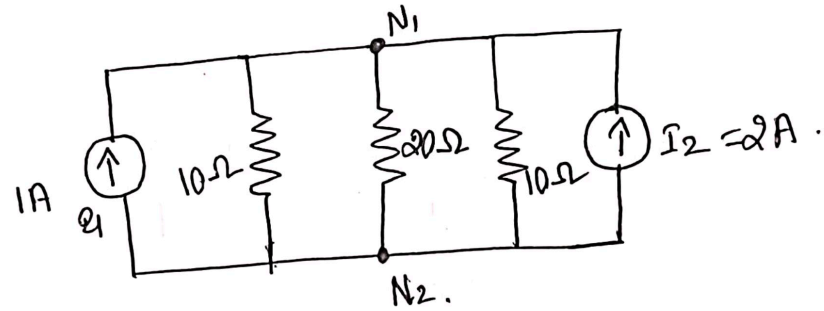
$$= \frac{\begin{vmatrix} 4 & -75 \\ -4 & 120 \end{vmatrix}}{72} = \frac{480 - 300}{72} = \frac{180}{72}$$

$$\boxed{V_2 = 2.5V}$$

4) using nodal analysis determine the current in the 20Ω resistor. (40)



Convert vtg source to current source



$I_1 = V/R.$

$= \frac{10V}{10\Omega} = 1A$

$I_2 = \frac{20V}{10\Omega} = 2A$

Apply KVL to N_1

$$1 + 2 = \frac{V_1}{10} + \frac{V_1}{20} + \frac{V_1}{10}$$

$$3 = V_1\left[\frac{1}{10} + \frac{1}{20} + \frac{1}{10}\right]$$

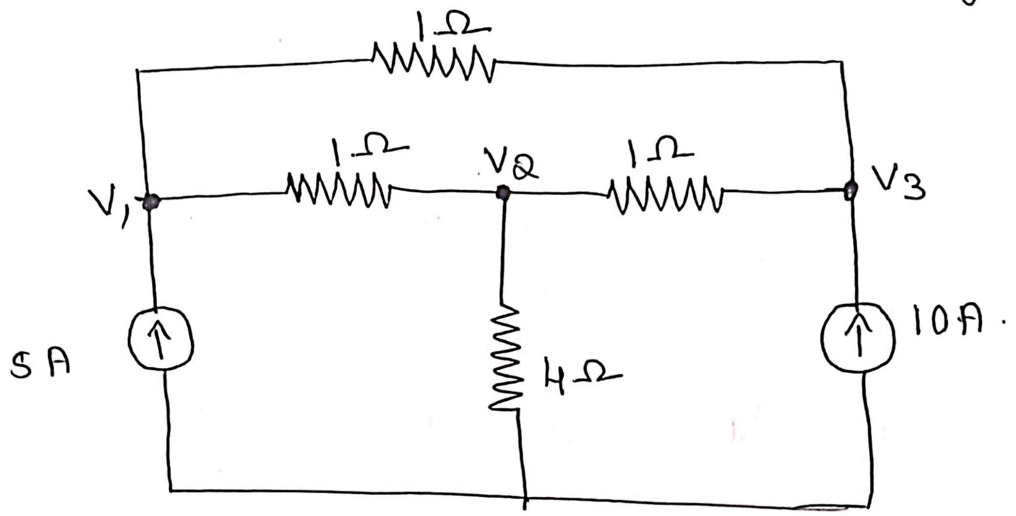
$$V_1 = \frac{3}{\frac{2+1+2}{20}}$$

$$V_1 = \frac{60}{\cancel{5}} = 12$$

$$I_{20\Omega} = \frac{12}{20} = 0.6A.$$

(41)

5) Find V_1, V_2, V_3 by the nodal method for the given circuit.



