ME3491 THEORY OF MACHINES

UNIT V NOTES

5.4.Vibratory Motion.

When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion. This is due to the reason that, when a body is displaced, the internal forces in the form of elastic or strain energy are present in the body. At release, these forces bring the body to its original position. When the body reaches the equilibrium position, the whole of the elastic or strain energy is converted into kinetic energy due to which the body continues to move in the opposite direction. The whole of the kinetic energy is again converted into strain energy due to which the body again returns to the equilibrium position. In this way, the vibratory motion is repeated indefinitely.

5.4.1 Terms Used in ms Used in Vibratory Motion

The following terms are commonly used in connection with the vibratory motion

1. Period of vibration or time period. It is the time interval after which the motion is repeated itself. The period of vibration is usually expressed in seconds.

2. Cycle. It is the motion completed during one time period.

3. Frequency. It is the number of cycles described in one second. In S.I. units, the frequency is expressed in hertz (briefly written as Hz) which is equal to one cycle per second.

5.4.2. Types of Vibratory Motion

The following types of vibratory motion are important from the subject point of view :

1. Free or natural vibrations.

When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibrations. The frequency of the free vibrations is called free or natural frequency.

2. Forced vibrations.

When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force. Note : When the frequency of the external force is same as that of the natural vibrations, resonance takes place.

3. Damped vibrations.

When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration. This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

5.4.3 Types of Free Vibrations

The following three types of free vibrations are important from the subject point of view :

1. Longitudinal vibrations,
2. Transverse vibrations, and
3. Torsional vibrations.

Consider a weightless constraint (spring or shaft) whose one end is fixed and the other end carrying a heavy disc, as shown in Fig. 23.1. This system may execute one of the three above mentioned types of vibrations.

1. Longitudinal vibrations.

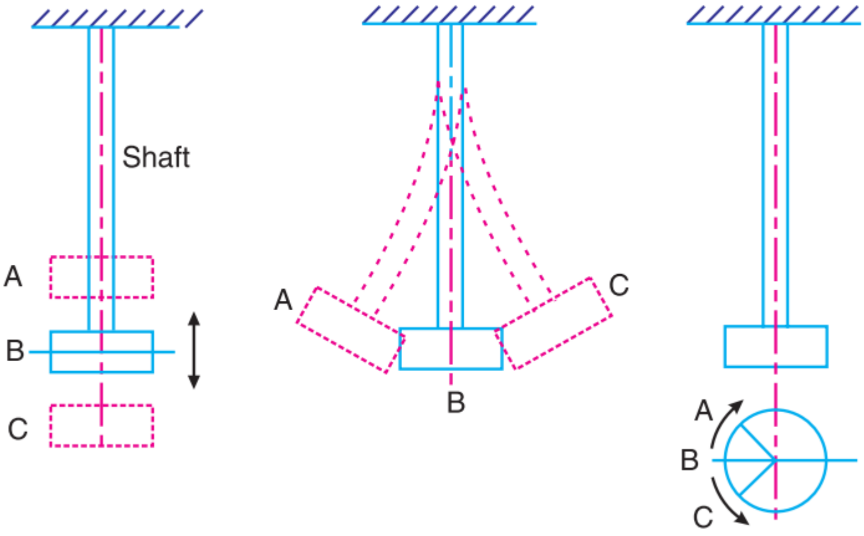
When the particles of the shaft or disc moves parallel to the axis of the shaft, as shown in Figure. then the vibrations are known as longitudinal vibrations. In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

2. Transverse vibrations.

When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, as shown in Figure, then the vibrations are known as transverse vibrations. In this case, the shaft is straight and bent alternately and bending stresses are induced in the shaft.

3. Torsional vibrations

When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Figure, then the vibrations are known as torsional vibrations. In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in the shaft.



B = Mean position ; *A* and *C* = Extreme positions.
(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

5.4.4 .Energy method for Natural Frequency of Free Longitudinal Vibrations

We know that the kinetic energy is due to the motion of the body and the poten.tial energy is with respect to a certain datum position which is equal to the amount of work required to move the body from the datum position. In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero

In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times.

and potential energy, $P.E. = \left(\frac{0 + s.x}{2}\right)x = \frac{1}{2} \times s.x^2$

. . . ($\because$ P.E. = Mean force × Displacement)

$$\therefore \quad \frac{d}{dt}\left[\frac{1}{2} \times m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2} \times s.x^2\right] = 0$$

$$\frac{1}{2} \times m \times 2 \times \frac{dx}{dt} \times \frac{d^2x}{dt^2} + \frac{1}{2} \times s \times 2x \times \frac{dx}{dt} = 0$$

or $\quad m \times \frac{d^2x}{dt^2} + s.x = 0 \quad$ or $\quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$ . . . (Same as before)

The time period and the natural frequency may be obtained as discussed in the previous method.

