DESIGNOF FIR FILTERS-SYMMETRIC AND ANTI-SYMMETRIC FIR FILTERS

Introduction:
- A digital filter is just a filter that operates on the digital signals.

Types:
- FIR filter design
- IIR filter design

FIR filter:

The digital filter which designed using finite number of response co-efficient is called as finite impulse response filters $h_o(n), h_1(n)..................h_{(N-1)}(n)$

Advantages:
1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non-recursive structure.
4. FIR filters with any arbitrary magnitude response can be tackled using FIR sequence.

Disadvantages:
1. For the same filter specification the order of the FIR filter design can be as high as 5 to 10 times   that of an IIR filter.
2. Large storage requirement needed.
3. Powerful computational facilities required for the implementation.

Linear Phase (LP) FIR Filters:

Derive the condition for Linear Phase (LP) FIR Filters. [Nov/Dec-2009]

The transfer function of a FIR causal filter is given by

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

Where $h(n)$ is the impulse response of the filter.

The Fourier transform of h(n) is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n},$$

Which is periodic in frequency with period 2π.

$$H(e^{j\omega}) = \pm |H(e^{j\omega})| e^{-j\theta(\omega)}$$

Where $H(e^{j\omega})$ is magnitude response and $\theta(\omega)$ is phase response.

We define the phase delay and group delay of a filter as

$$\tau_p = \frac{-\theta(\omega)}{\omega} \, and; \tau_g = \frac{-d\theta(\omega)}{d\omega} \text{-----------------------------------}>(1)$$

For FIR filters with linear phase we can define

$$\theta(\omega) = -\alpha\omega; -\pi \le \omega \le \pi \text{--}>(2)$$

Where α is a constant phase delay in samples.

Substitute: equation 2 in 1, we have $\tau_p = \tau_g = \alpha$, which means that α is independent of frequency. We can write,

$$\sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm |H(e^{j\omega})| e^{-j\theta(\omega)}$$

Which gives us,

$$\sum_{n=0}^{N-1} h(n)\cos \omega n = \pm |H(e^{j\omega})| \cos \theta(\omega) \text{ -----------------------------}>(3)$$

and
$$-\sum_{n=0}^{N-1} h(n)\sin \omega n = \pm |H(e^{j\omega})| \sin \theta(\omega) \text{ --------------------------------->(4)}$$

By taking ratio of equation (3) to equation (4), we obtain

$$\frac{\sum_{n=0}^{N-1} h(n)\sin \omega n}{\sum_{n=0}^{N-1} h(n)\cos \omega n} = \frac{\sin \alpha\omega}{\cos \alpha\omega}; [\theta(\omega) = -\alpha\omega] \text{ --------------------------->(5)}$$

After simplifying equation (5) we have

$$\sum_{n=0}^{N-1} h(n)\sin(\alpha - n)\omega = 0 \text{ --------------------------------------->(6)}$$

Equation (6) will be zero when

$$h(n) = h(N-1-n) \text{ --->(7)}$$

And
$$\alpha = \frac{N-1}{2} \text{ -->(8)}$$

Therefore, FIR filters will have constant phase and group delays when the impulse response is symmetrical about $\alpha = \frac{N-1}{2}$

The impulse response satisfying equation (7) & (8) for odd and even values of N. When N=7 the centre of symmetry of the sequence occurs at third sample and when N=6, the filter delay is $2\frac{1}{2}$ samples.

