5.2 RANKINE'S THEORY:

Rankine's theory of lateral earth pressure is applied to uniform cohesion less soils only. Later, it was extended to include cohesive soils, by Resal and by Bell. The theory has also been extended to stratified, partially immersed and submerged soils.

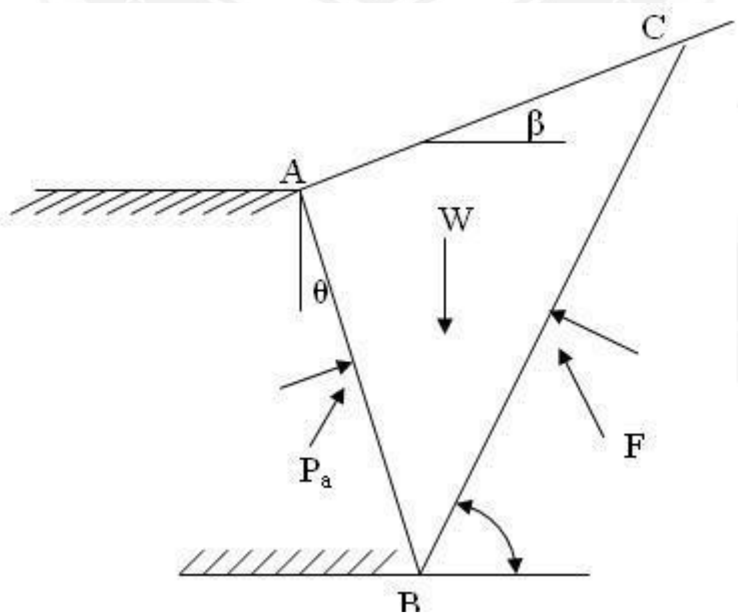
Following are theassumptions of the Rankine's theory:

- The soil mass semi-infinite, homogeneous, dry and cohesion less.
- The ground surface is a plane which may be horizontal and inclined
- The back of the wall is vertical and smooth. In other words, there are no shearing stresses between the wall and the soil and the stress relationship for any element adjacent to the wall is the same as for any other element fair away from the wall.
- The wall yields about the base and thus satisfies deformation condition for plastic equilibrium.

ACTIVE EARTH PRESSURE:

The following cases of cohesion less back fill will now be considered:

1. Dry or moist backfill with no surcharge.
2. Submerged backfill.
3. Backfill with uniform surcharge.
4. Backfill with sloping surface.
5. Inclined back and surcharge.



1.DRY OR MOIST BACKFILL WITH NO SURCHARGES:

Consider an element at a depth z below the ground surface. When the wall is at thepoint of moving outwards, the active state of plastic equilibrium is established.

Backfill is Cohesion less soil:

It is derived on the basis of the principal stress relationship on a failure plane.

$$\sigma_1 = 2Ctan\alpha + \sigma_3 \tan^2 \alpha$$

For active

$$\sigma_3 = \sigma_h$$

$$\sigma_1 = \sigma_v$$

Substitute in above equation

$$\sigma_v = 2Ctan\alpha + \sigma_h \tan^2 \alpha$$

Expression for active Pressure:

$$\sigma_v = \gamma Z$$

$$\sigma_h = P_a$$

According to principal stress relationship:

C=0

$$\sigma_v = \sigma_h \tan^2 \alpha - - - - - - (1)$$

Substitute σ_v and σ_h value in eqn(1)

$$\gamma Z = P_a \tan^2 \alpha$$

$$P_a = \frac{\gamma Z}{\tan^2 \alpha} = \gamma Z cot^2\alpha$$

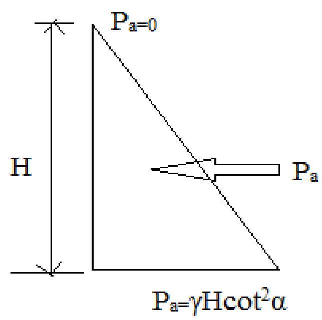
W.KT $K_a = \frac{1-sin\varphi}{1+sin\varphi} = Cot^2\alpha$

$$P_a = K_a \gamma Z - - - -(2)$$

Pressure Diagram:

At top ------- $Z=0, P_a=0$

At bottom ------- $Z=H, P_a=\gamma H\cot^2\alpha$



Consider for 1m run backfill

Total active earth pressure per m=Area of pressure diagram height

$$=\frac{1}{2}, K_a\gamma H.1.H$$

$$P_a=\frac{1}{2}, K_a\gamma H^2$$

P_a act as a distance H/3 from base.

