ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY
Approved by AICTE & Affliated to anna university
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DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

REGULATION 2021

UNIT I: BASIC & STATICS OF PARTICLES

Forces in space –Resultant and Equilibrium of particles in Three Dimensions [Vector approach]

Quantities:

Physical Quantities are

i) Scalar quantity

ii) Vector quantity

Scalar Quantity:

Scalar quantity are those which are completely defined by their magnitude only.

Ex. 2kg of mass

25℃ of temperature

10 m/s acceleration

Vector Quantity

The Quantity which are defined by their magnitude and direction is known as vector quantity.

Ex. 10 n force acting vertically downward direction

9.81 m/s^2acceleration directed towards is centre of the earth.

Types of Vectors

1. Free Vector
2. Fixed Vector
3. Sliding Vector

4. Unit Vector
5. Zero(or) Null Vector
6. Equal Vector
7. Like Vector

1. **Free Vector:**

If the vector may act at any point in space maintaining some magnitude and direction with no specific point of action is called Free vector.

2. **Fixed Vector:**

The vector whose point of action is same is called Fixed vector.

3. **Sliding vector:**

The vector may be applied at any point along its Line of action is called sliding vector.

4. **Unit vector:**

A vector whose magnitude is unity is called unit vector.

AB, n= $\frac{\vec{AB}}{|\vec{A}||\vec{B}|}$

5. **Zero (or) Null vector**

It is defind as the vector whose magnitude is zero.

6. **Equal vector**

Those vector which are similar to each other but have same magnitude and direction in same and equal is called equal vector.

7. **<u>Like vector:</u>**

These vector each are slimier to each other and have same direction and unequal magnitude is called like vector.

8. **<u>Vector Addition:</u>**

We Law of vector addition are

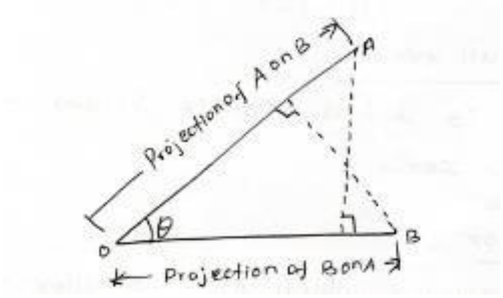
A+B=B+A [commutative Law]

A+ [B+C] =A+B+C [associative Law]

<u>Vector Product</u>

1. Scaler product(or) dot product
2. Vector product(or) cross product

1. **<u>Scaler product (or) dot product:</u>**



$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\, cos\theta$$

When the angle b/w two vector $\vec{A}\&\vec{B}$

$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|\,.|\vec{B}|}$$

(i) When $\theta = 0°$

Then $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|$

That is the two vector are in same direction.

(ii) when $\theta = 90° \vec{A}.\vec{B} = 0$

so the vectors are perpendicular

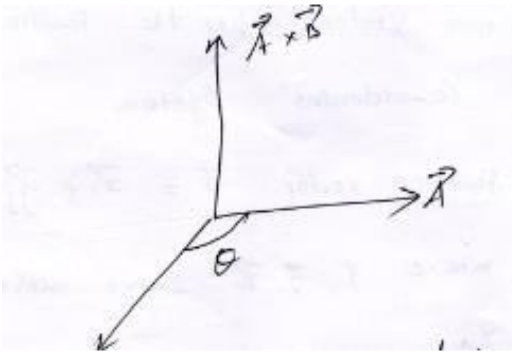
(iii) $\vec{A}.\vec{B} = |A|$

When the projection B or A

If the angle b/w the A & B given

$$cos\theta = \frac{\vec{A}.\vec{B}}{AB}$$

2. <u>Vector Product (or) cross product</u>



In forms of Rectangular component A=$A_{xi} + A_{yi} + A_{zk}$

B= $B_{xi} + B_{yi} + B_{zk}$

$$A \times B = \begin{vmatrix} i & j & k \\ Ax & Aj & Az \\ Bx & By & Bz \end{vmatrix}$$

The Angle b/w the vector is given by

$$sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

Dot product of force and displacement given workdone.

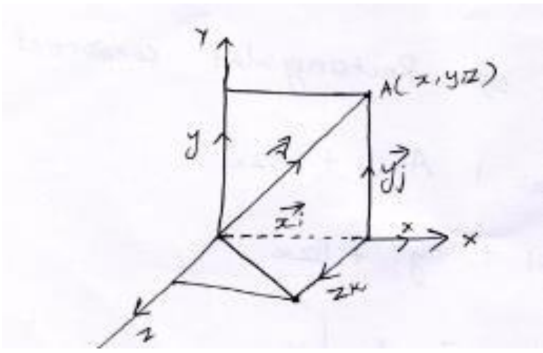
∴ Workdone=F_d

Position vector:

Position vector defines the position of points in any co-ordinate system.

