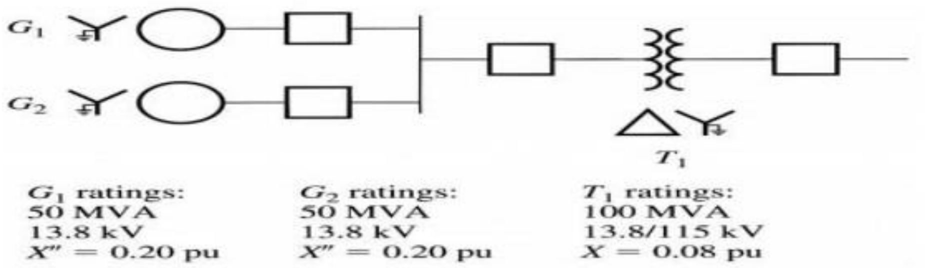
Symmetrical fault analysis through bus impedance matrix

Problems:

1.Two generators are connected in parallel to the low voltage side of a transformer. Generators G1 and G 2 are each rated at 50 MVA, 13.8 kV, with a subtransient resistance of 0.2 pu. Transformer T1 is rated at 100 MVA, 13.8/115 kV with a series reactance of 0.08 pu and negligible resistance.

Assume that initially the voltage on the high side of the transformer is 120 kV, that the transformer is unloaded, and that there are no circulating currents between the generators. Calculate the subtransient fault current that will flow if a 3 phase fault occurs at the high-voltage side of transformer



Solution:

Let choose the per-unit base values for this power system to be 100 MVA and 115 kV at the high-voltage side and 13.8 kV at the low-voltage side of the transformer. The subtransient reactance of the two generators to the system base is

$$X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$X_1" = X_2" = 0.2 \times \left(\frac{13{,}800}{13{,}800}\right)^2 \times \left(\frac{100{,}000}{50{,}000}\right) = j0.4p.u$$

The reactance of the transformer is already given on the system base, it will not change

$$XT = 0.08\ p.u$$

The per-unit voltage on the high-voltage side of the transformer is

$$V_{pu} = \frac{Actual\ value}{Base\ value} = \frac{120{,}000}{115{,}000} = j1.044\ p.u$$

Since there is no load on the system, the voltage at the terminals of each generator, and the internal generated voltage of each generator must also be 1.044 pu. To find this voltage, we must convert first the per-unit impedances to admittances, and the voltage sources to equivalent current sources. The Thevenin impedance of each generator is ZTh = j0.4, so the short circuit current of each generator is

$$I_{su} = \frac{V_{oc}}{Z_{th}} = \frac{1.044\angle 0^0}{j0.4} = 2.61\angle 90^0$$

Then the node equation for voltage V1

$$V1\ {-j2.5} + V1\ {-j2.5} + V1\ {-j12.5} = 2.61\angle - 90^0 + 2.61\angle - 90^o$$

$$V_1 = \frac{5.22\angle 90^0}{-j17.5} = 0.298\angle 0^0$$

Therefore, the subtransient current in the fault is

$IF = V1\ {-j12.5} = 3.729\angle{-90^o} p.$ U

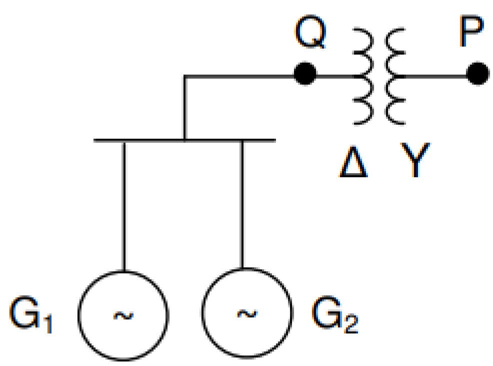
Since the base current at the high-voltage side of the transformer is

$$I_{base} = \frac{S_{3\Phi,base}}{\sqrt{3}V_{LL,base}} = \frac{100{,}000{,}000}{\sqrt{3}115{,}000} = 502\ A$$

the subtransient fault current will be

IF=IF,p.u I base=3.729×502=1872 A

2.Two synchronous generators are connected in parallel at the low voltage side of a three-phase -Y transformer as shown in Fig. 3.2. Machine 1 is rated 50 MVA, 13.8 kV. Machine 2 is rated 25 MVA, 13.8 kV. Each generator has subtransient reactance, transient reactance and direct axis synchronous reactance of 25%, 40% and 100% respectively. The transformer is rated 75 MVA, 13.8/69Y with a reactance of 10%. Before the fault occurs, the voltage on high voltage side of the transformer is 66 kV. The transformer is unloaded and there is no circulating current between the generators.



