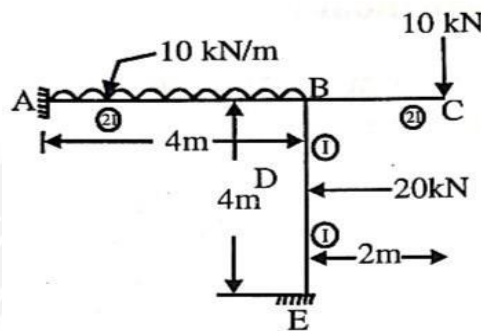


## 2.3 ANALYSIS OF RIGID FRAMES IN SLOPE DEFLECTION METHOD.

### 2.3.1 NUMERICAL EXAMPLES ON ( RIGID FRAMES ):

#### PROBLEM NO:01

Analysis the rigid frame shown in fig., Calculate the support moments using slope deflection method. Draw the SF and BM diagrams.



Solution:

- Fixed End Moments:**

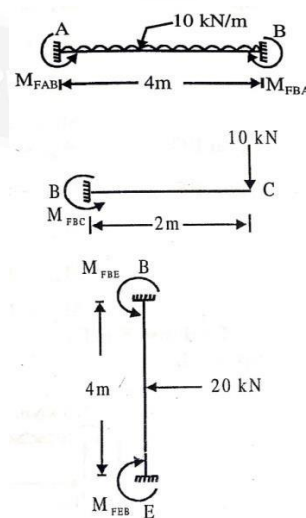
$$M_{FAB} = -Wl^2/12 = -10 \times 4^2/12 = -13.33 \text{ kNm};$$

$$M_{FBA} = Wl^2/12 = 10 \times 4^2/12 = 13.33 \text{ kNm};$$

$$M_{FBC} = -10 \times 2 = -20 \text{ kNm};$$

$$M_{FBE} = -Wl/8 = -20 \times 4/8 = -10 \text{ kNm};$$

$$M_{FEB} = Wl/8 = 20 \times 4/8 = 10 \text{ kNm};$$



- Slope Deflection Equations:**

$$M_{AB} = M_{FAB} + \frac{2E(2I)}{4}(2\theta_A + \theta_B + 3\delta/l)$$

$$= -13.33 + EI\theta_B \quad \text{--- (1)}$$

$$M_{BA} = M_{FBA} + \frac{2E(2I)}{4}(2\theta_B + \theta_A + 3\delta/l)$$

$$= 13.33 + EI\theta_B \quad \text{--- (2)}$$

$$M_{BE} = M_{FBE} + \frac{2EI}{3}(2\theta_B + \theta_E + 3\delta/l)$$

$$= -10 + EI\theta_B \quad \text{--- (3)}$$

$$M_{EB} = M_{FEB} + \frac{2EI}{3}(2\theta_E + \theta_B + 3\delta/l)$$

$$= 10 + 0.5EI\theta_B \quad \text{--- (4)}$$

- Joint Equilibrium Equations:**

Joint B,  $\Sigma M = 0$ ;

$$M_{BA} + M_{BC} + M_{BE} = 0$$

$$13.33 + 2EI\theta_B - 10 + EI\theta_B - 20 = 0$$

$$3EI\theta_B - 16.67 = 0$$

$$\theta_B = 5.557/EI;$$

- Final Moments:**

$$M_{AB} = -7.773 \text{ kNm};$$

$$M_{BA} = 24.44 \text{ kNm};$$

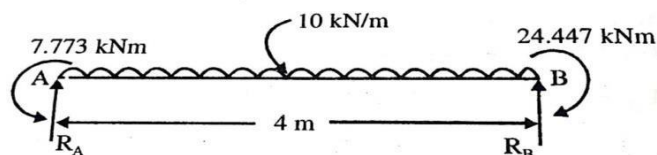
$$M_{BC} = -20 \text{ kNm};$$

$$M_{CD} = -4.33 \text{ kNm};$$

$$M_{DC} = 12.78 \text{ kNm};$$

- To Draw S.F.D:**

Span AB:

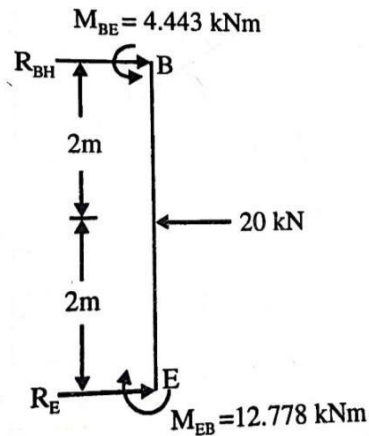


Taking moments about A.

$$R_A \times 4 - 10 \times 4 \times 4/2 - 7.773 + 24.447 = 0; R_A = 15.83 \text{ KN}$$

$$R_B = 10 \times 4 - R_A; R_B = 24.168 \text{ KN}$$

Span BE:

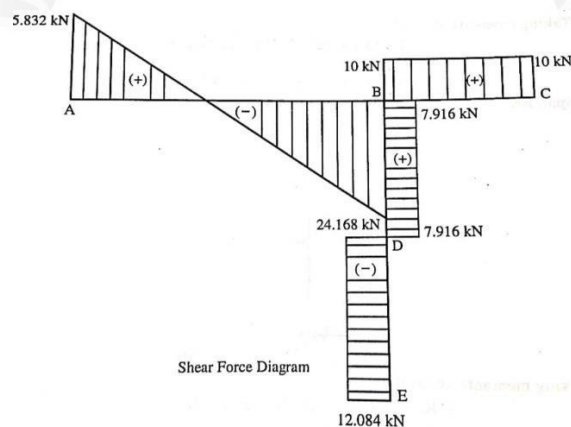
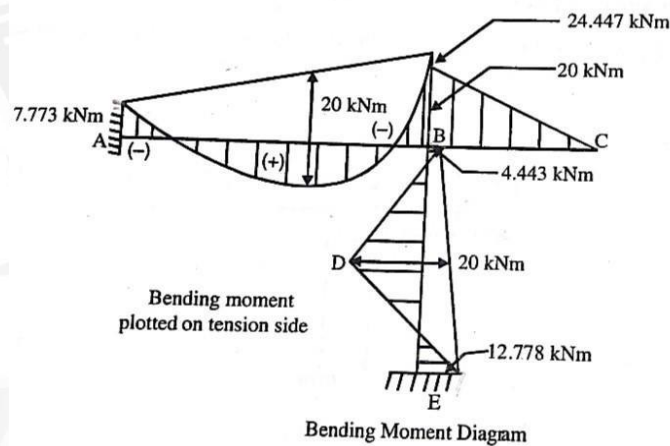


Taking moments about B.

$$-R_E \times 4 - 4.443 + 12.778 + 20 \times 2 = 0; R_E = 12.083 \text{ kN}$$

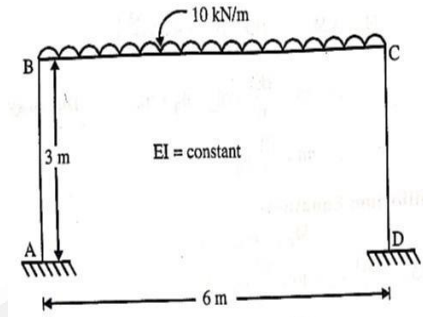
$$R_{BH} = \text{Total load} - R_E = 7.916 \text{ kN}$$