Position vector $\vec{r} = \vec{x}\vec{i} + \vec{y}\vec{j} + \vec{z}\vec{k}$

Where $\vec{i}, \vec{j}, \vec{k}$ – are unit vector



Magnitude $r = \vec{r} = \sqrt{x^2 + y^2 + z^2}$

Formula

Resultant vector $r = \vec{A} + \vec{B} + \vec{C}$

Unit vector to resultant vector $n = \frac{\vec{r}}{|\vec{R}|}$

$|R| = \sqrt{x^2 + y^2 + z^2}$

Magnitude $= \sqrt{x^2 + y^2 + z^2}$

Unit vector $n = \frac{\vec{AB}}{|\vec{AB}|}$

$\vec{AB} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$

$|AB| = \sqrt{x^2 + y^2 + z^2}$

Dot product vector $\vec{A}.\vec{B} = [\vec{i} + \vec{j} + \vec{k}].[\vec{i} + \vec{j} + \vec{k}]$

Angle b/w the vector $s\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

Cross Product vector $A \times B = \begin{vmatrix} i & j & k \\ x1 & y1 & z1 \\ x2 & y2 & z2 \end{vmatrix}$

Angle b/w the vector $sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

Problems:

1. Three vectors A, B, C are given as $A = 3\vec{i} + 2\vec{j} + 4\vec{k}, B = 4\vec{i} - 2\vec{j} + 6\vec{k}$

   $C = 2\vec{i} - 3\vec{j} - \vec{k}$, find

   1. The resultant vector
   2. A unit vector || er top resultant vector

   Given:

   $A = 3\vec{i} + 2\vec{j} + 4\vec{k}$

   $B = 4\vec{i} - 2\vec{j} + 6\vec{k}$

   $C = 2\vec{i} - 3\vec{j} - \vec{k}$

   To find:

   1. The resultant vector
   2. A unit vector || [er] top resultant vector

   Solution:

   1. The resultant vector

$$R = \vec{A} + \vec{B} + \vec{C}$$

$$R = 3\vec{i} + 2\vec{j} + 4\vec{k} + 4\vec{i} - 2\vec{j} + 6\vec{k} + 2\vec{i} - 3\vec{j} - \vec{k}$$

$$R = 9\vec{i} - 3\vec{j} + 9\vec{k}$$

2. A unit vector || er top resultant vector

Unit Vector $n = \frac{\vec{R}}{|\vec{R}|}$

$$\vec{R} = 9\vec{i} - 3\vec{j} + 9\vec{k}$$

$$|R| = \sqrt{9^2 + (-3)^2 + 9^2} = \sqrt{81 + 9 + 81}$$

$$|R| = \sqrt{171}$$

$$R = 13.08$$

$$n = \frac{9\vec{i} - 3\vec{j} + 9\vec{k}}{13.08}$$

$$n = \frac{9}{13.08}\vec{i} - \frac{3}{13.08}\vec{j} + \frac{9}{13.08}\vec{k}$$

Unit vector$n = 0.68i - 0.22j + 0.68k$

2.If $A = \vec{i} - \vec{j} - 2\vec{k}, B = 3\vec{i} + 2\vec{j} - 2\vec{k} C = 2\vec{i} + 3\vec{j} - 4\vec{k}$, find

$2A - 2B + 3C$ n terms of i, j, k and its magnitude.

Given :

$$A = \vec{i} - \vec{j} - 2\vec{k}$$

$$B = 3\vec{i} + 2\vec{j} - 2\vec{k}$$

$C = 2\vec{i} + 3\vec{j} - 4\vec{k}$

To find:

$2A - 2B + 3C$ =? magnitude

Solution:

$2A - 2B + 3C = ?$

$2A = 2[\vec{i} - \vec{j} - 2\vec{k}]$

$2A = 2\vec{i} - 2\vec{j} - 4\vec{k}$

$2B = 2[3\vec{i} + 2\vec{j} - 2\vec{k}]$

$2B = 6\vec{i} + 4\vec{j} - 4\vec{k}$

$3C = 3[2\vec{i} + 3\vec{j} - 4\vec{k}]$

$3C = 6\vec{i} + 9\vec{j} - 12\vec{k}$

$2A - 2B + 3C = [2\vec{i} - 2\vec{j} - 4\vec{k}] - [6\vec{i} + 4\vec{j} - 4\vec{k}] + 6\vec{i} + 9\vec{j} - 12\vec{k}$

$= 2\vec{i} - 2\vec{j} - 4\vec{k} - 4\vec{j} + 4\vec{k} + 9\vec{j} - 12\vec{k}$

$2A - 2B + 3C = 2\vec{i} + 3\vec{j} - 12\vec{k}$

$2A - 2B + 3C = \sqrt{2^2 + 3^2 + (-12)^2} = \sqrt{4 + 9 + 144}$

$$|2A - 2B + 3C| = \sqrt{157} = 12.53$$

$$|2A - 2B + 3C| = 12.53$$

3. Find the unit vector along the line which ordinates at point (2,3,-2) and passes through the point(1,0,5)

Given:

At point (2,3,-2)=(x_1,y_1,z_1)(1,0,5) = (x_2,y_2,z_2)

To find:Unit vector 'n'=?