If the soil is dry, γ is the dry weight of the soil, if wet, γ is the moist weight.

Backfill is Cohesive soil:

For active Pressure:

$$\sigma_v=\gamma Z$$

$$\sigma_h=P_a$$

According to principal stress relationship:

$$\sigma_v=2Ctan\alpha+\sigma_h\tan^2\alpha-----(1)$$

Substitute σ_v and σ_h value in eqn(1)

$$\gamma Z=2Ctan\alpha+P_a\tan^2\alpha$$

$$P_a\tan^2\alpha=\gamma Z-2Ctan\alpha$$

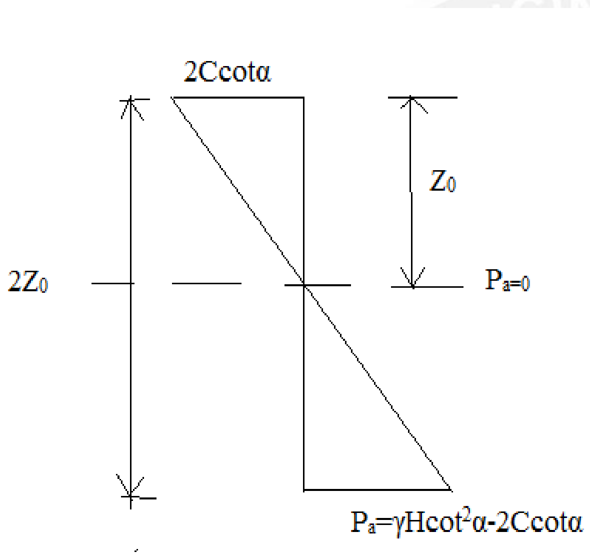
$$P_a = \frac{\gamma Z}{\tan^2 \alpha} - \frac{2Ctan\alpha}{\tan^2 \alpha}$$

$$P_a = \gamma Z \cot^2\alpha - 2C\cot\alpha - - - - (2)$$

Pressure Diagram:

At top ------- $Z=0, P_a=-2C\cot\alpha$

At bottom ------- $Z=H, P_a=\gamma H\cot^2\alpha-2C\cot\alpha$



If $P_a=0, Z=Z_0$

$$0 = \gamma Z_0 \cot^2 \alpha - 2C\cot\alpha$$

$$\gamma Z_0 \cot^2 \alpha = 2C\cot\alpha$$

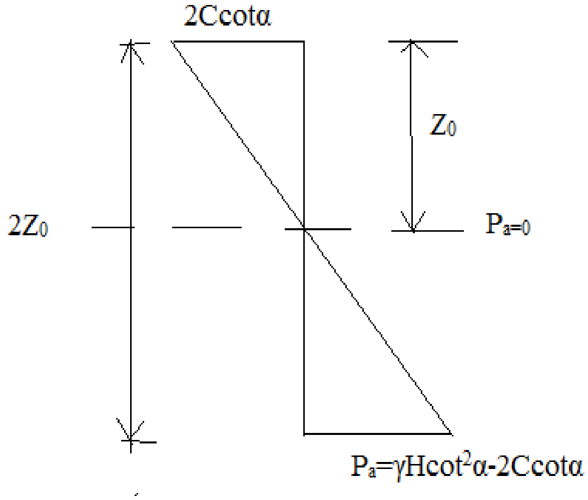
$$Z_0 c = \frac{2C\cot\alpha}{\gamma \cot^2 \alpha} = \frac{2C}{\gamma \cot \alpha} = \frac{2C\tan\alpha}{\gamma}$$

Z_0 indicates the soil can withstand comfortably without slip.