- BMD and SFD:**



**PROBLEM NO:02**

Analysis the rigid frame shown in fig., Calculate the support moments using slope deflection method. Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

$$M_{FAB} = M_{FBA} = 0$$

$$M_{FBC} = -Wl^2/12 = -10 \times 6^2/12 = -30 \text{ kNm}$$

$$M_{FCB} = Wl^2/12 = 10 \times 6^2/12 = 30 \text{ kNm}$$

$$M_{FCD} = M_{FDC} = 0$$

- **Slope Deflection Equations:**

$$\begin{aligned} M_{AB} &= M_{FAB} + 2EI/3(2\theta_A + \theta_B + 3\delta/l) \\ &= 2/3EI\theta_B \end{aligned} \quad \text{--- (1)}$$

$$\begin{aligned} M_{BA} &= M_{FBA} + 2EI/3(2\theta_B + \theta_A + 3\delta/l) \\ &= 4/3EI\theta_B \end{aligned} \quad \text{--- (2)}$$

$$\begin{aligned} M_{BC} &= M_{FBC} + 2EI/6(2\theta_B + \theta_C + 3\delta/l) \\ &= -30 + 1/3EI\theta_B \end{aligned} \quad \text{--- (3)}$$

$$\begin{aligned} M_{CB} &= M_{FCB} + 2EI/6(2\theta_C + \theta_B + 3\delta/l) \\ &= 30 + 1/3EI\theta_B \end{aligned} \quad \text{--- (4)}$$

$$\begin{aligned} M_{CD} &= M_{FCD} + 2EI/6(2\theta_C + \theta_D + 3\delta/l) \\ &= 4/3EI\theta_B \end{aligned} \quad \text{--- (5)}$$

$$\begin{aligned} M_{DC} &= M_{FDC} + 2EI/6(2\theta_D + \theta_C + 3\delta/l) \\ &= 2/3EI\theta_B \end{aligned} \quad \text{--- (6)}$$

- **Joint Equilibrium Equations:**

Joint B:

$$M_{BA} + M_{BC} = 0$$

$$4/3\theta_B - 30 + 1/3EI\theta_B = 0 \quad \text{--- (7)}$$

Joint C:

$$M_{CB} + M_{CD} = 0$$

$$4/3\theta_C - 30 + 1/3EI\theta_C = 0 \quad \text{--- (8)}$$

Equating (7 & 8); we get

$$\theta_C = -18/EI; \quad \theta_B = 18/EI;$$

- **Final Moments:**

$$M_{AB} = 12 \text{ kNm};$$

$$M_{BA} = 24 \text{ kNm};$$

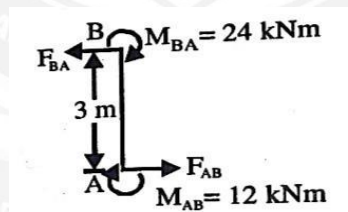
$$M_{BC} = -24 \text{ kNm};$$

$$M_{CB} = 24 \text{ kNm};$$

$$M_{CD} = -24 \text{ kNm};$$

$$M_{DC} = -12 \text{ kNm};$$

- **To Draw S.F.D:**



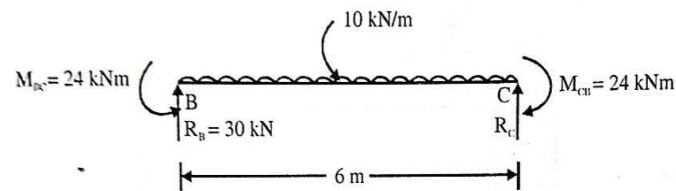
Span AB:

Taking moments about A.

$$-F_{BA} \times 3 + M_{BA} + M_{AB} = 0;$$

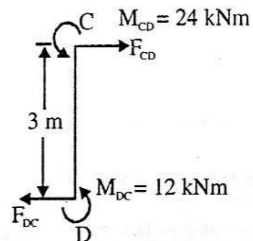
$$24 + 12 = F_{BA} \times 3; \quad F_{BA} = F_{AB} = 12 \text{ KN}$$

Span BC:



$$R_B = R_C = \text{Total load}/2 = 10 \times 6/2 = 30 \text{ kN}$$

Span CD:

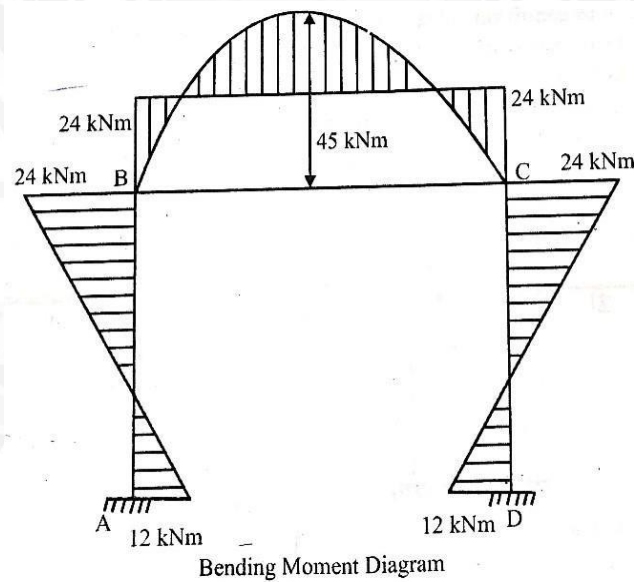


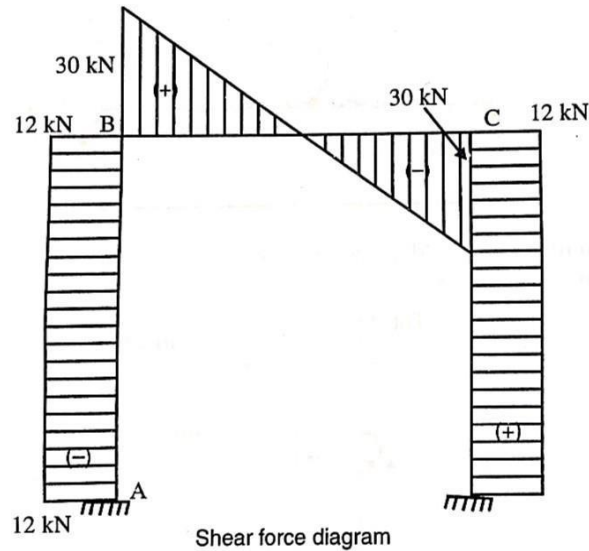
$$F_{CD} = F_{DC} = 12 \text{ kN} \quad (\text{by symmetry})$$