(a) Find the current supplied by the generators.

(b) A three-phase short circuit occurs at P. Determine the subtransient, transient and steady state short circuit current in each generator.

(c) A three-phase short circuit occurs at Q. Determine the subtransient, transient and steady state short circuit current in each generator.

Select a base of 75 MVA and 69 kV in the high tension circuit.
Summarize the results in a tabular form.

Solution:

Base voltage at the low tension circuit = 13.8 kV

$$\text{Prefault voltage at the LV side} = \frac{13.8}{69} \times 66 = 13.2 \text{ kV}$$

$$\text{Base current at the LV side} = \frac{75 \times 10^3}{\sqrt{3} \times 13.8} = 3137.77 \text{ amp.}$$

On the selected base

Generator 1: $X_d'' = 0.25 \times \frac{75}{50} = 0.375$ p.u. $\quad X_d' = 0.4 \times \frac{75}{50} = 0.6$ p.u.

$X_d = 1.0 \times \frac{75}{50} = 1.5$ p.u. $\quad E_{g1} = \frac{13.2}{13.8} = 0.9565$ p.u.

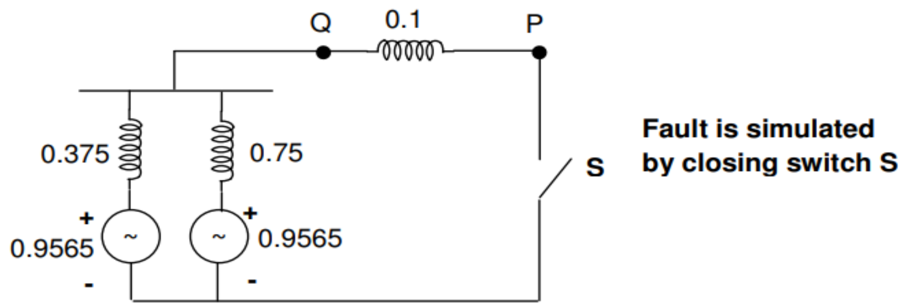
Generator 2: $X_d'' = 0.25 \times \frac{75}{25} = 0.75$ p.u. $\quad X_d' = 0.4 \times \frac{75}{25} = 1.2$ p.u.

$X_d = 1.0 \times \frac{75}{25} = 3.0$ p.u. $\quad E_{g2} = \frac{13.2}{13.8} = 0.9565$ p.u.

Transformer: X = 0.1 p.u.

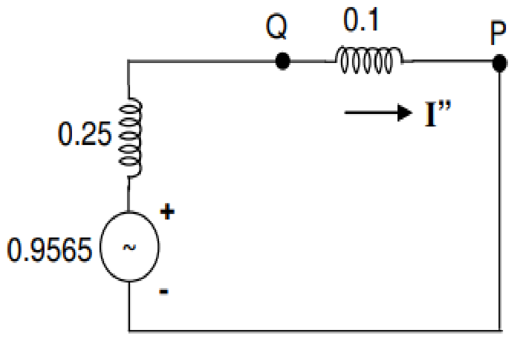
(a) Transformer is unloaded. Therefore, Ig1 = Ig2 = 0

(b) per unit subtransient reactance diagram is shown in Fig.