If only constant group delay is required, and not the phase delay we can write
$$\theta(\omega) = \beta - \alpha\omega$$

Now we have $H(e^{j\omega}) = \pm |H(e^{j\omega})| e^{j(\beta - \alpha\omega)}$

Equation (9) can be expressed as

$$\sum_{n=0}^{N-1} h(n)e^{-j\omega n} = \pm |H(e^{j\omega})| e^{j(\beta - \alpha\omega)} \text{ ----------------------------------->(9)}$$

which gives us

$$\sum_{n=0}^{N-1} h(n)\cos \omega n = \pm |H(e^{j\omega})| \cos(\beta - \alpha\omega) \text{ ----------------------->(10)}$$

and
$$-\sum_{n=0}^{N-1} h(n)\sin \omega n = \pm |H(e^{j\omega})| \sin(\beta - \alpha\omega) \text{ ----------------------->(11)}$$

By taking ratio of equation (11) to (10), we get

$$\frac{-\sum_{n=0}^{N-1} h(n)\sin \omega n}{\sum_{n=0}^{N-1} h(n)\cos \omega n} = \frac{\sin(\beta - \alpha\omega)}{\cos(\beta - \alpha\omega)}$$

From which we obtain

$$\sum_{n=0}^{N-1} h(n)\sin[\beta - (\alpha - n)\omega] = 0 \text{ --------------------------------->(12)}$$

If $\beta = \frac{\pi}{2}$, Equation (12) becomes,

$$\sum_{n=0}^{N-1} h(n)\cos(\alpha - n)\omega = 0 \text{ --------------------------------------->(13)}$$
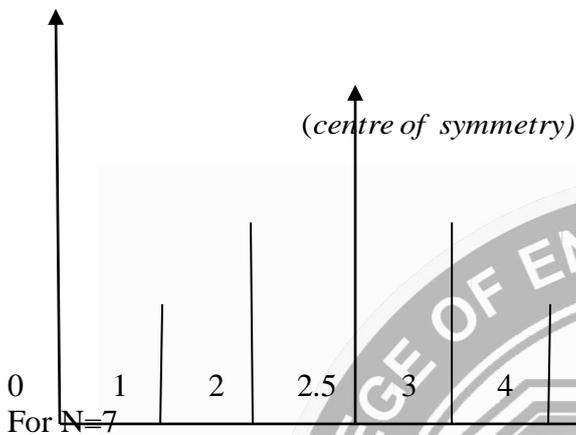
The equation 13 will be satisfied when $h(n) = h(N-1-n)$

And $\alpha = \frac{N-1}{2}$

Therefore, FIR filters have constant group delay, τ_g and not constant phase delay when the impulse response is anti-symmetrical about $\alpha = \dfrac{N-1}{2}$.

Example:

For N=6 $\alpha = \dfrac{N-1}{2} = \dfrac{6-1}{2} = 2\dfrac{1}{2}$ (centre of symmetry)

For N=7 $\alpha = \dfrac{N-1}{2} = \dfrac{7-1}{2} = 3$ (centre of symmetry)

centre of symmetry

Linear Phase FIR Filter:

An FIR filter has linear phase if its unit sample response satisfies the condition

$$h(n) = \pm h(M-1-n); \qquad n = 0,1,2........N-1$$

Case (i): Symmetric impulse response for "N is ODD":

> **Determine the frequency response of FIR filter with symmetric impulse response and the order of the filter is "N is Odd".**

The frequency response of impulse response can be written as,

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-\frac{j\omega(M-1)}{2}} + \sum_{n=\frac{N+1}{2}}^{M-1} h(n)e^{-j\omega n} \text{--------------------------->(1)}$$

Let $n = M-1-n$, where $z = e^{j\omega}\big|H(e^{j\omega}) = H(z)\big|z = e^{j\omega}$

$$H\left(e^{j\omega}\right) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-\frac{j\omega(M-1)}{2}} + \sum_{n=0}^{\frac{M-3}{2}} h(M-1-n)\,e^{-j\omega(M-1-n)} \text{--------->(2)}$$

For a symmetrical impulse response, $h(n) = h(M-1-n)$, substituting this relation in above equation (2)

$$H\left(e^{j\omega}\right) = e^{-\frac{j\omega(M-1)}{2}}\left[\sum_{n=0}^{\frac{M-3}{2}} h(n)e^{j\omega\left(\frac{M-1}{2-n}\right)} + \sum_{n=0}^{\frac{M-1}{2}} h(n)\,e^{-\frac{j\omega(M-1)}{2-n}} + h\left(\frac{M-1}{2}\right)\right]$$

$$H\left(e^{j\omega}\right) = e^{-\frac{j\omega(M-1)}{2}}\left[\left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\frac{M-1}{2}\right)-n\right] + h\left(\frac{M-1}{2}\right)\right]$$