- **Free BMD:**

$$M_{BC} = Wl^2/8 = 10 \times 6^2/8 = 45 \text{ kNm}$$

- **BMD and SFD:**

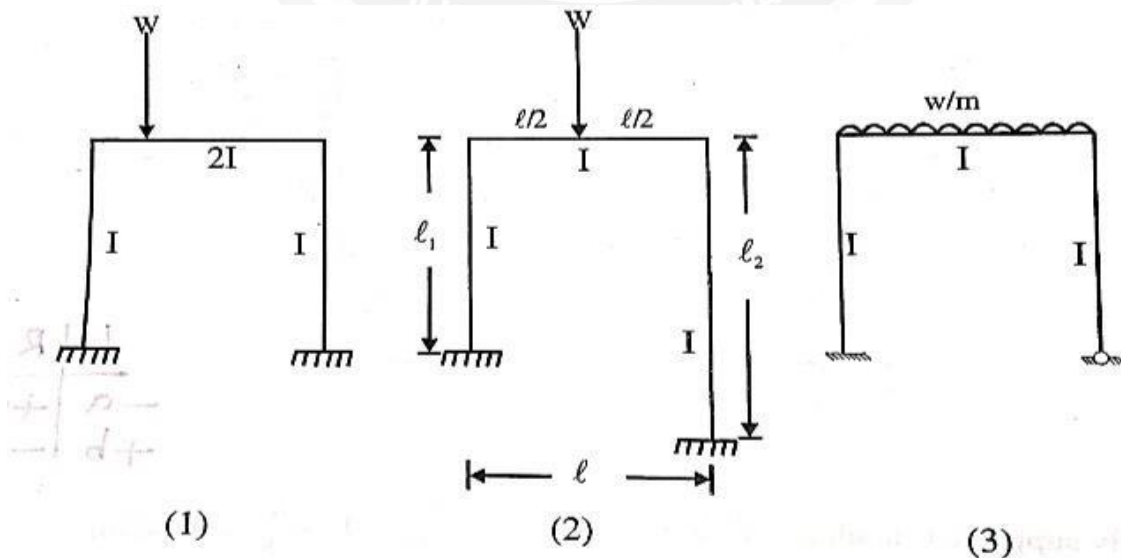


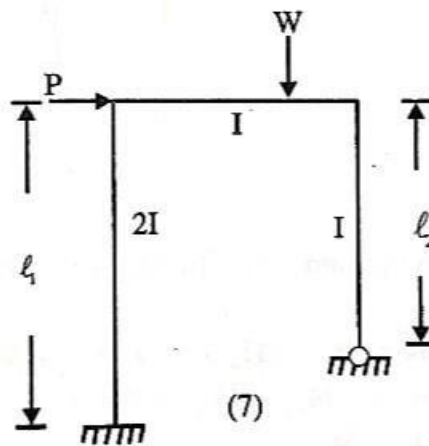
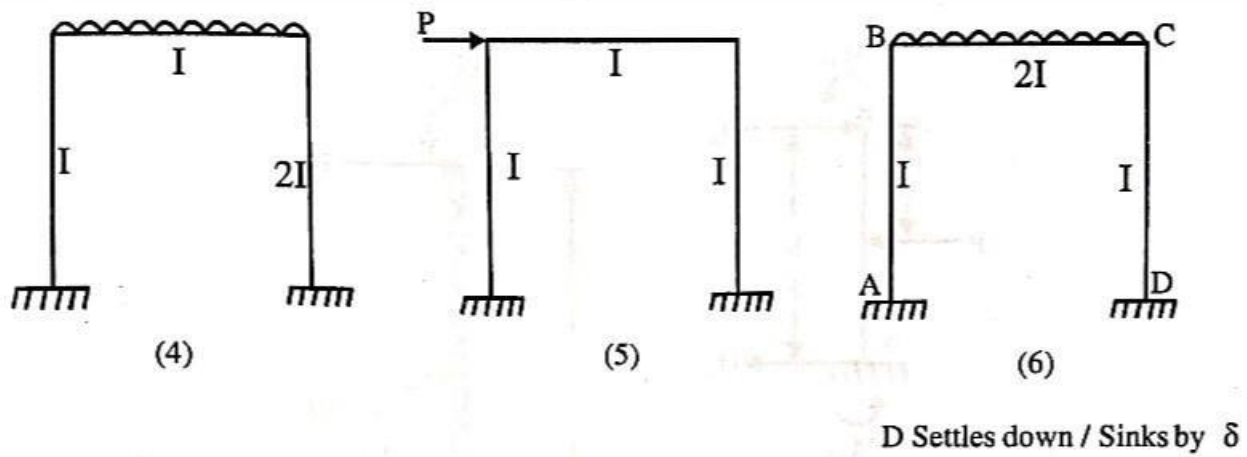


### 2.3.2. RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.

Portal frames may sway due to one of the following reasons:

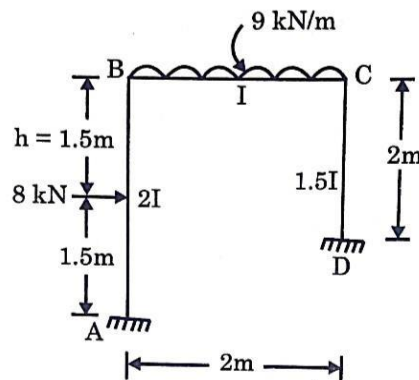
- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.