The depth upto which the cohesive soil can withstand without any support is known as critical height (H_c)

$$H_c = 2Z_0$$

$$H_c = 2x\frac{2C}{\gamma}tan\alpha$$



$$H_c = \frac{4C}{\gamma}\sqrt{N\varphi} \qquad [\alpha = N\varphi]$$

Height of backfill below P_a=0,H-Z_0

Consider for 1m run backfill

Total active earth pressure per m=Area of pressure diagram x Height

$$= \frac{1}{2}, (\gamma H cot^2\alpha - 2C\ cot\alpha) x (H - Z_0)$$

2.SUBMERGED BACKFILL:

In this case, the sand fill behind the retaining wall is saturated with water. The lateral pressure is made up of two components:

For Active Pressure:

a)Backfill is fully submerged

Lateral pressure due to two component

Due to submerged unit weight of soil

Due to pore water

Consider a cohesionless soil with unit weight γ' and height Z

$$P_a = K_a\gamma' Z --- for\ submerged\ soil$$

$$P_a = \gamma_w Z --- pore\ water$$

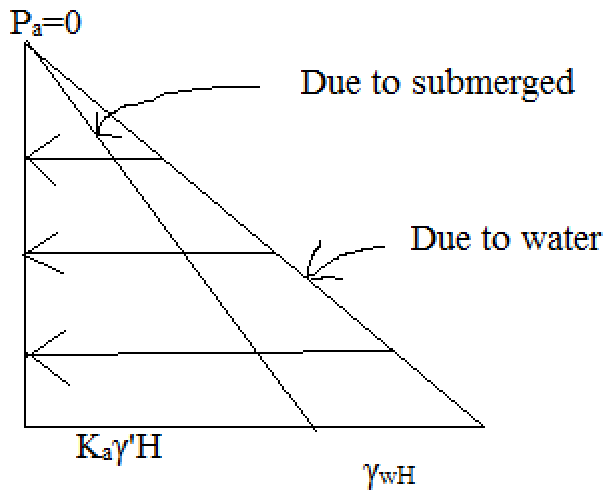
Active earth pressure,

$$P_a = K_a \gamma' Z + \gamma_w Z$$

Pressure diagram,

At top ------- Z=0,P_a=0

At bottom ------- $Z = H, P_a = K_a \gamma' H + \gamma_w H$



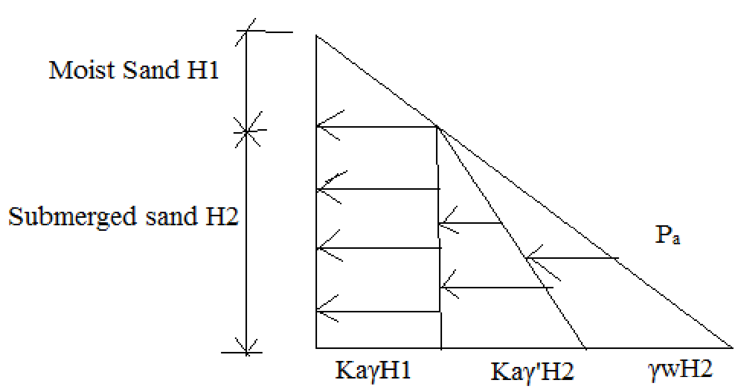
Total Pressure$P_a = \frac{1}{2} K_a \gamma' H^2 + \frac{1}{2} \gamma_w H^2$

Case b) If backfill is partially submerged

The pressure diagram

γ=unit weight of moist sand having depth H1

γ'=unit weight of moist sand having depth H2



$$P_a = K_a \gamma H_1 + K_a \gamma' H + \gamma_w H$$

Total Pressure per m

$$P_a = \frac{1}{2}K_a\gamma H_1^2 + K_a\gamma' H_2^2 + \frac{1}{2}\gamma_w H^2$$

$$\bar{Z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_a}$$

3.BACKFILL WITH UNIFORM SURCHARGE:

If the backfill is horizontal and carries a surcharge of uniform intensity q per unit area. The vertical pressure increment, at any depth z, will increase by q. the increase in the lateral pressure due to this will be K_aq.

For active pressure:

Consider a surcharge load (q) is acting on the top of backfill. It act as vertical stress[σ_v=q]

For surcharge load alone C=0

According to principal stress relationship,

$$\sigma_v = 2Ctan\alpha + \sigma_h \tan^2 \alpha - - - - - - (1)$$

$$\sigma_v = q$$

$$\sigma_h = P_a$$

Substitute σ_v and σ_h value in eqn(1)

$$q = P_a \tan^2 \alpha$$

$$P_a \tan^2 \alpha = \gamma Z - 2Ctan\alpha$$

$$P_a = \frac{q}{\tan^2 \alpha}$$

$$P_a = q\cot^2\alpha = \mathrm{qK_a} - - - -(2)$$

Pressure Diagram:

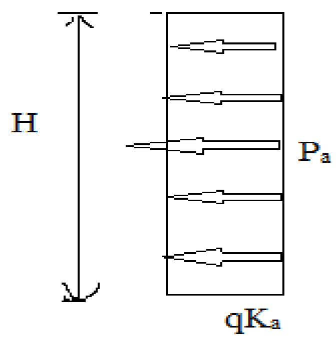
At top ------- Z=0,P_a=qK_a

At bottom ------- Z=H,P_a= qK_a

Total active pressure per m run

P_a=Area of pressure diagram X height

$= qK_a \, x1xH = q\cot^2 \alpha$



Total active pressure=Pressure due to surcharge + pressure due to backfill

$$= q\cot^2\alpha + \frac{1}{2}, K_a \gamma H^2$$

At the base of the wall, the pressure intensity is $P_a = 1/2K_a\gamma H + K_a q$

4.BACKFILL WITH SLOPING SURFACE:

Let the sloping surface behind the wall be inclined at the angle β with the horizontal; β is called the surcharge angle. In finding out the active earth pressure for this case by Rankine's theory, an additional assumption that the vertical and lateral stresses are conjugate is made. It can be shown that if the stress on the given plane at a given point is parallel to the another plane, the stress on the latter plane at the same point must be parallel to the first plane. such planes are called the conjugate planes the stresses acting on them are called conjugate stresses.

Consider a soil element at point A at depth z with in a backfill with a sloping surface. The top plane of the element is parallel to the ground plane and the other plane conjugate to this is vertical. Let σ and p be the conjugate stresses, σ being vertical and p being the parallel to the sloping backfill. Being conjugate, both the vertical pressure and lateral pressure have the same angle of obliquity β, which is equal to the surcharge angle.

$$\sigma_1 - \sigma_3 / \sigma_1 + \sigma_3 = \sin \phi \text{----(1)}$$

Mohr circle corresponding to the principal stress intensity σ_1 and σ_3 at A. OA_1 represents the resultant stress p and OA_2 represents the resultant stress σ. Draw OB perpendicular to $A_1 A_2$.

$$OB = OC \cos\beta \text{-------(2)}$$

$$BC = OC \sin\beta = \sin\beta\, (\sigma 1 + \sigma 3)/2 \text{------(3)}$$

$$A_1B = BA_2 = \sqrt{(A_1C^2 - BC^2)} = \sqrt{((\sigma_1 - \sigma_3)/2)^2 - (\sigma_1 + \sigma_3)/2)^2 \sin^2\beta)}$$

From (1) ,

$A_1B = BA_2 = (\sigma1 + \sigma3)/2 \sqrt{}(\sin^2\phi-\sin^2\beta)$------(4)

Now stress

$\sigma = OB+BA_2$

$= (\sigma1 + \sigma3)/2 \cos\beta + (\sigma1 + \sigma3)/2 \sqrt{}(\sin^2\phi-\sin^2\beta)$ ------ (5)

And stress $p= OB-A_1B$

$= (\sigma1 + \sigma3)/2 \cos\beta - (\sigma1 + \sigma3)/2 \sqrt{}(\sin^2\phi-\sin^2\beta)$ ------ (6)

Dividing (5) & (6) we get,

$p/\sigma=K=\cos\beta - \sqrt{}(\sin^2\phi-\sin^2\beta)/\cos\beta+ \sqrt{}(\sin^2\phi-\sin^2\beta)$

$K= \cos\beta - \sqrt{}(\cos^2\beta-\cos^2\phi)/\cos\beta+ \sqrt{}(\cos^2\beta-\cos^2\phi)$

The ratio K is called conjugate ratio.

For the present case,

$\sigma = (\gamma.z.b\cos\beta/b)$

$=\gamma.z.\cos\beta$

$P_a=\gamma.z.\cos\beta\ (\cos\beta - \sqrt{}(\cos^2\beta-\cos^2\phi)/\cos\beta+ \sqrt{}(\cos^2\beta-\cos^2\phi))$

$$P_a = K_a \gamma Z$$

$$K_a = cos\beta \left[\frac{cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}\right]$$

$$K_a=(1-\sin\phi)/(1+\sin\phi)$$

The total active pressure P_a for the wall of height H is given by$P_a=1/2\ K_a\gamma.H^2$

If the backfill is submerged, the lateral pressure due to the submerged weight of the soil will act at β with horizontal, while the lateral pressure due to water will act normal to the wall.