The polar form of $H(e^{j\omega})$ can be expressed as

$$H\left(e^{j\omega}\right) = \left|H\left(e^{j\omega}\right)\right|e^{j\angle H(e^{j\omega})}$$

$\therefore$ *Magnitude of* $H(e^{j\omega})$ *is given as* $\quad \left|H(e^{j\omega})\right| = h\left(\dfrac{M-1}{2}\right) + 2\displaystyle\sum_{n=0}^{\frac{M-3}{2}} h(n)\cos\omega\left(\dfrac{M-1}{2} - n\right)$

Angle of $H(e^{j\omega})$ *is given as* $\quad \angle H(e^{j\omega}) = \begin{cases} -\omega\left(\dfrac{M-1}{2}\right), & for\ \left|H(e^{j\omega})\right| > 0 \\[2mm] -\omega\left(\dfrac{M-1}{2}\right) + \pi, & for\ \left|H(e^{j\omega})\right| < 0 \end{cases}$

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Case (ii) : Symmetric Impulse Response For –"N is EVEN":

> **Determine the frequency response of FIR filter with symmetric impulse response and the order of the filter N is Even.**

The **"N is Even"** frequency response of impulse response can be written as,

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} \quad\text{---} >(1)$$

Let $n = M - 1 - n$, *where* $z = e^{j\omega}$, $H(e^{j\omega}) = H(z)|z = e^{j\omega}$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-2}{2}} h(M-1-n)\,e^{-j\omega(M-1-n)} \quad\text{-----------------------} >(2)$$

For a symmetrical impulse response, $h(n) = h(M-1-n)$, *substituting this relation in above equation (2)*

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + h\left(\frac{M-1}{2}\right)e^{-\frac{j\omega(M-1)}{2}} + \sum_{n=0}^{\frac{M-1}{2}} h(n)\,e^{-j\omega(M-1-n)} \quad\text{--------------} >(3)$$

$$H(e^{j\omega}) = e^{-\frac{j\omega(M-1)}{2}}\left[\sum_{n=0}^{\frac{M-2}{2}} h(n)e^{j\omega\left(\frac{M-1}{2}-n\right)} + \sum_{n=0}^{\frac{M-2}{2}} h(n)\,e^{-j\omega\left(\frac{M-1}{2}-n\right)}\right]$$

$$H(e^{j\omega}) = e^{-\frac{j\omega(M-1)}{2}}\left[2\sum_{n=0}^{\frac{M-2}{2}} h(n)\cos\omega\left(\frac{M-1}{2}-n\right)\right]$$

The polar form of $H(e^{j\omega})$ *can be expressed as* $H(e^{j\omega}) = \left|H(e^{j\omega})\right|e^{j\angle H(e^{j\omega})}$

$\therefore$ *Magnitude* $\left|H(e^{j\omega})\right| = 2\displaystyle\sum_{n=0}^{\frac{M-2}{2}} h(n)\cos\omega\left(\dfrac{M-1}{2}-n\right)$

Angle of $H(e^{j\omega})$ *is given as* $\angle H(e^{j\omega}) = \begin{cases} -\omega\left(\dfrac{M-1}{2}\right), & for\ \left|H(e^{j\omega})\right| > 0 \\[2mm] -\omega\left(\dfrac{M-1}{2}\right) + \pi, & for\ \left|H(e^{j\omega})\right| < 0 \end{cases}$

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Case (iii) : Antisymmetric for "N is ODD":

> **Determine the frequency response of FIR filter with Antisymmetric impulse response and the order of the filter N is Odd.**