KCL at N_1

$$5 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1}$$

$$= V_1 - V_2 + V_1 - V_3 = 2V_1 - V_2 - V_3.$$

$$2V_1 - V_2 - V_3 = 5 \quad \text{—— (1)}$$

KCL at N_2

$$\frac{V_2 - V_1}{1} + \frac{V_2}{4} + \frac{V_2 - V_3}{1}$$

$$0 = V_2 - V_1 + \frac{V_2}{4} + V_2 - V_3$$

$$= -V_1 + 2V_2 + \frac{V_2}{4} - V_3$$

$$= -V_1 + \frac{9}{4}V_2 - V_3. \quad \text{—— (2)}$$

KCL at N3.

$$10 = \frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{1}$$

$$= V_3 - V_1 + V_3 - V_2 \Rightarrow -V_1 - V_2 + 2V_3 = 10$$

—— (3)

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & \begin{matrix}9/4 \\ (2.25)\end{matrix} & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

$$\Delta = 2\left[4.5-1\right] - (-1)\left[-2-1\right] + (-1)\left[1+2.25\right]$$

$$= 7-3-3.25 \Rightarrow \boxed{\Delta = 0.75}$$

$$\Delta V_1 = \begin{bmatrix} 5 & -1 & -1 \\ 0 & 2.25 & -1 \\ 10 & -1 & 2 \end{bmatrix}$$

$$= 5\left[4.5-1\right] - (-1)(0+10) + (-1)(0-22.5)$$

$$= 17.5 + 10 + 22.5 = 50$$

$$V_1 = \frac{\Delta V_1}{\Delta} = \frac{50}{0.75} \Rightarrow \boxed{V_1 = 66.66\ V}$$

$$\Delta V_2 = \begin{bmatrix} 2 & 5 & -1 \\ -1 & 0 & -1 \\ -1 & 10 & 2 \end{bmatrix}$$

$$= 2\left[10\right] - 5\left[-2-1\right] + (-1)(-10)$$

$$= 20 + 15 + 10 = 45 \Rightarrow V_2 = \frac{\Delta V_2}{\Delta} = \frac{45}{0.75} = 60$$

(43)

$\boxed{V_2 = 60}$

$$\Delta V_3 = \begin{bmatrix} 2 & -1 & 5 \\ -1 & 2.25 & 0 \\ -1 & -1 & 10 \end{bmatrix}$$

