

**Equivalent Circuit of Induction Motor**

We have already seen that the induction motor can be treated as generalized transformer. Transformer works on the principle of electromagnetic induction. The induction motor also works on the same principle. The energy transfer from stator to rotor of the induction motor takes place entirely with the help of a flux mutually linking the two. Thus stator acts as a primary while the rotor acts as a rotating secondary when induction motor is treated as a transformer.

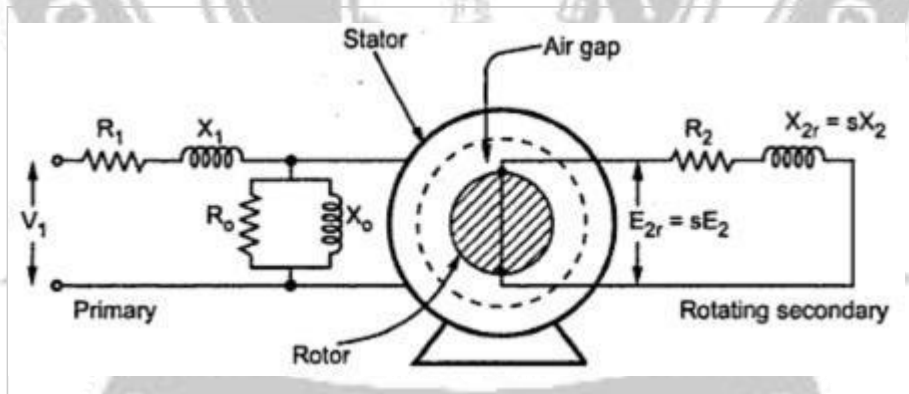
If  $E_1$  = Induced voltage in stator per phase  
 $E_2$  = Rotor induced e.m.f. per phase on standstill  $k =$   
 Rotor turns / Stator turns

then  $k = E_2 / E_1$

Thus if  $V_1$  is the supply voltage per phase to stator, it produces the flux which links with both stator and rotor. Due to self induction  $E_1$ , is the induced e.m.f. in stator per phase while  $E_2$  is the induced e.m.f. in rotor due to mutual induction, at standstill. In running condition the induced e.m.f. in rotor becomes  $E_{2r}$  which is  $s E_2$ .

Now  $E_{2r}$  = Rotor induced e.m.f. in running condition per phase  $R_2 =$   
 Rotor resistance per phase  
 $X_{2r}$  = Rotor reactance per phase in running condition  $R_1 =$   
 Stator resistance per phase  
 $X_1$  = Stator reactance per phase

So induction motor can be represented as a transformer as shown in the Fig. 3.25.



**Fig. 3.25 Induction motor as a transformer**

When induction motor is on no load, it draws a current from the supply to produce the flux in air gap and to supply iron losses.

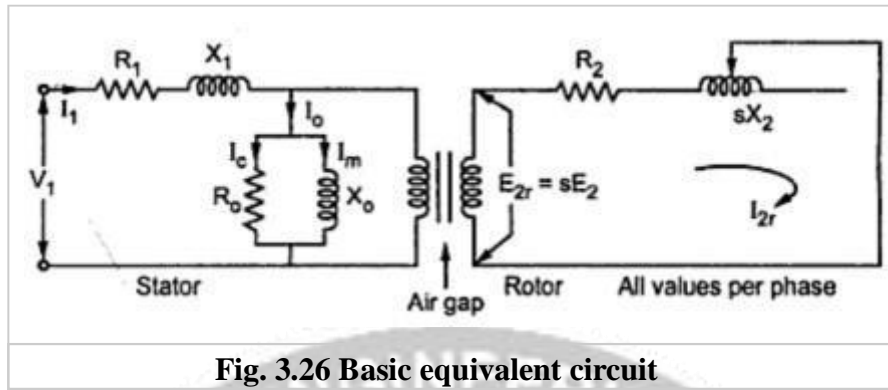
1.  $I_c$  = Active component which supplies no load losses
2.  $I_m$  = Magnetizing component which sets up flux in core and air gap These two currents give us the elements of an exciting branch as,

$R_0$  = Representing no load losses =  $V_1 / I_c$  and

$X_0$  = Representing flux set up =  $V_1 / I_m$  Thus,

$$\bar{I}_0 = \bar{I}_c + \bar{I}_m$$

The equivalent circuit of induction motor thus can be represented as shown in the Fig. 3.26.