Soln:

Unit vector $n = \frac{\vec{}}{|AB|}$

$AB = -1$

$A = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$

$B = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$

$$AB = (x_2 - x_1)\vec{i} + (y_2\text{-}y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$AB = (1 - 2)\vec{} + (0\text{-}3)\vec{j} + (5 - (-2))\vec{k}$$

$$AB = -1\vec{i} - 3\vec{j} + 7\vec{k}$$

$$|AB| = \sqrt{(-1)^2 + (-3)^2 + (7)^2}$$

$$|AB| = \sqrt{1 + 9 + 49} = \sqrt{59}$$

$$|AB| = 7.68$$

$$n = \frac{\vec{AB}}{|\vec{AB}|} \frac{= -1\vec{i} - 3\vec{j} + 7\vec{k}}{7.68}$$

$$n = \frac{-1}{7.68}\vec{i} - \frac{3}{7.68}\vec{j} - \frac{7}{7.68}\vec{k}$$

$$\boldsymbol{ans}\; n = -0.13\hat{i} - 0.39\hat{j} + 0.91\vec{k}$$

4. Find the dot product of two vector $A = 2\hat{i} - 6\hat{j} - 3\vec{k}$,

   $B = 4\hat{i} + 3\hat{j} - \vec{k}$ also find the angle b/w the angle b/w them.

   <u>Given Data:</u>

   $A = 2\hat{i} - 6\hat{j} - 3\vec{k}$

   $b = 4\hat{i} + 3\hat{j} - \vec{k}$

   <u>To find:</u>

   1. Dot product of Two vector
   2. Angle b/w the vector

<u>Soln:</u>

1. Dot product of two vector

   $A.B = [2\hat{i} - 6\hat{j} - 3\vec{k}].[4\hat{i} + 3\hat{j} - \vec{k}]$

   $= 2 \times 4 + [-6] \times 3 + [-3] \times [-1]$

   $= 8 - 18 + 3$

$A.B = -7$

2. Angle b/w two vector

   $$Cos\theta = \frac{A.B}{|A||B|}$$

   $|\vec{A}| = \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

   $|\vec{A}| = \sqrt{4 + 36 + 9}$

   $|\vec{A}| = 7$

   $|\vec{B}| = \sqrt{(4)^2 + (3)^2 + (-1)^2} = 16 + 9 + 1 = \sqrt{16 + 9 + 1}$

$|\vec{B}| = \sqrt{26}$

$$cos\theta = \frac{-7}{7\sqrt{26}} = \frac{-1}{\sqrt{26}}$$

$$\theta = cos^{-1}\left[\frac{-1}{\sqrt{26}}\right]$$

5. Find the cross product of vector $A = 2\vec{i} - 6\vec{j} - 3\vec{k}, B = 4\vec{i} + 3\vec{j} - \vec{k}$ and the angle b/w them.

Given:

$A = 2\vec{i} - 6\vec{j} - 3\vec{k}$

$B = 4\vec{i} + 3\vec{j} - \vec{k}$

To find:

1. Cross product of vector
2. Angle b/w hem two vector

Soln:

Cross product: $\times B$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$= \vec{i}[(-6 \times -1) - (-3 \times 3)] - \vec{j}[(2 \times -1) - (4 \times -3)]$

$+\vec{k}[(2 \times 3) - (4 \times -6)$

$\vec{i}[6 + 9] - \vec{j}[-2 + 12] + \vec{k}[6 + 24]$

$\vec{A} \times \vec{B} = 15\vec{i} - 10\vec{j} + 30\vec{k}$

$$sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

$|\vec{A} \times \vec{B}| = \sqrt{15^2 + (-10)^2 + (30)^2}$

$$|\vec{A} \times \vec{B}| = \sqrt{1225}$$

$$|\vec{A} \times \vec{B}| = 35$$

$$|\vec{A}| = \sqrt{2^2 + (-6)^2 + (-3)^2} = \sqrt{4 + 36 + 9}$$

$$|\vec{A}| = \sqrt{49}$$

$$|\vec{A}| = 7$$

$$|\vec{B}| = 4^2 + 3^2 + (-1)^2 = \sqrt{16 + 9 + 1} = \sqrt{26}$$

$$|\vec{B}| = \sqrt{26}$$

$$sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|} = \frac{35}{7 \times \sqrt{26}}$$

