Power-Angle Curve

Consider a synchronous machine connected to an infinite bus through a transmission line of reactance X_1 shown in a figure below. Let us assume that the resistance and capacitance are neglected.



Equivalent diagram of synchronous machine connected to an infinite bus through a transmission line of series reactance X_i is shown below:



Let, $V = V < 0^0$ – voltage of infinite bus, $E = E < \delta$ – voltage behind the direct axis synchronous reactance of the machine, X_d = synchronous / transient resistance of the machine. The complex power delivered by the generator to the system is

S = VI

$$S = V \left[\frac{E < \delta - V < 0^{0}}{j(X_{d} + X_{l})} \right]$$
$$X_{d} + X_{l} = X$$
Let,

$$S = V \left[\frac{E < \delta}{X < 90^{0}} + j \frac{V}{X} \right]$$
$$S = \frac{EV}{X} < (90^{0} - \delta) - j \frac{V^{2}}{X}$$
$$S = V \left[\frac{EV}{X} \sin\delta + j \frac{EV}{X} \cos\delta - j \frac{V^{2}}{X} \right]$$

$$P_e + jQ_e = \frac{EV}{X}\sin\delta + j\left(\frac{EV}{X}\cos\delta - \frac{V^2}{X}\right)$$

Active power transferred to the system

$$P_e = \frac{EV}{X} \sin\delta$$

The reactive power transferred to the system

$$Q_e = \frac{EV}{X}\cos\delta - \frac{V^2}{X}$$

The maximum steady-state power transfers occur when $\delta = 0$

$$P_e = \frac{EV}{X} \sin 90^{\circ}$$
$$(\sin 90^{\circ} = 1)$$

$$P = \frac{EV}{X} \quad P_e = P_{emax} sin\delta$$

The graphical representation of P_e and the load angle δ is called the power angle curve. It is widely used in power system stability studies. The power angle curve is shown below



Power-angle diagram

Maximum power is transferred when $\delta = 90^{\circ}$. As the value of load angle δ is above 90, P_e decrease and becomes zero at $\delta = 180^{\circ}$. Above 180° , Pe becomes negative, which show that the direction of power flow is reversed, and the power is supplied from infinite bus to the generator. The value of P_e is often called pull out power. It is also called the steady-state limit.

The total reactance between two voltage sources E and X is called the transfer reactance. The maximum power limit is inversely proportion to the transfer reactance.