5.4.5.Rayleigh's method

In this method, the maximum kinetic energy at the mean position is equal to the maximum potential energy (or strain energy) at the extreme position. Assuming the motion executed by the vibration to be simple harmonic, then

$$x = X \sin \omega.t \quad \text{. . . (i)}$$

where x = Displacement of the body from the mean position after time t seconds, and

X = Maximum displacement from mean position to extreme position.

Now, differentiating equation (*i*), we have

$$\frac{dx}{dt} = \omega \times X \cos \omega.t$$

Since at the mean position, $t = 0$, therefore maximum velocity at the mean position,

$$v = \frac{dx}{dt} = \omega.X$$

$\therefore$ Maximum kinetic energy at mean position

$$= \frac{1}{2} \times m.v^2 = \frac{1}{2} \times m.\omega^2.X^2 \quad \text{. . . (ii)}$$

and maximum potential energy at the extreme position

$$= \left(\frac{0 + s.X}{2}\right)X = \frac{1}{2} \times s.X^2 \qquad \ldots (iii)$$

Equating equations (*ii*) and (*iii*),

$$\frac{1}{2} \times m.\omega^2.X^2 = \frac{1}{2} \times s.X^2 \quad \text{or} \quad \omega^2 = \frac{s}{m}, \text{ and } \omega = \sqrt{\frac{s}{m}}$$

$\therefore$ Time period, $$t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{s}{m}} \qquad \ldots \text{(Same as before)}$$

and natural frequency, $$f_n = \frac{1}{t_p} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

5.4.6. Natural Frequency of Free Transverse Vibrations

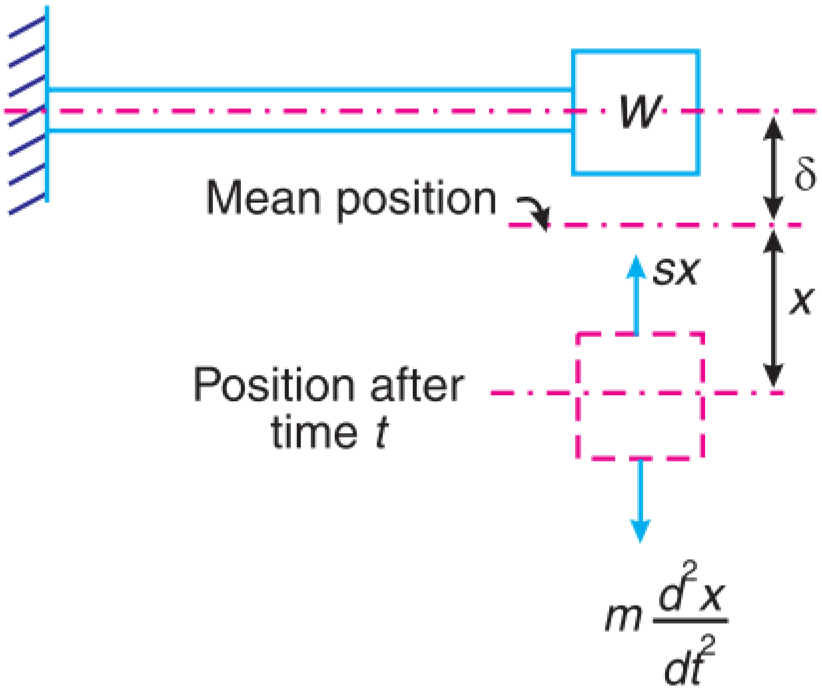
Consider a shaft of negligible mass, whose one end is fixed and the other end carries a body of weight W, as shown in Figure

s = Stiffness of shaft,

δ = Static deflection due to weight of the body,

x = Displacement of body from mean position after time t.

m = Mass of body = W/g



As discussed in the previous article,

Restoring force $= -s.x$ . . . (*i*)

and accelerating force $= m \times \frac{d^2x}{dt^2}$ . . . (*ii*)

Equating equations (*i*) and (*ii*), the equation of motion becomes

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \quad \text{. . . (Same as before)}$$

Hence, the time period and the natural frequency of the transverse vibrations are same as that of longitudinal vibrations. Therefore

Time period, $t_p = 2\pi\sqrt{\frac{m}{s}}$

and natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

Problem

A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m2 . Determine the frequency of longitudinal and transverse vibrations of the shaft.

Solution.

Given : d = 50 mm = 0.05 m ; l = 300 mm = 0.03 m ; m = 100 kg ; E = 200 GN/m2 = 200 ×109 N/m

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} \text{m}^2$$

and moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.3 \times 10^{-6} \text{m}^4$$

Frequency of longitudinal vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l}{A.E} = \frac{100 \times 9.81 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9} = 0.751 \times 10^{-6} \text{ m}$$

$$\ldots(\because W = m.g)$$

$\therefore$ Frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575 \text{ Hz}$$

Frequency of transverse vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l^3}{3E.I} = \frac{100 \times 9.81 \times (0.3)^3}{3 \times 200 \times 10^9 \times 0.3 \times 10^{-6}} = 0.147 \times 10^{-3} \text{ m}$$

$\therefore$ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.147 \times 10^{-3}}} = 41 \text{ Hz}$$