$$sin\theta = \frac{5}{\sqrt{26}}$$

$$\theta = sin^{-1} \frac{5}{\sqrt{26}}$$

$$\theta = 78.69'$$

Formula used for three dimension force analysis

Force vector $\vec{F} = \lambda \times F$

$$\lambda = \frac{\overrightarrow{OA}}{|\overrightarrow{OA}|}$$

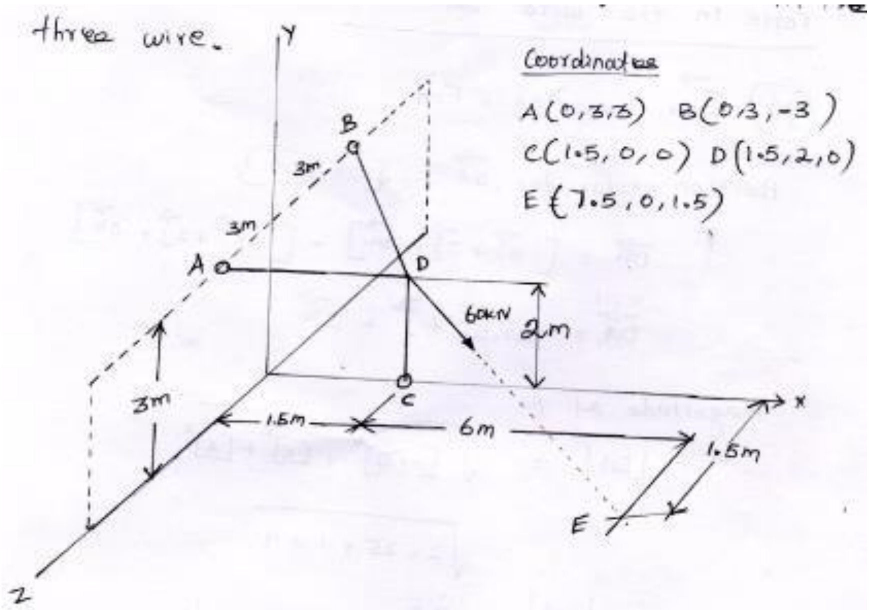
Magnitude $|OA| = \sqrt{(x)^2 + (y)^2 + (z)^2}$

$$\vec{R} = \vec{F_A} + \vec{F_B} + \vec{F_C}$$

$$R = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$$

$$\theta = cos^{-1}\left(\frac{Rx}{R}\right)$$

1. In the figures shown, three wire jointed at D. The Two ends A and B are on the wall and the other end C is on the ground. The wire CD is vertical. A force of 60 KN is applied at ‘D’ and it passes through a point E on the ground as shown in fig. Find the forces in all the three wire.



Given:

$F_{DE} = 60KN$

To find:

F_{DA} =? F_{DB} =? F_{DC} =?

Soln:

$OA = 0\vec{i} + 3\vec{j} + 3\vec{k}$

$OB = 0\vec{i} + 3\vec{j} - 3\vec{k}$

$OC = 1.5\vec{i} + 0\vec{j} + 0\vec{k}$

$OD = 1.5\vec{i} + 2\vec{j} + 0\vec{k}$

$OE = 7.5\vec{i} + 0\vec{j} + 1.5\vec{k}$

Force in the wire DA ‘F_{DE}’

$\vec{F_{DA}} = \lambda_{DA} \times F_{DA}$

Position vector for $\vec{DA} = [\vec{OA} - \vec{OD}]$

$\vec{DA} = [0\vec{i} + 3\vec{j} + 3\vec{k}] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$

$\vec{DA} = -1.5\vec{i} + \vec{j} + 3\vec{k}$

Magnitude of DA

$|DA| = \sqrt{(-1.5)^2 + (1)^2 + (3)^2}$

$= \sqrt{2.25 + 1 + 9}$

$|DA| = 3.5$

$\lambda_{DA} = \frac{\vec{DA}}{|DA|} = \frac{-1.5\vec{i} + \vec{j} + 3\vec{k}}{3.5}$

$\lambda_{DA} = -0.428\vec{i} + 0.285\vec{j} + 0.857\vec{k}$

$\vec{F_{DA}} = \lambda_{DA} \times F_{DA}$

$\vec{F_{DA}} = -0.428\vec{i}F + 0.285\vec{j}F_{DA} + 0.857\vec{k}F_{DA}$

$\vec{F_{DA}} = -0.428\vec{i}F + 0.285\vec{j}F_{DA} + 0.857\vec{k}F_{DA}$------- (1)

Force in the wire DB

Force from D to B coordinates

$\vec{F_{DB}} = \lambda_{DB} \times F_{DB}$

$$\lambda_{DB} = \frac{\vec{DB}}{|DB|}$$

$$\vec{DB} = [\vec{OB} - \vec{OD}]$$

$= [0\vec{i} + 3\vec{j} - 3\vec{k}] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$

$\vec{DB} = -1.5\vec{i} + \vec{j} - 3\vec{k}$

$$|\vec{DB}| = \sqrt{(1.5)^2 + (1)^2 + (-3)^2}$$

$$|\vec{DB}| = 3.5$$

$$\lambda_{DB} = \frac{DB}{|DB|} = \frac{-1.5\vec{i} + \vec{j} - 3\vec{k}}{3.5}$$