### 2.3.3. NUMERICAL EXAMPLES ON ( RIGID FRAMES WITH SWAY ): PROBLEM NO:03

Analysis the rigid frame shown in fig., Calculate the support moments using slope deflection method. Draw the SF and BM diagrams.





Solution:

- Fixed End Moments:**

$$MF_{AB} = -Wl/8 = -8 \times 3/8 = -3 \text{ kNm};$$

$$MF_{BA} = Wl/8 = 8 \times 3/8 = 3 \text{ kNm};$$

$$MF_{AB} = -Wl^2/12 = -20 \times 4^2/12 = -26.67 \text{ kNm};$$

$$MF_{BA} = Wl^2/12 = 20 \times 4^2/12 = 26.67 \text{ kNm};$$

$$MF_{CD} = 0;$$

$$MF_{DC} = 0;$$

- Slope Deflection Equations:**

$$\begin{aligned} M_{AB} &= MF_{AB} + 2E(2I)/3(2\theta_A + \theta_B + 3\delta/l) \\ &= -3 + 4/3EI(\theta_B - \delta) \end{aligned} \quad \text{--- (1)}$$

$$\begin{aligned} M_{BA} &= MF_{BA} + 2E(2I)/3(2\theta_B + \theta_A + 3\delta/l) \\ &= 3 + 4/3EI(2\theta_B - \delta) \end{aligned} \quad \text{--- (2)}$$

$$\begin{aligned} M_{BC} &= MF_{BC} + 2EI/2(2\theta_B + \theta_C + 3\delta/l) \\ &= -3 + EI(2\theta_B + \theta_C) \end{aligned} \quad \text{--- (3)}$$

$$\begin{aligned} M_{CB} &= MF_{CB} + 2EI/2(2\theta_C + \theta_B + 3\delta/l) \\ &= 3 + EI(2\theta_C + \theta_B) \end{aligned} \quad \text{--- (4)}$$

$$\begin{aligned} M_{CD} &= MF_{CD} + 2E(1.5I)/2(2\theta_C + \theta_D + 3\delta/l) \\ &= 1.5EI(2\theta_C - 3\delta/2) \end{aligned} \quad \text{--- (5)}$$

$$\begin{aligned} M_{DC} &= MF_{DC} + 2EI/6(2\theta_D + \theta_C + 3\delta/l) \\ &= 1.5EI(2\theta_C - 3\delta/2) \end{aligned} \quad \text{--- (6)}$$

- Equilibrium and Shear Equations:**

$$M_{BA} + M_{BC} = 0$$

$$14\theta_B - 4\delta + 3\theta_C = 0 \quad \text{---(7)}$$

$$M_{CB} + M_{CD} = 0$$

$$\theta_B - 2.25\delta + 5\theta_C = 0 \quad \text{---(8)}$$

Using Shear Equations, we get;

$$M_{AB} + M_{BA} - Ph/l + M_{CD} + M_{DC}/l + P = 0$$

$$(\theta_C = -0.044/EI; \theta_B = 0.414/EI).$$

- **Final Moments:**

$$M_{AB} = -4.34 \text{ kNm};$$

$$M_{BA} = 2.24 \text{ kNm};$$

$$M_{BC} = -2.24 \text{ kNm};$$

$$M_{CB} = 3.33 \text{ kNm};$$

$$M_{CD} = -3.33 \text{ kNm};$$

$$M_{DC} = -3.26 \text{ kNm};$$

- **Free Bending Moments:**

$$AB = Wl/4 = 8 \times 3/4 = 6 \text{ kNm}$$

$$BC = Wl^2/8 = 9 \times 2^2/8 = 4.5 \text{ kNm}$$

- **BMD:**