**Fig. 3.26 Basic equivalent circuit**

The stator and rotor sides are shown separated by an air gap.

$$I_{2r} = \text{Rotor current in running condition} \\ = E_{2r} / Z_{2r} = (s E_2) / \sqrt{(R_2)^2 + (s X_2)^2}$$

It is important to note that as load on the motor changes, the motor speed changes. Thus slip changes. As slip changes the reactance  $X_{2r}$  changes. Hence  $X_{2r} = sX_2$  is shown variable.

**Representing of rotor impedance :**

It is shown that,  $I_{2r} = (sE_2) / \sqrt{(R_2)^2 + (s X_2)^2} = E_2 / \sqrt{(R_2/s)^2 + X_2^2}$

So it can be assumed that equivalent rotor circuit in the running condition has fixed reactance  $X_2$ , fixed voltage  $E_2$  but a variable resistance  $R_2/s$ , as indicated in the above equation.

Now  $R_2/s = R_2 + (R_2/s) - R_2$

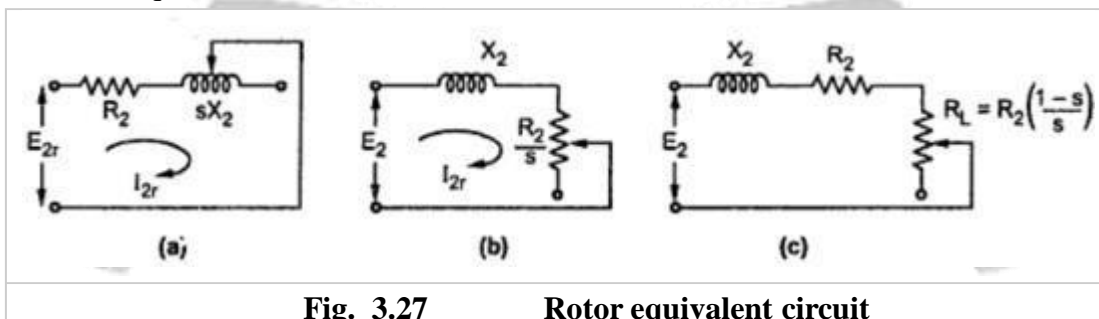
$$\therefore \frac{R_2}{s} = R_2 + R_2 \left( \frac{1}{s} - 1 \right) = R_2 + R_2 \left( \frac{1-s}{s} \right)$$

So the variable rotor resistance  $R_2/s$  has two parts.

1. Rotor resistance  $R_2$  itself which represents copper loss.
2.  $R_2(1 - s)/s$  which represents load resistance  $R_L$ . So it is electrical equivalent of mechanical load on the motor.

**Key Point :** Thus the mechanical load on the motor is represented by the pure resistance of value  $R_2(1 - s)/s$ .

So rotor equivalent circuit can be shown as,



**Fig. 3.27 Rotor equivalent circuit**

Now let us obtain equivalent circuit referred to stator side.

Equivalent circuit referred to stator :

Transfer all the rotor parameters to stator,  $k = E_2/E_1 =$  Transformation ratio

$$E_2' = E_2/k$$

The rotor current has its reflected component on the stator side which is  $I_{2r}'$ .

$$I_{2r}' = k I_{2r} = (k s E_2) / \sqrt{(R_2^2 + (s X_2)^2)}$$

$$X_2' = X_2 / K^2 = \text{Reflected rotor reactance } R_2' =$$

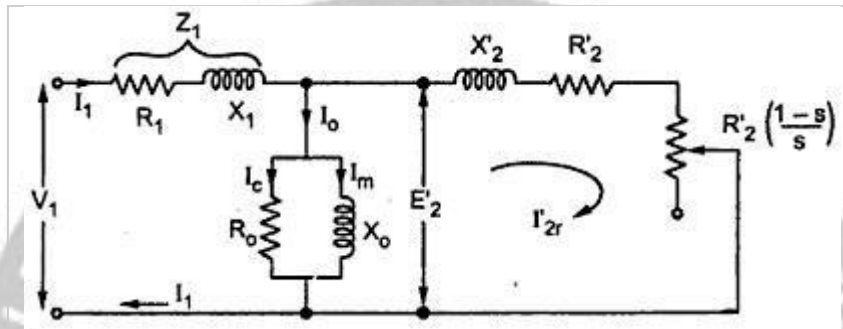
$$R_2 / K^2 = \text{Reflected rotor resistance } R_L' =$$

$$R_L / K^2 = (R_2 / K^2)(1-s / s)$$

$$= R_2' (1-s / s)$$

Thus  $R_L'$  is reflected mechanical load on stator.