Using Thevenin's equivalent above reactance diagram for the faulted condition can be reduced as shown in Fig.

$$\frac{0.375 \times 0.75}{0.375 + 0.75} = 0.25 \text{ p.u.}$$



$$\text{Subtransient current } I'' = \frac{0.9565}{j0.35} = -j\,2.7329 \text{ p.u.}$$

$$\text{Voltage at Q} = j\,0.1 \times (-j\,2.7329) = 0.27329 \text{ p.u.}$$

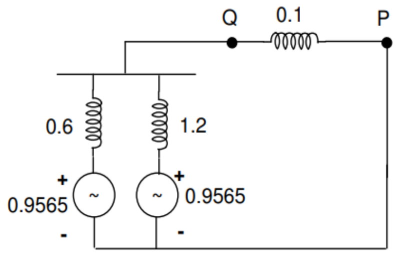
$$\text{Current supplied by generator 1} = \frac{0.9565 - 0.27329}{j0.375} = -j\,1.8219 \text{ p.u.}$$

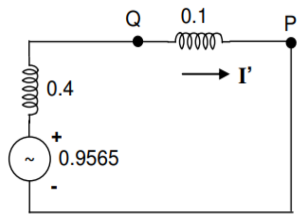
$$\text{Subtransient current in machine 1} \quad |I_1''| = 5716.7 \text{ A}$$

$$\text{Current supplied by generator 2} = \frac{0.9565 - 0.27329}{j0.75} = -j\,0.9109 \text{ p.u.}$$

$$\text{Subtransient current in machine 2} \quad |I_2''| = 2858.3 \text{ A}$$

per unit transient reactance diagram is shown in Fig





$$\text{Subtransient current } I' = \frac{0.9565}{j0.5} = -j\,1.913 \text{ p.u.}$$

$$\text{Voltage at Q} = j\,0.1 \times (-j\,1.913) = 0.1913 \text{ p.u.}$$

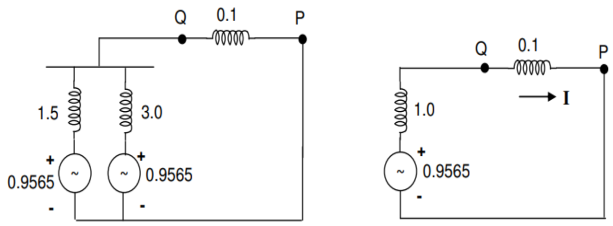
$$\text{Current supplied by generator 1} = \frac{0.9565 - 0.1913}{j0.6} = -j\,1.275 \text{ p.u.}$$

$$\text{Transient current in machine 1} \quad |I_1'| = 4001.7 \text{ A}$$

$$\text{Current supplied by generator 2} = \frac{0.9565 - 0.1913}{j1.2} = -j\,0.6377 \text{ p.u.}$$

$$\text{Transient current in machine 2} \quad |I_2'| = 2000.9 \text{ A}$$

per unit direct axis reactance diagram is shown in Fig.



Steady state short circuit current $I = \dfrac{0.9565}{j1.1} = -j\,0.8695$ p.u.

Voltage at Q = j 0.1 x (- j 0.8695) = 0.08695 p.u.

Current supplied by generator 1 $= \dfrac{0.9565 - 0.08695}{j1.5} = -j\,0.5797$ p.u.

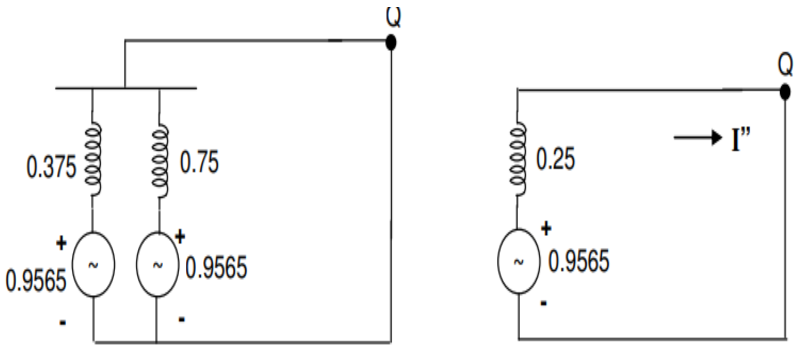
Steady state short circuit current in machine 1 $|I_1| = 1819$ A

Current supplied by generator 2 $= \dfrac{0.9565 - 0.08695}{j3.0} = -j\,0.2899$ p.u.

Steady state short circuit current in machine 2 $|I_2| = 909.48$ A

(b) Fault occurs at point Q.

per unit subtransient reactance diagram is shown in Fig.



