Magnetic field intensity on the axis of circular loop:

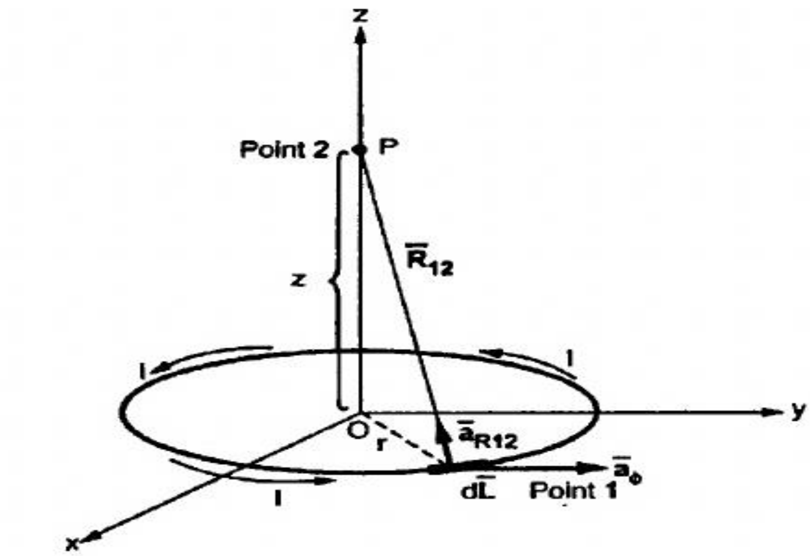

Fig 7: H on the axis of the circular loop.

Consider a circular loop carrying a direct current I placed in xy plane with z-axis. The magnetic field intensity $\overline{H}$ at point P is to be obtained. The point P is at a distance 'z' from the plane of the circular loop along its axis. The radius of the circular loop is 'r' consider the differential length dl of the loop.

In cylindrical co-ordinate system,

$$d\bar{l} = dr\, \overline{ar} + rd\emptyset\, a\overline{\emptyset} + dz\, \overline{az}$$

But dl is in the plane for which 'r' is constant and z=0 for constant plane. The I dl is tangential at point 1 in aΦ direction.

$$\text{I}\ d\bar{l} = Ir\ d\emptyset\ \overline{a\emptyset}$$

The unit vector a$\overline{R_{12}}$is in the direction along the line joining differential current element to the point P.

$$aR_{12} = \frac{\overline{R_{12}}}{|R_{12}|}$$

$$R_{12} = -r\overline{ar} + z\overline{az}$$

$$|R_{12}| = \sqrt{(-r)^2 + z^2}$$

$$aR_{12} = \frac{-r\ \overline{ar} + z\ \overline{az}}{\sqrt{r^2 + z^2}}$$

Now $d\bar{l} \times R_{12} = \begin{vmatrix} \overline{ar} & \overline{a\emptyset} & \overline{az} \\ 0 & rd\emptyset & 0 \\ -r & 0 & z \end{vmatrix}$

$$= z\, r\, d\emptyset\, \overline{ar} + r^2\, d\emptyset\, \overline{az}$$

According to Biot- savart law, the differential field strength d$\overline{H}$ at point P is given by

$$d\overline{H} = \frac{Id\bar{l} \times aR_{12}}{4\pi {R_{12}}^2} = \frac{I[zrd\emptyset\, \overline{ar} + r^2\, d\emptyset \overline{az}]}{4\pi\left(\sqrt{r^2 + z^2}\right)^3}$$

$|\overline{R_{12}}|$is neglected while obtaining the cross product. The total H is to be obtained by integrating Φ from 0 to 2π.

$$\overline{H} = \int_0^{2\pi} \frac{I\,[zr\,\overline{ar} + r^2\overline{az}]\, d\emptyset}{4\pi(r^2 + z^2)^{\frac{3}{2}}}$$

$$= \frac{I}{4\pi}\left\{\int_0^{2\pi} \frac{zr\, ar\, d\emptyset}{(r^2 + z^2)^{\frac{3}{2}}} + \int_0^{2\pi} \frac{r^2\, d\emptyset\, \overline{az}}{(r^2 + z^2)^{\frac{3}{2}}}\right\}$$

Consider 1st integral to prove that its value is zero due to radial symmetry.

$$\int_0^{2\pi} \frac{zr\, d\emptyset\, \overline{ar}}{(r^2 + z^2)^{\frac{3}{2}}} = \int_0^{2\pi} \frac{zr\, d\emptyset}{(r^2 + z^2)^{\frac{3}{2}}}[\cos\emptyset\overline{ax} + \sin\emptyset\overline{ay}]$$

The unit vector ar is expressed in rectangular co-ordinate system,
$\cos\emptyset a\bar{x} + \sin\emptyset a\bar{y}$

Now $\int_0^{2\pi} \cos\emptyset\, d\emptyset = [\sin\emptyset]_0^{2\pi} = 0$

$$\int_0^{2\pi} \sin\emptyset\, d\emptyset = [-\cos\emptyset]_0^{2\pi} = -\cos 2\pi + \cos 0 = -1 + 1 = 0$$

$$\int_0^{2\pi} \frac{zrd\emptyset\overline{ar}}{(r^2 + z^2)^{\frac{3}{2}}} = 0$$

$$\overline{H} = \frac{I}{4\pi}\int_0^{2\pi} \frac{r^2\, d\emptyset\, az}{(r^2 + z^2)^{\frac{3}{2}}} = \frac{I}{4\pi}\frac{r^2\, az}{(r^2 + z^2)^{\frac{3}{2}}}\int_0^{2\pi} d\emptyset = \frac{I}{4\pi}\frac{r^2\, az}{(r^2 + z^2)^{\frac{3}{2}}}[\emptyset]_0^{2\pi}$$

$$\overline{H} = \frac{I\, 2\pi\, r^2\, a\bar{z}}{4\pi\, (r^2 + z^2)^{\frac{3}{2}}} = \frac{I\, r^2\, a\bar{z}}{2(r^2 + z^2)^{\frac{3}{2}}} A/m$$

r→ Radius of the circular loop.
z→ Distance of point p along the axis
Magnetic field intensity at the center of the circular loop

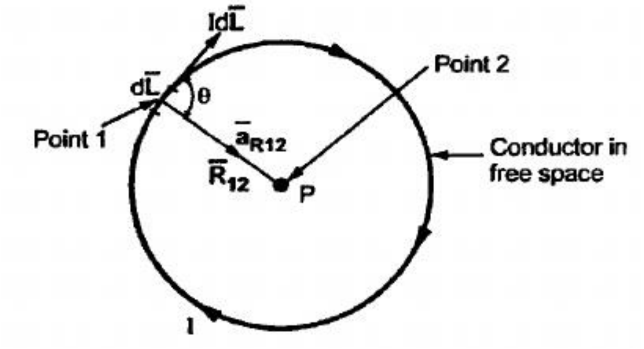

Fig8: Magnetic field intensity at the centre of the circular loop

Substitute z=0 in the above equation .

$\overline{H} = \frac{I\, r^2\ az}{2r^3} = \frac{I\,\overline{az}}{2r}$ A/m.