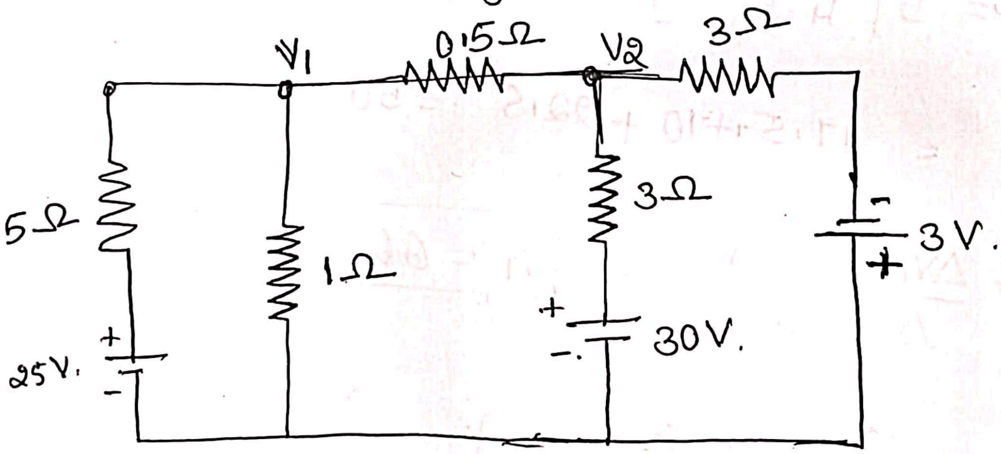
$$= 2[22.5) - (-1)(-10) + 5(1 + 2.25)$$

$$= 45 - 10 + 16.25$$

$$= 51.25$$

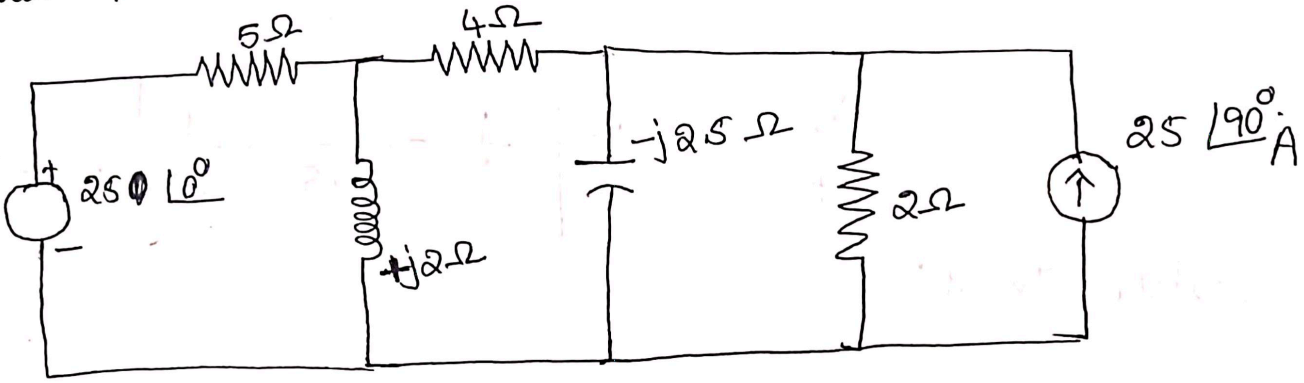
$$V_3 = \frac{\Delta V_3}{\Delta} = \frac{51.25}{0.75} = 68.33V.$$

Find the voltage of the nodes 1 and 2 in the network shown in figure

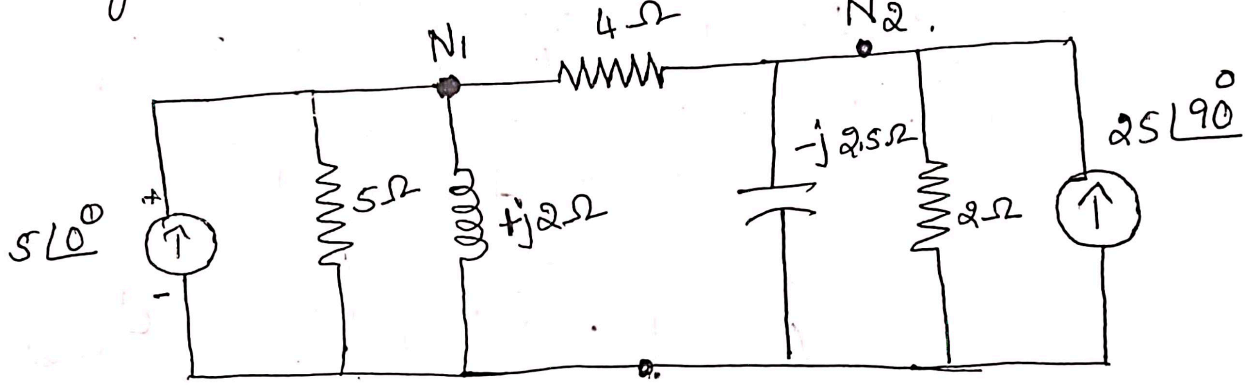


(44)

using nodal analysis, find the current through 4 Ω resistor in the circuit shown in figure.



1) Convert vtg source to ct source.