Subtransient current $I'' = \dfrac{0.9565}{j0.25} = -j\,3.826$ p.u.

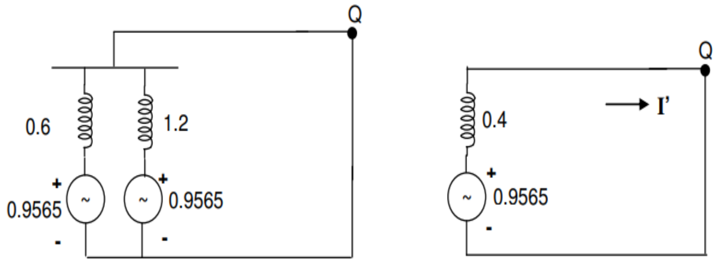
Current supplied by generator 1 $= \dfrac{0.9565 - 0}{j0.375} = -j\,2.5507$ p.u.

Subtransient current in machine 1 $|I_1''| = 8003.4$ A

Current supplied by generator 2 $= \dfrac{0.9565 - 0}{j0.75} = -j\,1.2753$ p.u.

Subtransient current in machine 2 $|I_2''| = 4001.7$ A

per unit transient reactance diagram is shown in Fig



Transient current $I' = \dfrac{0.9565}{j0.4} = -j\,2.3913$ p.u.

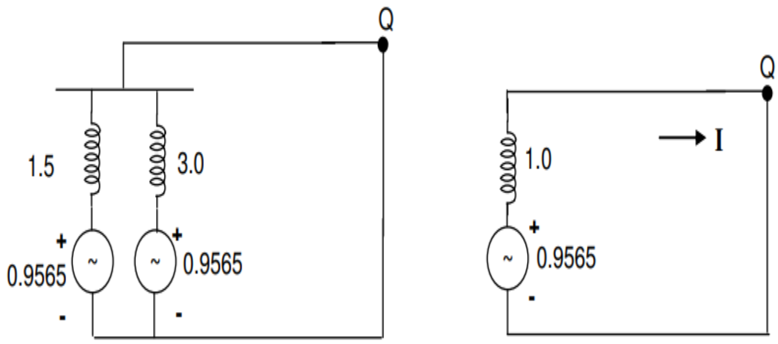
Current supplied by generator 1 $= \dfrac{0.9565 - 0}{j0.6} = -j\,1.5942$ p.u.

Transient current in machine 1 $|I_1'| = 5002.2$ A

Current supplied by generator 2 $= \dfrac{0.9565 - 0}{j1.2} = -j\,0.7971$ p.u.

Subtransient current in machine 2 $|I_2'| = 2501.1$ A

per unit direct axis transient reactance diagram is shown in Fig.



Direct axis steady state short circuit current $I = \dfrac{0.9565}{j1.0} = -j\,0.9565$ p.u.

Current supplied by generator 1 $= \dfrac{0.9565 - 0}{j1.5} = -j\,0.6377$ p.u.

Steady state short current in machine 1 $|I_1| = 2000.9$ A

Current supplied by generator 2 $= \dfrac{0.9565 - 0}{j3.0} = -j\,0.3188$ p.u.

Steady state short circuit current in machine 2 $|I_2| = 1000.4$ A

In the prefault condition, since the transformer is not loaded Ig1 = Ig2 = 0

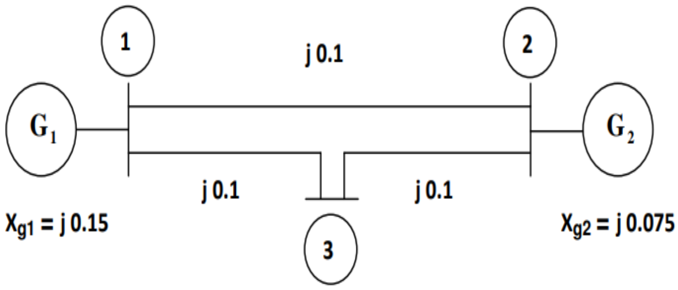
Fault occurs at the HV side of the transformer:

Subtransient		Transient		Steady state	
$\lvert I_1''\rvert$	$\lvert I_2''\rvert$	$\lvert I_1'\rvert$	$\lvert I_2'\rvert$	$\lvert I_1\rvert$	$\lvert I_2\rvert$
5717 A	2858 A	4002 A	2001 A	1819 A	909 A