So equivalent circuit referred to stator can be shown as in the Fig. 4

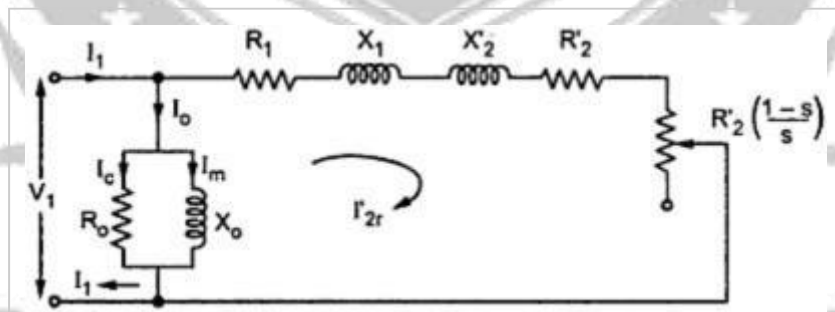


**Fig. 3.28 Rotor equivalent circuit**

The resistance  $R_2' (1-s) / s = R_L'$  is fictitious resistance representing the mechanical load on the motor.

**Approximate Equivalent Circuit**

Similar to the transformer the equivalent circuit can be modified by shifting the exciting current ( $R_o$  and  $X_o$ ) purely across the supply, to the left of  $R_1$  and  $X_1$ . Due to this, we are neglecting the drop across  $R_1$  and  $X_1$  due to  $I_o$ , which is very small. Hence the circuit is called approximate equivalent circuit. The circuit is shown in the Fig.3.29.



**Fig. 3.29 Approximate equivalent circuit**

Now the resistance  $R_1$  and  $R_2'$  while reactance  $X_1$  and  $X_2'$  can be combined. So we get,

$$R_{1e} = \text{Equivalent resistance referred to stator} = R_1 + R_2'$$

$$X_{1e} = \text{Equivalent reactance referred to stator} = X_1 + X_2'$$

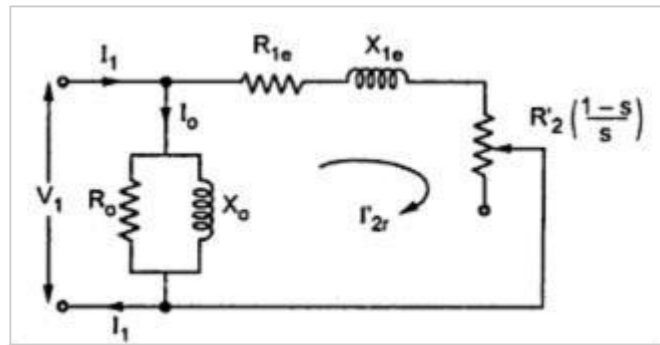
$$R_{1e} = R_1 + (R_2 / K^2)$$

and  $X_{1e} = X_1 + (X_2 / K^2)$

While  $\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}'$  ..... phasor diagram

and  $\bar{I}_o = \bar{I}_c + \bar{I}_m$

Thus the equivalent circuit can be shown in the Fig.3.30.



**Fig. 3.30 equivalent circuit**

**Power Equations from Equivalent Circuit**

With reference to approximate equivalent circuit shown in the Fig. 3.30, we can write various power equations as,

$$P_{in} = \text{input power} = 3 V_1 I_1 \cos \Phi$$

where  $V_1$  = Stator voltage per phase

$I_1$  = Current drawn by stator per phase  $\cos \Phi$

= Power factor of stator

$$\text{Stator core loss} = I^2 R_m$$

$$\text{Stator copper loss} = 3 I_1^2 R_o$$

where  $R_1$  = Stator resistance per phase

$$P_2 = \text{Rotor input} = (3 I_{2r}^2 R_2')/s$$

$$P_c = \text{Rotor copper loss} = 3 I_{2r}^2 R_2'$$

Thus  $P_c = s P_2$

$P_m$  = Gross mechanical power developed

$$\therefore P_m = P_2 - P_c = \frac{3 (I_{2r}')^2 R_2'}{s} - 3 (I_{2r}')^2 R_2' = 3 (I_{2r}')^2 R_2' \left( \frac{1-s}{s} \right)$$

T = Torque developed

$$\therefore T = \frac{P_m}{\omega} = \frac{3 (I_{2r}')^2 R_2' \left( \frac{1-s}{s} \right)}{\frac{2\pi N}{60}}$$

where  $N$  = Speed of motor

But  $N = N_s (1-s)$ , so substituting in above

$$T = \frac{3(I_{2r}')^2 R_2'}{\frac{s}{2\pi N_s} \cdot 60} = 9.55 \times \frac{3(I_{2r}')^2 R_2'}{N_s} \text{ N-m}$$

and  $I_{2r}' = V_1 / ((R_{1e} + R_L') + j X_{1e})$

where  $R_L' = R_2' (1-s)/s$

$$I_{2r}' = V_1 / \sqrt{(R_{1e} + R_L')^2 + X_{1e}^2}$$

**Key Point :** Remember that in all the above formula all the values per phase values.

### Maximum Power Output

Consider the approximate equivalent circuit as shown in the Fig.7

In this circuit, the exciting current  $I_0$  is neglected hence the exciting no load branch is not shown.