Apply KCL to node 1,

$$5 = \frac{V_1}{5} + \frac{V_1}{-j2} + \frac{V_1 - V_2}{4}$$

$$5 = \frac{V_1}{5} + \frac{V_1}{4} + \frac{V_1}{-2j} - \frac{V_2}{4}.$$

$$5\angle 0^\circ = V_1\left[\frac{1}{5} + \frac{1}{2j} + \frac{1}{4}\right] - \frac{1}{4}V_2 \quad \text{—— (1).}$$

Apply KCL to N2.

$$25\angle 90^\circ = \frac{V_2}{2} + \frac{V_2}{-j2.5} + \frac{V_2 - V_1}{4}.$$

(4b)

$$\frac{V_2}{2} - \frac{V_2}{j2.5} + \frac{V_2}{4} - \frac{V_1}{4} = 25\angle 90^\circ.$$

$$\frac{-V_1}{4} \quad V_2\left[\frac{1}{2} - \frac{1}{j2.5} + \frac{1}{4}\right] = 25\angle 90^\circ$$

matrix form :-

$$\begin{bmatrix} \frac{1}{5}+\frac{1}{j2}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{-j2.5}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5\angle 0^\circ \\ 25\angle 90^\circ \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 \mp 0.5j + 0.25 & -0.25 \\ -0.25 & 0.25 + 0.4j + 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.45 \mp 0.5j & -0.25 \\ -0.25 & 0.75 + 0.4j \end{bmatrix}$$

$$= \cancel{0.1375 + 0.555j \mp 0.0625}$$

$$= (0.45 \mp 0.5j)(0.75 + 0.4j) - (0.25)^2$$

$$= 0.3375 + 0.18j \mp 0.375j + 0.2 - 0.0625$$

$$= 0.5375 - j0.195 - 0.0625 = 0.475 - j0.19$$

$$= \sqrt{x^2+y^2} \tan^{-1}\frac{0.195}{0.537}$$

$$\Delta = 0.513\angle -22.3^\circ$$

$$\Delta_1 = \begin{vmatrix} 5\angle 0^\circ & -0.25 \\ 25\angle 90^\circ & 0.75+0.4j \end{vmatrix}$$

$\sqrt{0.5625+0.16}$

$\sqrt{0.7225}$

$\tan^{-1} 0.53 =$

$$= \begin{vmatrix} 5\angle 0^\circ & 0.25\angle 0^\circ \\ 25\angle 90^\circ & 0.85\angle 27.9^\circ \end{vmatrix}$$

$$= 4.25\angle 27.9^\circ - 6.25\angle 90^\circ$$

$$= 5(0.75+0.4j) + 0.25(25\angle 90^\circ)$$

$$= 3.75 + j2 + 0.25(25i)$$

$$= 3.75 + j2 + j6.25$$

$$\Delta_1 = 3.75 + j8.25 = 9.06\angle 65.5^\circ$$

shift + Pol =, shift =

$$\Delta_2 = \begin{vmatrix} 0.45 - j0.5 & 5\angle 0^\circ \\ -0.25 & 25\angle 90^\circ \end{vmatrix}$$

$$= 25\angle 90^\circ (0.45 - j0.5) + 0.25 \times 5\angle 0^\circ$$

$$= j25(0.45 - j0.5) + 1.25$$

$$= 11.25j + 12.5 + 1.25$$

$$= 13.75 + j11.25$$

$$\boxed{\Delta_2 = 17.77\angle 39.3^\circ}$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{9.06\angle 65.5^\circ}{0.513\angle -22.3^\circ}$$

$$\boxed{V_1 = 17.66\angle 87.8^\circ \text{ volts}} = 0.677 + j17.6.$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{17.77\angle 39.3^\circ}{0.513\angle -22.3^\circ} = 34.64\angle 61.6^\circ \text{ volts}$$