<u>5.INCLINED BACK AND SURCHARGE</u>:

A retaining wall with an inclined back supporting a backfill with horizontal ground surface. The total active pressure P_1 is first calculated on a vertical plane BC passing through the heel. The total pressure P is the resultant of the horizontal pressure P_1 and the weight W of thewedge ABC:

Where,

$$P = \sqrt{P_1^2 + W^2}$$

$$P_1 = 1/2\, K_a \gamma . H^2$$

The active earth pressure is first calculated on a vertical plane passing through the heel and intersecting the surface of the backfill or it's extension in point C. the height H of vertical plane is represented by BC. The resultant of P is the vector sum of P_1 and W, where W is the weight of the soil contained in the triangle ABC.

For Passive Pressure:

Case i) Dry or moisture backfill with no surcharge:

Backfill is Cohesion less soil:

$\sigma_1 = \sigma_h, \sigma_3 = \sigma_v$

$$\sigma_v = \gamma Z$$

$$\sigma_h = P_p$$

According to principal stress relationship:

C=0

$$\sigma_h = 2Ctan\alpha + \sigma_v \tan^2 \alpha - - - - - - (1)$$

Substitute σ_v and σ_h value in eqn(1)

$$P_p = 2Ctan\alpha + \gamma Z \tan^2 \alpha$$

$$P_p = \gamma Z \tan^2 \alpha$$

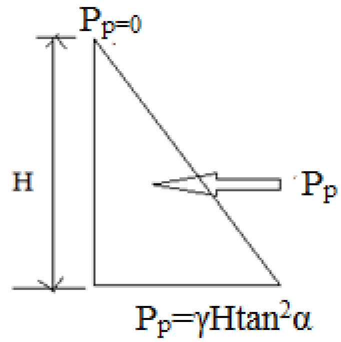
W.KT $K_p = \frac{1+sin\varphi}{1-sin\varphi} = tan^2\alpha$

$$P_p = K_p \gamma Z - - - -(2)$$

Pressure Diagram:

At top ------- Z=0,P_p=0

At bottom ------- Z=H,P_p=$\gamma H \tan^2\alpha$



Consider for 1m run backfill

Total passive pressure per m=Area of pressure diagram height

$$= \frac{1}{2}, K_p \gamma H.1.H$$

$$P_p = \frac{1}{2}, K_p \gamma H^2$$

P_p act as a distance H/3 from base.

Backfill is Cohesive soil:

$\sigma_1 = \sigma_h, \sigma_3 = \sigma_v$

$$\sigma_v = \gamma Z$$

$$\sigma_h = P_p$$

According to principal stress relationship:

$$\sigma_h = 2Ctan\alpha + \sigma_v \tan^2 \alpha ------(1)$$

Substitute σ_v and σ_h value in eqn(1)

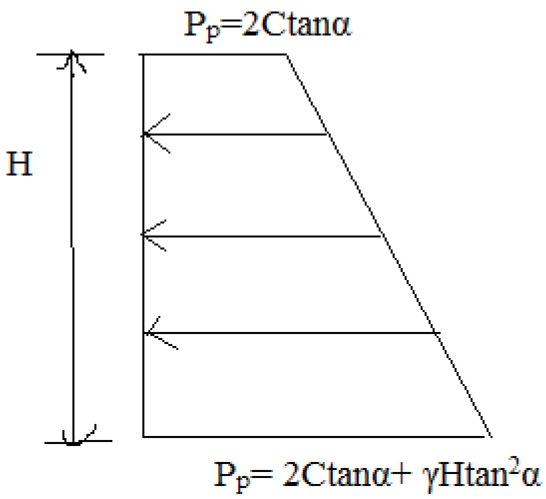
$$P_p = 2Ctan\alpha + \gamma Z \tan^2 \alpha$$

$$P_p = 2\text{Ctan}\alpha + \gamma Z \tan^2\alpha ----(2)$$

Pressure Diagram:

At top ------- Z=0,P_p=2Ctanα

At bottom ------- Z=H,P_p= 2Ctanα+ $\gamma H\tan^2\alpha$



Consider for 1m run backfill

Total passive earth pressure per m=Area of pressure diagram x Height

$$= \frac{1}{2}(2C\ tan\alpha + \gamma Htan^2\alpha + 2C\ tan\alpha)xHx1$$

$$= \frac{1}{2}(4C\ tan\alpha + \gamma Htan^2\alpha)xHx1$$

Case iii) Surcharge Load:

For passive pressure:

Consider a surcharge load (q) is acting on the top of backfill. It act as vertical stress[σ_v=q]

For surcharge load alone C,

$\sigma_1 = \sigma_h$,$\sigma_3 = \sigma_v$

According to principal stress relationship,

$$\sigma_v = 2Ctan\alpha + \sigma_h \tan^2\alpha - - - - - - (1)$$

$$\sigma_v = q$$

$$\sigma_h = P_p$$

Substitute σ_v and σ_h value in eqn(1)

$$P_p = q\tan^2\alpha$$

$$P_a = q\tan^2\alpha = qK_p - - - -(2)$$

Pressure Diagram:

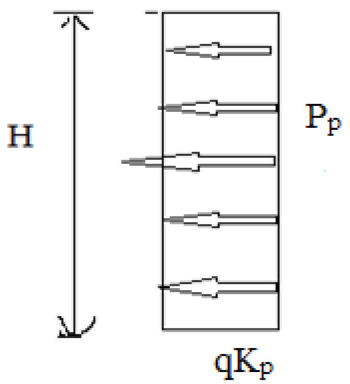
At top ------- $Z=0, P_p=qK_p$

At bottom ------- $Z=H, P_a= qK_p$

Total active pressure per m run

P_p=Area of pressure diagram X height

$= qK_p \, x1xH=q\tan^2 \alpha$



Case iv) Effect of inclined surcharge or sloping backfill (or) Expression for earth pressure in case of sub charge angle.

$$\sigma_3 = \sigma_v$$

$$\sigma_1 = \sigma_h$$

For passive pressure:

$$P_p = K_p \gamma Z$$

$$K_p = cos\beta \left[\frac{cos\beta + \sqrt{\cos^2 \beta - \cos^2 \varphi}}{cos\beta - \sqrt{\cos^2 \beta - \cos^2 \varphi}}\right]$$

Case v) Effect of inclined surcharge or sloping backfill (or) Expression for earth pressure in case of sub charge angle.

$$\sigma_3 = \sigma_h$$

$$\sigma_1 = \sigma_v$$

For active pressure:

$$P_a = K_a \gamma Z$$

$$K_a = cos\beta \left[\frac{cos\beta - \sqrt{\cos^2 \beta - \cos^2 \varphi}}{cos\beta + \sqrt{\cos^2 \beta - \cos^2 \varphi}}\right]$$

For passive pressure:

$$P_p = K_p \gamma Z$$

$$K_p = cos\beta \left[\frac{cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}{cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}\right]$$

Problems:

1. A gravity retaining wall retains 10 m of a backfill, unit weight of soil =18 kN/m^{3,} angle of shearing resistance =30° with a horizontal surface. Assume the wall interface to be vertical, determine (i) the magnitude and point of application of the total active pressure (ii) if the water table is at a height of 5m, and how far do the magnitude and the point of the application of active pressure changed. Take submerged unit weight = 10kN/m^3.

H= 10m, $\emptyset$ = 30°, $\gamma = 18kN/m^2$, γ_{sub} =10kN/m^2

$$K_a = \frac{1 - \sin\varphi}{1 + \sin\varphi} = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}$$

$$\text{Active Pressure at base of wall} = \frac{1}{3}x18x10 = 60KN/m^2$$

$$Total\ active\ thrust\ , P_a = \frac{1}{2}K_a \gamma H^2$$

$$P_a = \frac{1}{2}x\frac{1}{3}x18x10 = \frac{60KN}{m^2}$$

ii)Water – table at 5m from surface:

$P_a = K_a\ \gamma\ H_1 + K_a\ \gamma_{sub}\ H_2 + \gamma_w H$

$$= \frac{1}{3}x18x5 + \frac{1}{2}x10x5 + 9.81x5$$

= 104.05 kN/m^2 at base

P_1 = 1/2 x 1/3 x 18 x 5 x 5 = 75 kN,Y_1 =5/3 + 5 = 6.67m

P_2 =5 x 1/3 x18 x 5 = 150 kN, Y_2 = 5/2 = 2.5m

P_3 = ½ x 1/3 x 10 x 5 x 5 =41.67 kN,Y_3 = H / 3 =5/3 = 1.67m

P_4 = 1/3 X 9.81 X 5 X 5 =122.63 kNY_4 = H/3 = 1.67 m

Total thrust, $p_a = p_1 + p_2 + p_3 + p_4$

= 75 +150+41.67 +122.63

P_a = 389.3 kN per meter length of wall.