$\lambda = 0.428\vec{i} + 0.285\vec{j} - 0.857\vec{k}$

$\vec{F_{DB}} = \lambda_{DB} \times F_{DB}$

$F = -0.428F_{DB}\vec{i} + 0.285F_{DB}\vec{j} - 0.857F_{DB}\vec{k}$-------------- (2) Force

in the wire DC

Force from D to C coordinate

$\vec{F_{DC}} = \lambda_{DC}.F_{DC}$

$\lambda_{DB} = \frac{\vec{DC}}{|\vec{DC}|}$

$$\vec{DC} = [\vec{OC} - \vec{OD}]$$

$= [1.5\vec{i} + 0\vec{j} - 0\vec{k}] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$

$\vec{DC} = 0\vec{i} - 2\vec{j} + 0\vec{k}$

$$|\vec{DC}| = \sqrt{(0)^2 + (-2)^2 + (0)^2} = \sqrt{4}$$

$$|\vec{DC}| = 2$$

$\lambda_{DC} = \frac{\vec{DB}}{|\vec{DB}|} = \frac{o\vec{i} - 2\vec{j} + 0\vec{k}}{2}$

$\lambda_{DC} = \vec{-j}$

$$\vec{F_{DC}} = \lambda_{DC} \times F_{DC}$$

$\vec{F_{DC}} = -F_{DC}\vec{j}$------------------ (3)

Force in the wire DE

Force from D to E

$$\vec{F_{DE}} = \lambda_{DE} . F_{DE}$$

$$\lambda_{DE} = \frac{\vec{DE}}{|\vec{DE}|}$$

$$\vec{DE} = [\vec{OE} - \vec{OD}]$$

$$= [7.5\vec{i} + 0\vec{j} - 1.5\vec{k}] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$$

$$\vec{DE} = 6\vec{i} - 2\vec{j} + 1.5\vec{k}$$

$$|\vec{DE}| = \sqrt{(6)^2 + (-2)^2 + (1.5)^2} = \sqrt{36 + 4 + 2.25}$$

$$|\vec{DE}| = 6.5$$

$$\lambda = 0.923\vec{i} + 0.307\vec{j} + 0.23\vec{k}$$

$$\vec{F_{DE}} = \lambda_{DE} \times F_{DE}$$

$$F = -0.923\vec{i} \times F_{DE} - 0.307\vec{j} \times F_{DE} + 0.23\vec{k} \times F_{DE}$$

$$F = -0.923F_{DE}\vec{i} - 0.307F_{DE}\vec{j} + 0.23F_{DE}\vec{k}$$

$$F_{DE} = 60KN$$

$$F = -0.923 \times 60\vec{i} - 0.307 \times 60\vec{j} + 0.23 \times 60\vec{k}$$

$\vec{F_{DE}} = 55.38\vec{i} - 18.42\vec{j} + 13.84\vec{k}$------------ (4)

$$F = -0.428\vec{i}F_{DA} + 0.285\vec{j}F_{DA} + 0.857\vec{k}F_{DA}$$

$$F = -0.428F_{DB}\vec{i} + 0.285F_{DB}\vec{j} - 0.857F_{DB}\vec{k}$$

$$F_{DC} = -F_{DC}\vec{j}$$

$$F = 55.38\vec{i} - 18.42\vec{j} + 13.84\vec{k}$$

Applying equilibrium condition

$\sum F_x = 0, F_{DA}.F_{DB}.F_{DC}.F_{DE} = 0$

$-0.428F_{DA} - 0.428F_{DB} + 55.38 = 0$

$$-0.428[F_{DA} + F_{DB}] + 55.38 = 0$$

$F_{DA} + F_{DB} = -55.38$

$F_{DA} + F_{DB} = \frac{-55.38}{-0.428}$

$F_{DA} + F_{DB} = 129.39$-------------- (5)

$\sum F_y = 0$

$-0.285F_{DA} + 0.285F_{DB} - F_{DC} - 18.42 = 0$

$-0.285F_{DA} + 0.285F_{DB} - F_{DC} = 18.42$

$\sum F_Z = 0$

$-0.857F_{DA} - 0.857F_{DB} + 18.42 = 0$

$0.857[F_{DA} - F] + 13.84 = 0$

$F_{DA} - F_{DB} = \frac{-13.84}{0.857}$

$F_{DA} - F_{DB} = -16.15$------------- (7)