$$= 16.47 + j30.47\,V$$

Voltage across $4\,\Omega$ = $V_2 - V_1$

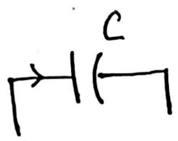
$$= 16.47 + j30.47 - 0.677 - j17.6.$$

$$= 15.793 + j12.87$$

$$= 20.37\angle 39.17^\circ \text{ volts}.$$

current $= \frac{V}{R} = \frac{20.37\angle 39.17^\circ}{4}$

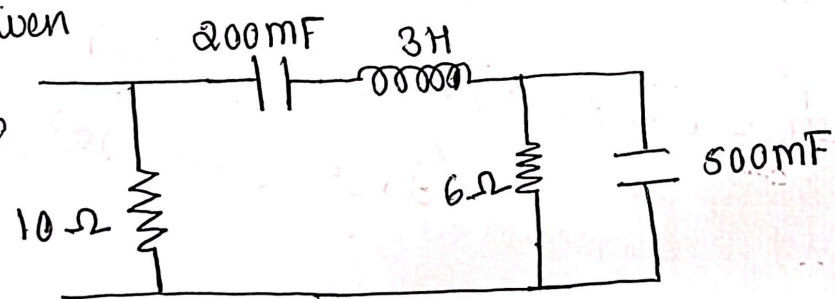
$$= 5.09\angle 39.17^\circ\ A$$

		Time domain	Frequency domain
Resistor		$V = iR$	$V = IR$
Inductor		$V = L \cdot \frac{di}{dt}$	$V = j\omega L \cdot I$
capacitor		$V = \frac{1}{C}\int i\,dt$	$V = \left(\frac{1}{j\omega C}\right) I$

Impedance

$$Z_R = R, \quad Z_L = j\omega L, \quad Z_C = \frac{1}{j\omega C}.$$

Find the equivalent impedance of the network shown for a given operating frequency of 5 rad/s



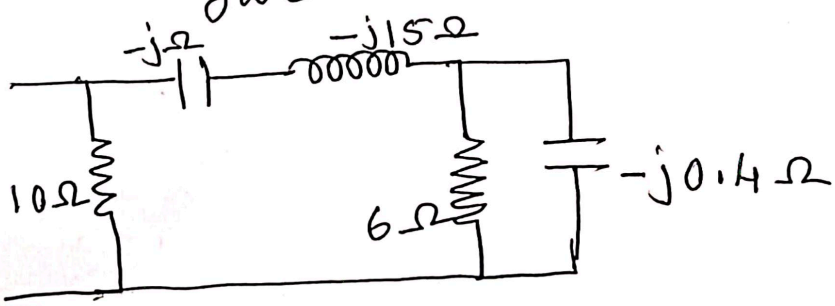
Soln

$Z_R = 10\,\Omega$

$Z_L = j\omega L = j \times 5 \times 3 = j15\,\Omega.$

$Z_R = 6\,\Omega$

$$Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{5 \times 500 \times 10^{-3}} = -j0.4\,\Omega.$$



(47)

$$Z_{eq} = \frac{6(-j0.4)}{6-j0.4} =$$

$$\boxed{Z_{eq} = 0.02655 - j0.3984\ \Omega.}$$

series img $Z_{eq} = Z_L + Z_C$

Parallel imp $Z_{eq} = \frac{Z_L Z_C}{Z_L + Z_C}$

$Z_{eq} = -j\Omega$ and $j15\ \Omega$.

$Z_{eq} = 0.02655 - j0.3982 - j + j15 = 0.02655 + j13.6\Omega$

This Z_{eq} is now in parallel with 10Ω.

$$Z_{eq} = \frac{10(0.02655 + j13.6)}{10 + 0.02655 + j13.6}$$

$= 6.488 + j4.763\ \Omega$.

$Z_{eq} = 6.488 + j4.763\ \Omega$ (or)

$8.05 \angle 36.28^\circ \Omega$).

Admittance $Y = G + jB = \frac{1}{Z} = \frac{1}{R + jX}$. $\left[\because Y = \frac{1}{Z}\right]$

mesh equation derivation.