$$\therefore I_1 = I_{2r}'$$

The total impedance is given by,

$$Z_T = (R_{1e} + R_L') + j X_{1e} \quad \text{where } R_L' = R_2' (1-s)/s$$

$$= V_1 / \sqrt{(R_{1e} + R_L')^2 + (X_{1e})^2}$$

The power supplied to the load i.e.  $P_{out}$  per phase is, Per phase

$$P_{out} = I_1^2 R_L' \text{ watts per phase}$$

$$\therefore \text{Total} = 3 I_1^2 R_L'$$

$$\therefore P_{out} = 3 \frac{V_1^2}{[(R_{1e} + R_L')^2 + (X_{1e})^2]} (R_L')$$

To obtain maximum output power, differentiate the equation of total  $P_{out}$  with respect to variable  $R_L'$  and equal to zero.

$$\therefore \frac{d}{dR_L'} \left[ \frac{3 V_1^2 (R_L')}{[(R_{1e} + R_L')^2 + (X_{1e})^2]} \right] = 0$$

$$\therefore [(R_{1e} + R_L')^2 + (X_{1e})^2] [3 V_1^2] - 3 V_1^2 (R_L') [2 (R_{1e} + R_L')] = 0$$

$$\therefore (R_{1e} + R_L')^2 + (X_{1e})^2 - 2 (R_L') (R_{1e} + R_L') = 0 \quad \dots \text{Taking } 3 V_1^2 \text{ common}$$

$$\therefore R_{1e}^2 + (R_L')^2 + 2 R_{1e} R_L' + X_{1e}^2 - 2 R_{1e} R_L' - 2 (R_L')^2 = 0$$

$$\therefore R_{1e}^2 + X_{1e}^2 = (R_L')^2$$

But  $Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2} = \text{Leakage impedance referred to stator}$

$$\therefore Z_{1e}^2 = R_L'^2$$

$$Z_{1e} = R_L'$$

Thus the mechanical load on the induction motor should be such that the equivalent load

resistance referred to stator is equal to the total leakage impedance of motor referred to stator.

**Slip at maximum  $P_{out}$**  : This can be obtained as,

$$R_L' = Z_{1e} = R_2'(1-s)/s \quad \text{where } R_L' = R_2/K^2$$

$$\therefore s Z_{1e} = R_2' - sR_2'$$

$$\therefore s(Z_{1e} + R_2') = R_2'$$

$$s = \frac{R_2'}{R_2' + Z_{1e}}$$

This is slip at maximum output.

**Expression for maximum  $P_{out}$**  : Using the condition obtained in expression of total  $P_{out}$  , we can get maximum  $P_{out}$ .

$$\begin{aligned} \therefore (P_{out})_{max} &= 3 I_1^2 Z_{1e} \\ &= 3 \frac{V_1^2}{(R_{1e} + Z_{1e})^2 + (X_{1e})^2} \cdot Z_{1e} \quad \text{as } R_L' = Z_{1e} \\ &= 3 \frac{V_1^2}{(R_{1e}^2 + 2 R_{1e} Z_{1e} + Z_{1e}^2 + X_{1e}^2)} \cdot Z_{1e} \end{aligned}$$

But  $R^2 + X^2 = Z^2$

$$\therefore (P_{out})_{max} = 3 \frac{V_1^2}{2 Z_{1e}^2 + 2 R_{1e} Z_{1e}} \cdot Z_{1e} = 3 \frac{V_1^2}{2 Z_{1e} + (R_{1e} + Z_{1e})} \cdot Z_{1e}$$

$$\therefore (P_{out})_{max} = \frac{3 V_1^2}{2 (R_{1e} + Z_{1e})} \text{ watts}$$

The condition for maximum torque can be obtained from maximum power transfer theorem. When  $I_{2r}^2 R_2'/s$  is maximum consider the approximate equivalent circuit of induction motor as shown in The Fig. 8.