(5)⇒ $F_{DA} + F_{DB} = 129.39$

(7)⇒ $F_{DA} - F_{DB} = -16.15$

$2F_{DA} = 113.24$

$F_{DA} = \frac{113.24}{2}$

$F_{DA} = 56.62\ KN$

(5) $\Rightarrow F_{DA} + F_{DB} = 129.39$

$56.62 + F_{DB} = 129.39 \Rightarrow F_{DB} = 129.39 - 56.62$

$F_{DB} = 72.76\ KN$

(6) $\Rightarrow -0.285 \times F_{DA} + 0.285 F_{DB} - F_{DC} = 18.42$

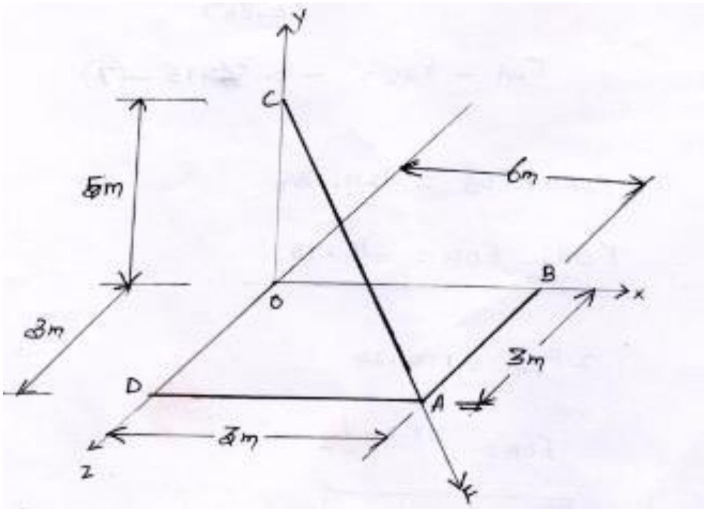
$$-0.285 \times 56.62 - 0.285 \times 72.76 - F_{DC} = 18.42$$

$$16.138 + 20.71 - F_{DA} = 18.42$$

$$F_{DA} = 16.138 + 20.71 - 18.42$$

$$F_{DA} = 18.428\ N$$

1. Fig shows three cables AB, AC,& AD that are used to support the end of a sign which exerts a force of $F = (250\vec{i} + 450\vec{j} - 150\vec{k})N$ at A. Determine the force develop in each cable.



Given:

$F = (250\vec{i} + 450\vec{j} - 150\vec{k})$ at A

To find:

Force in AB, AC &AD

Soln:

$A = (3,0,3)$

$B = (6,0,0)$

$C = (0,5,0)$

$D = (0,0,3)$

$OA = 3\vec{i} + 0\vec{j} + 3\vec{K}$

$OB = 6\vec{i} + 0\vec{j} + 3\vec{k}$

$OC = 0\vec{i} + 5\vec{j} + 0\vec{k}$

$OD = 0\vec{i} + 0\vec{j} + 3\vec{k}$

Force of AB

$F_{AB} = \lambda_{AB} \times F_{AB}$

$$\lambda_{AB} = \frac{AB}{|AB|}$$

Position vector for AB

$$AB = OB - OA = [6\vec{i} + 0\vec{j} + 0\vec{k}] - [3\vec{i} + 0\vec{j} + 3\vec{k}]$$

$$|\vec{AB}| = 3\vec{i} + 0\vec{j} - 3\vec{k}$$

$$|\vec{AB}| = \sqrt{3^2 + 0^2 + [-3]^2} = \sqrt{9 + 0 + 9}$$

$$|\vec{AB}| = \sqrt{18}$$

$$|\vec{AB}| = 4.2$$

$$\lambda_{AB} = \frac{3\vec{i} + 0\vec{y} - 3\vec{k}}{4.2}$$

$$\lambda_{AB} = 0.714\vec{i} + 0\vec{j} - 0.714\vec{k}$$

$$\vec{F_{AB}} = \lambda_{AB} \times F_{AB}$$

$$\vec{F_{AB}} = 0.714F_{AB}\vec{i} + 0\vec{j} - 0.714F_{AB}\vec{k} \text{-------------------- (1)}$$

Force on AC

$$\vec{F_{AC}} = \lambda_{AC} \times F_{AC}$$

$$\lambda_{AC} = \frac{\vec{AC}}{|\vec{AC}|}$$

Position vector of $AC = OC - OA$

$$[0\vec{i} + 5\vec{j} + 0\vec{k}] - [3\vec{i} + 0\vec{j} + 3\vec{k}]$$

$$AC = -3\vec{i} + 5\vec{j} - 3\vec{k}$$

$$|\vec{AC}| = \sqrt{(-3)^2 + (5)^2 + (-3)^2} = \sqrt{9 + 25 + 9} = \sqrt{43}$$

$$|\vec{AC}| = 6.5$$

$$\lambda_{AC} = \frac{\vec{AC}}{|\vec{AC}|} = \frac{-3\vec{i} + 5\vec{j} - 3\vec{k}}{6.5}$$

$$\lambda = -0.461\vec{i} + 0.769\vec{j} - 0.461\vec{k}$$

$$\vec{F_{AC}} = \lambda_{AC} \times F_{AC}$$

$\vec{F_{AC}} = -0.461F_{AC}\vec{i} + 0.769F_{AC}\vec{j} - 0.461F_{AC}\vec{k}$------------- (2)