Taking moments about base

$$P \times \bar{Y} = p_1 y_1 + p_2 y_2 + p_3 y_3 + p_4 y_4$$

$$389.3\,\bar{y} = (75 \times 6.67) + (150 \times 2.5) + (41.67 \times 1.67) + (122.63 \times 1.67)$$

$$\therefore \bar{y} = 2.95 \text{ m}$$

∴ Total thrust of 389.3 kN per meter length of wall will act at 2.95 m from base of wall.

2.A retaining wall is 4 m high. Its back is vertical and it has got sandy backfill up to its top. The top of the fill is horizontal and carries a uniform surcharge of 85 kN/m^2. Determine the active pressure on the wall per meter length of wall. Water table is 1m below the top of the fill. Dry density of soil = 18.5 kN/m^3. Moisture content of soil above water table =12%. Angle of internal friction of soil = 30°, specific gravity of soil particles = 2.65. Porosity of backfill = 30°. The wall friction may be neglected.

$$e = \frac{n}{1-n}$$

$$= 0.43$$

$$\gamma = (1 + w) \times \gamma_d = (1+0.12) \times 18.5 = 20.7 \text{ kN/m}^3$$

$$K_a = \frac{1 - sin\varphi}{1 + sin\varphi}$$

$$= \frac{1 - 05}{1 + 0.5} = 0.3$$

$$\gamma_{sub} = \gamma_{sat} - \gamma_w = 20.7 - 9.81 = 11.52 KN/m^3$$

(i) Due to soil above W.T

$P_1 = ½ K_a \gamma H^2 + K_a \gamma H$

$=1/2 \times 0.33 \times 18.5 \times 4^2 + 0.33 \times 18.5 \times 4$

= 21.85 kN/m

(ii) Due to submerged soil

$P_2 = ½ \times 0.333 \times 11.52 \times 3^2 = 17.3$ kN/m.

Due to water pressure, $P_3 = ½ \times 9.81 \times 3^2 = 45$ kN/m.

∴ Total active pressure = $P1 + P_2 + P_3 + P_4$

$P_a = 21.58 + 17.3 + 45 + 113.2$

=197.08 kN/meter length of wall.

3. A retaining wall is 9 m high retain on a cohesion less soil with an angle internal friction 33^0.The surface is level with the level with the top of wall. The unit weight of the top 3m of the fill is $21KN/m^3$ and that the rest is $27KN/m^3$.Find the magnitude and point of application of resultant active thrust.

$$K_a = \frac{1 - sin\varphi}{1 + sin\varphi}$$

$$= \frac{1 - sin33^o}{1 + sin33^0} = 0.295$$

P_a @3m= $K_a \gamma H$=0.295x21x3=18.6KN/m^3

P_a @3m= $K_a \gamma H$=0.295x(18.6+27x6) = 66.4KN/m^3

Total active thrust,

P_a=1/2x3x18.6+18.6x6+1/2x6x47.8=283KN

$$\bar{z} = \frac{27.9(6+1) + 111.6x3 + 143.4x2}{283} = 2.89m$$

4.A wall of6m height sand having a unit weight of $20KN/m^3$ and angle of internal friction of 30^0.If the surface of the backfill slope upwards @15^0 to the horizontal. Find the active thrust per unit length of the wall using Rankine's theory. Solve the problem both analytically.

$$P_a = \frac{K_a \gamma H^2}{2}$$

$$K_a = cos\beta \left[\frac{cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}\right]$$

Cos15=0.96, $\cos^2 15$=0.93, $\cos^2 30$=0.75

$$K_a = cos15 \left[\frac{cos15 - \sqrt{\cos^2 15 - \cos^2 30}}{cos15 + \sqrt{\cos^2 15 - \cos^2 30}}\right]$$

$$K_a = 0.96 \left[\frac{0.96 - \sqrt{0.93 - 0.75}}{0.96 + \sqrt{0.93 - 0.75}}\right]$$

$$= 0.96x\frac{0.5}{1.418} = 0.37$$

$$P_a = \frac{0.37x20x6^2}{2} = 131.2KN/m$$