For this type of sequence

$$h\left(\frac{M-1}{2}\right) = 0$$

$$H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

The frequency response of impulse response can be written as,

$$H\left(e^{j\omega}\right)= \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-3}{2}} h(M-1-n)e^{-j\omega(M-1-n)}$$

for antisymmetric impulse response, $h(n) = -h(M-1-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega n} - \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} \left[\sum_{n=0}^{\frac{M-3}{2}} h(n)e^{j\omega\left(\frac{M-1}{2}-n\right)} - \sum_{n=0}^{\frac{M-3}{2}} h(n)e^{-j\omega\left(\frac{M-1}{2}-n\right)} \right]$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} j \left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin \omega\left(\frac{M-1}{2}-n\right) \right]$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} e^{j\frac{\pi}{2}} \left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin \omega\left(\frac{M-1}{2}-n\right) \right]$$

$$H\left(e^{j\omega}\right) = e^{j\left[\frac{\pi}{2}-\omega\left(\frac{M-1}{2}\right)\right]} \left[2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin \omega\left(\frac{M-1}{2}-n\right) \right]$$

Magnitude of $H(e^{j\omega})$ is given as, $\quad \left|H(e^{j\omega})\right| = 2\sum_{n=0}^{\frac{M-3}{2}} h(n)\sin \omega\left(\frac{M-1}{2}-n\right)$

Angle of $H(e^{j\omega})$ is given as, $\quad \angle H(e^{j\omega}) = \begin{cases} \dfrac{\pi}{2} - \omega\left(\dfrac{M-1}{2}\right) & for \left|H(e^{j\omega})\right| > 0 \\ \dfrac{3\pi}{2} - \omega\left(\dfrac{M-1}{2}\right) & for \left|H(e^{j\omega})\right| < 0 \end{cases}$

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Case (iv) : Antisymmetric For –"N is EVEN":

> **Determine the frequency response of FIR filter with Antisymmetric impulse response and the order of the filter N is Even.**

The frequency response of impulse response can be written as,

$$H\left(e^{j\omega}\right)= \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{M-2}{2}} h(M-1-n)e^{-j\omega(M-1-n)}$$

for antisymmetric impulse response, $h(n) = -h(M-1-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega n} - \sum_{n=0}^{\frac{M-2}{2}} h(n)e^{-j\omega(M-1-n)}$$

$$= e^{-j\omega\left(\frac{M-1}{2}\right)} j \left[2\sum_{n=0}^{\frac{M-2}{2}} h(n)\sin \omega\left(\frac{M-1}{2}-n\right) \right]$$

$$H\left(e^{j\omega}\right)=e^{-j\omega\left(\frac{M-1}{2}\right)}e^{j\frac{\pi}{2}}\left[2\sum_{n=0}^{\frac{M-2}{2}}h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$H\left(e^{j\omega}\right)=e^{j\left[\frac{\pi}{2}-\omega\left(\frac{M-1}{2}\right)\right]}\left[2\sum_{n=0}^{\frac{M-2}{2}}h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$=e^{-j\omega\left(\frac{M-1}{2}\right)}j\left[2\sum_{n=0}^{\frac{M-2}{2}}h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

$$=e^{j\left[\frac{\pi}{2}-\omega\left(\frac{M-1}{2}\right)\right]}\left[2\sum_{n=0}^{\frac{M-2}{2}}h(n)\sin\omega\left(\frac{M-1}{2}-n\right)\right]$$

Magnitude of $H(e^{j\omega})$ is given as, $\left|H(e^{j\omega})\right|=2\sum_{n=0}^{\frac{M-2}{2}}h(n)\sin\omega\left(\frac{M-1}{2}-n\right)$

Angle of $H(e^{j\omega})$ is given as, $\angle H(e^{j\omega})=\begin{cases}\dfrac{\pi}{2}-\omega\left(\dfrac{M-1}{2}\right) & for\left|H(e^{j\omega})\right|>0\\[3mm]\dfrac{3\pi}{2}-\omega\left(\dfrac{M-1}{2}\right) & for\left|H(e^{j\omega})\right|<0\end{cases}$

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