Force on AD

$$\vec{F_{AD}} = \lambda_{AD} \times F_{AD}$$

$$\vec{AD} = \vec{OA} - \vec{OA} = [0\vec{i} + 0\vec{j} + 3\vec{k}] - [3\vec{i} + 0\vec{j} + 3\vec{k}]$$

$$\vec{AD} = -3\vec{i} + 0\vec{j} + 0\vec{k}$$

Magnitude of AD

$$|\vec{AD}| = \sqrt{(-3)^2} = \sqrt{9}$$

$$|\vec{AD}| = 3$$

$$\lambda_{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{-3\vec{i} + 0\vec{j} + 0\vec{k}}{3}$$

$\lambda_{AD} = \vec{i}$

$\overrightarrow{F_{AD}} = -F_{AD}\vec{i}$---------------- (3)

$F = 250\vec{i} + 450\vec{j} - 150\vec{k}$ [*is given*]

Applying the equilibrium Eqn

$\sum F_x = 0$

$0.714F_{AB} - 0.461F_{AC} - F_{AD} + 250 = 0$

$0.714F_{AB} - 0.461F_{AC} - F_{AD} = -250$-------------- (4)

$\sum F_y = 0$

$0F_{AB} + 0.769F_{AC} + 450 = 0$---------------- (5)

$0.769F_{AC} = -450$

$F_{AC} = \frac{-450}{0.769}$

$F_{AC} = -585.17\ N$

$\sum F_z = 0$

$-0.714F_{AB} - 0.461\ F_{AC} - 150 = 0$---------------- (6)

$-0.714F_{AB} - 0.461\ \times (-585.17) - 150 = 0$

$-0.714F_{AB} + 269.76 - 150 = 0$

$-0.714F_{AB} + 119.76 = 0$

$F_{AB} = \frac{-119.76}{-0.714}$

$F_{AB} = 167.73N$

(4)$\Rightarrow$ $0.714F_{AB} - 0.461F_{AC} - F_{AD} = -250$

$0.714 \times 167.73 - 0.461 \times -585.17 - F_{AD} = -250$

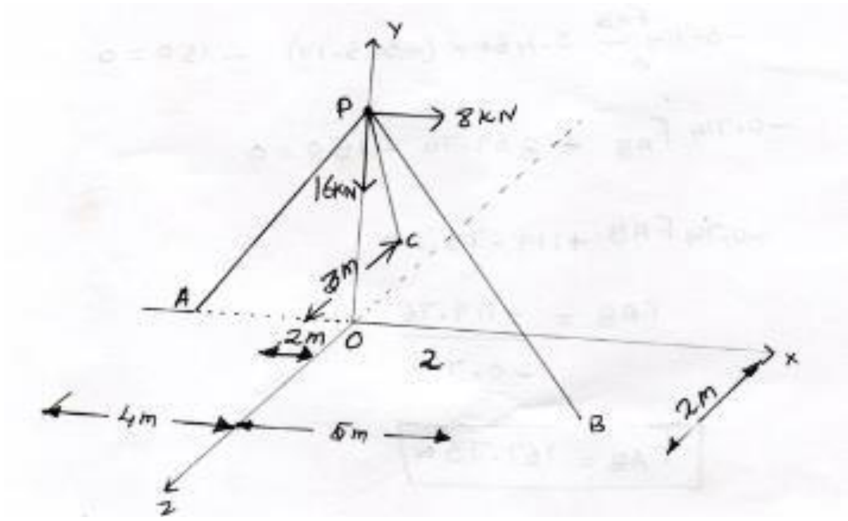
$119.76 + 269.76 - F_{AD} = -250$

$389.52 - F_{AD} = -250$

$-F_{AD} = -250 - 389.52$

$-F_{AD} = -639.52 F_{AD} = 639.52\, N$

2. Two force act upon the tripod at point P as shown in fig. The force 8 KN is parallel to X axis & the force 16 KN is parallel to Y axis. Determine the force acting at the legs of tripod if the rest on legs on ground at A, B, &C whose coordinates with respect to O are given the height of the P above the origin is 10m.



Given:

8 KN at point ‘P’ in horizontal

16 KN at point ‘P’ in vertical

Height of point P=10m from 0

To Find:

F_{PA}, F_{PB}, F_{PC}

Soln:

Coordinates

$A = (-4,0,0), B = (5,0,2), C = (-2,0,-3), P(0,10,0)$

$OA = -4\vec{i} + 0\vec{j} + 0\vec{k}, OB = 5\vec{i} + 0\vec{j} + 2\vec{k}, OC = -2\vec{i} + 0\vec{j} - 3\vec{k}$

$$OP = 0\vec{i} + 10\vec{j} + 0\vec{k}$$

Force on F_{PA}

$$\vec{F}_{PA} = \lambda_{PA} \times F_{PA} \quad \lambda_{PA} = \frac{A}{|\vec{PA}|}$$

$$|\vec{PA}| = \vec{OA} - \vec{OP}$$

$$= -4\vec{i} - [10\vec{j}]$$

$$|\vec{PA}| = -4\vec{i} - 10\vec{j}$$

$$|\vec{PA}| = \sqrt{(-4)^2 + (-10)^2} = \sqrt{16 + 100} = \sqrt{116}$$

$$|\vec{PA}| = 10.77$$

$$\lambda_{PA} = \frac{\vec{PA}}{|\vec{PA}|} = \frac{-4\vec{i} - 10\vec{j}}{10.77}$$

$$\lambda_{PA} = -0.371\vec{i} - 0.928\vec{j}$$

$$\vec{F}_{PA} = \lambda_{PA} \times F = -0.371\vec{i} \times F_{PA} - 0.928\vec{j} \times F_{PA}$$

$$\vec{F}_{PA} = -0.371F_{PA}\vec{i} - 0.928F_{PA}\vec{j} \text{------------- (1)}$$