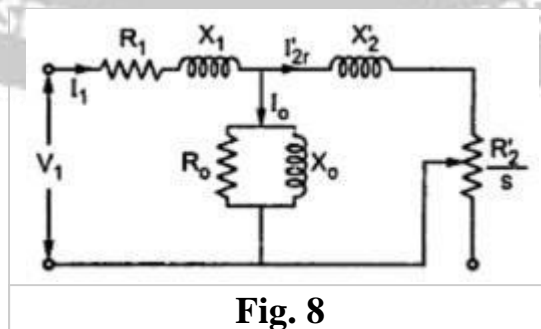
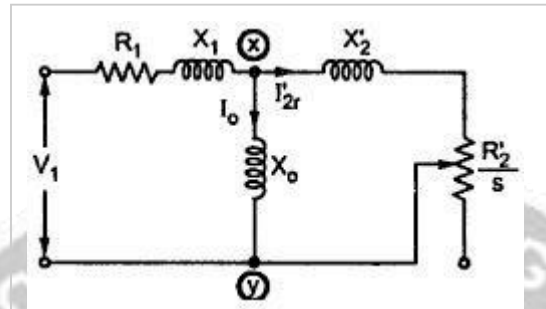


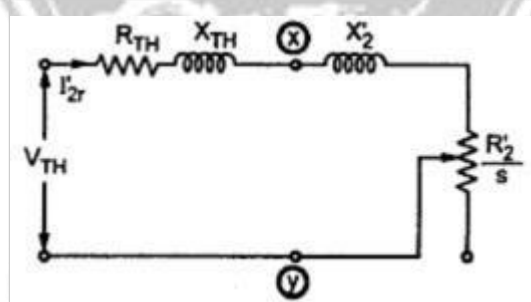
Fig. 8

The value of  $R_o$  is assumed to be negligible. Hence the circuit will be reduced as shown below.



**Fig. 3.31 Thevenin's Equivalent Circuit**

The thevenin's equivalent circuit for the above network is shown in the Fig.10 across the terminals x and y.



**Fig. 3.32 Equivalent Circuit**

$$\text{Now, } Z_{TH} = R_{TH} + jX_{TH} = (R_1 + jX_1) \parallel jX_o$$

$$V_{TH} = \frac{V_1(jX_o)}{R_1 + j(X_{TH} + X_o)}$$

The mechanical torque developed by rotor is maximum if there is maximum power transfer to the resistor  $R_2'/s$ . This takes place when  $R_2'/s$  equals to impedance looking back into the supply source.

$$\frac{R_2'}{s} = R_{TH} + j(X_{TH} + X_2')$$

$$\frac{R_2'}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}$$

$$\therefore s = s_m = \frac{R_2'}{\sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}}$$

This is the slip corresponding to the maximum torque. The maximum torque is given by,

$$T_m = \frac{3}{\omega_s} \cdot \frac{R'_2}{s_m} \cdot (I'_{2r})^2$$

$$\frac{R'_2}{s_m} = \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}$$

$$\therefore I'_{2r} = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R'_2}{s_m}\right)^2 + (X_{TH} + X'_2)^2}}$$

$$I'^2_{2r} = \frac{V^2_{TH}}{\left(R_{TH} + \frac{R'_2}{s_m}\right)^2 + (X_{TH} + X'_2)^2}$$

Substituting,

$$T_m = \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{V^2_{TH}}{\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\right]^2 + (X_{TH} + X'_2)^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{V^2_{TH}}{2R_{TH}^2 + 2R_{TH}\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} + 2(X_{TH} + X'_2)^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{R_{TH}^2}{2\left[R_{TH}^2 + (X_{TH} + X'_2)^2 + R_{TH}\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\right]}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{0.5 V^2_{TH}}{\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\right]}$$

$$\therefore T_m = \frac{3}{\omega_s} \cdot \frac{0.5 V^2_{TH}}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}}$$

From the above expression, it can be seen that the maximum torque is independent of rotor resistance.

### Synchronous Watt

The torque produced in the induction motor is given by,

$$T = \frac{3(I'_{2r})^2 R'_2}{\frac{s}{2\pi N_s}} = \frac{P_2}{\frac{60}{2\pi N_s}} \text{ N-m}$$

Thus torque is directly proportional to the rotor input. By defining new unit of torque which is synchronous watt we can write,

$$T = P_2 \text{ synchronous-watts}$$

If torque is given in synchronous-watts then it can be obtained in N-m as,



$$1 \text{ syn-watt} = \frac{60}{2\pi N_s} \text{ N-m}$$

i.e.

$$1 \text{ N-m} = \frac{2\pi N_s}{60} \text{ syn-watt}$$

**Key Point :** Unit synchronous watt can be defined as the torque developed by the motor such that the power input to the rotor across the air gap is 1 W while running at synchronous speed.