Force of PB

$$\vec{F}_{PB} = \lambda_{PB} \times F_{PB} \quad \lambda_{PB} = \frac{\vec{PB}}{|\vec{PB}|}$$

$$\vec{PB} = \vec{OB} - \vec{OP}$$

$$= [5\vec{i} + 2\vec{k}] - [10\vec{j}]$$

$$\vec{PB} = 5\vec{i} - 10\vec{j} + 2\vec{k}$$

$$|\vec{PB}| = 11.35$$

$$\lambda_{PB} = \frac{\vec{PB}}{|\vec{PB}|} = \frac{5\vec{i} - 10\vec{j} + 2\vec{k}}{11.35}$$

$$\lambda_{PB} = 0.44\vec{i} - 0.88\vec{j} + 0.176\vec{k}$$

$$\vec{F}_{PB} = \lambda_{PB} \times F_{PB}$$

$$\vec{F_{PB}} = 0.44F_{PB}\vec{i} - 0.88F_{PB}\vec{j} + 0.176F_{PB}\vec{k}$$ -------- (2)

Force on PC

$$\vec{F_{PC}} = \lambda_{PC} \times F_{PC} \quad \lambda_{PC} = \frac{\vec{PC}}{|\vec{PC}|}$$

$$\vec{PC} = \vec{OC} - \vec{OP}$$

$$= [2\vec{i} - 3\vec{k}] - 10\vec{j}$$

$$\vec{PC} = -2\vec{i} - 10\vec{j} - 3\vec{k}$$

$$|\vec{PC}| = \sqrt{(-2)^2 + (-10)^2 + (-3)^2} = \sqrt{4 + 100 + 9} = \sqrt{113}$$

$$|\vec{PC}| = 10.63$$

$$\lambda_{PC} = \frac{\vec{PC}}{|\vec{PC}|} = \frac{-2\vec{i} - 10\vec{j} - 3\vec{k}}{10.63}$$

$$\lambda = -0.188\vec{i} - 0.94\vec{j} - 0.282\vec{k}$$

$$\vec{F_{PC}} = \lambda_{PC} \times F_{PC}$$

$$\vec{F_{PC}} = -0.188F_{PC}\vec{i} - 0.94F_{PC}\vec{j} - 0.282\vec{k}$$

$$P = 0\vec{i} + 10\vec{j} + 0\vec{k}$$ ------------ (4)

Apply Equilibrium condition

$$\sum F_x = 0$$

$$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} = 0$$ ------------- (5)

$$\sum F_y = 0$$

$$-0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} + 10 = 0$$

$$-0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} = -10$$

$$0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} = 10$$ -------- (6)

$$\sum F_Z = 0$$

$$-0.282F_{PC} + 0.178F_{PB} = 0$$

$0.178F_{PB} - 0.282F_{PC} = 0$--------- (7)

Solve Eqn(5)&(6)

(5)× 0.928 $\quad - 0.344\, F_{PA} + 0.4F_{PB} - 0.174F_{PC} = 0$

(6) × 0.371 $\quad 0.344F_{PA} + 0.326F_{PB} + 0.348F_{PC} = 3.71$

$0.726F_{PB} - 0.174F_{PC} = 3.71$--------- (8)

Solve Eqn (7) &(8)

(7)⇒0.726⇒ $\quad 0.127F_{PB} - 0.2F_{PC} = 0$

(8)⇒0.176⇒ $\quad 0.127F_{PB} + 0.03F_{PC} = 0.652$

$-0.23F_{PC} = -0.652$

$F_{PC} = \frac{-0.652}{-0.23}$

Eqn(7) becomes $0.176F_{PB}$-$0.282F_{PC} = 0$

$0.176 \times F_{PB} - 0.282 \times 2.834 = 0$

$F_{PB} = \frac{0.282 \times 2.834}{0.176}$

$F_{PB} = 4.539N$

Eqn (5) becomes

$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} = 0$

$-0.371F_{PA} + 0.44 \times 4.539 - 0.188 \times 2.834 = 0$

$-0.371 \times F_{PA} + 1.997 - 0.532 = 0$

$-0.371 \times F_{PA} + 1.465 = 0$

$F_{PA} = \frac{-1.465}{-0.371}$

$F_{PA} = 3.94N$