Fault occurs at the LV side of the transformer i.e. at the generator terminals:

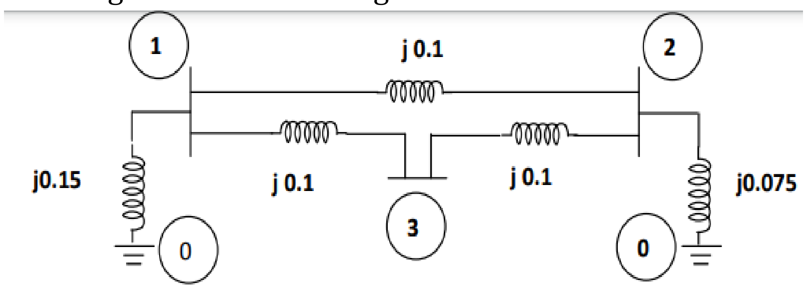
Subtransient		Transient		Steady state	
$\lvert I_1''\rvert$	$\lvert I_2''\rvert$	$\lvert I_1'\rvert$	$\lvert I_2'\rvert$	$\lvert I_1\rvert$	$\lvert I_2\rvert$
8003 A	4002 A	5002 A	2501 A	2001 A	1000 A

3.Consider the power system shown in Fig. The values marked are p.u. impedances. The p.u. reactances of the generator 1 and 2 are 0.15 and 0.075 respectively. Compute the bus impedance matrix of the generator – transmission network.



Solution:

The ground bus is numbered as 0 and it is taken as reference bus. The p.u. impedance diagram is shown in Fig.



When element $0-1$ is included

$$Z_{bus} = j \begin{array}{c|c} & 1 \\ 1 & [0.15] \end{array}$$; When element $0-2$ is included $$Z_{bus} = j \begin{array}{c|cc} & 1 & 2 \\ 1 & 0.15 & 0 \\ 2 & 0 & 0.075 \end{array}$$

Element $1-2$ is added; it is a link between buses 1 and 2. With bus ℓ

$$Z_{bus} = j \begin{array}{c|ccc} & 1 & 2 & \ell \\ 1 & 0.15 & 0 & 0.15 \\ 2 & 0 & 0.075 & -0.075 \\ \ell & 0.15 & -0.075 & 0.325 \end{array} ;$$

Eliminating the ℓ^{th} bus

$$Z_{bus} = j \begin{array}{c|cc} & 1 & 2 \\ 1 & 0.08077 & 0.034615 \\ 2 & 0.034615 & 0.05769 \end{array}$$

Add element $1-3$. It is a branch from bus 1 and it creates bus 3.

$$Z_{bus} = j \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0.08077 & 0.034615 & 0.08077 \\ 2 & 0.034615 & 0.05769 & 0.034615 \\ 3 & 0.08077 & 0.034615 & 0.18077 \end{array}$$

Finally add element $2-3$. It is a link between buses 2 and 3. With bus ℓ

$$Z_{bus} = j \begin{array}{c|cccc} & 1 & 2 & 3 & \ell \\ 1 & 0.08077 & 0.034615 & 0.08077 & -0.046155 \\ 2 & 0.034615 & 0.05769 & 0.034615 & 0.023075 \\ 3 & 0.08077 & 0.034615 & 0.18077 & -0.146155 \\ \ell & -0.046155 & 0.023075 & -0.146155 & 0.26923 \end{array}$$

$$Z_{bus} = j \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0.07286 & 0.03857 & 0.05571 \\ 2 & 0.03857 & 0.05571 & 0.04714 \\ 3 & 0.05571 & 0.04714 & 0.10143 \end{array}$$

Symmetrical fault analysis through bus impedance matrix

4.

Consider the power system discussed in Example 3.2. The p.u. impedances are on a base of 50 MVA and 12 kV. Symmetrical short circuit occurs at bus 3 with zero fault impedance. Using Z_{bus} matrix determine the fault current, bus voltages and also the currents contributed by the generators.

Solution

As seen in example 3.2, Z_{bus} matrix of the transmission-generator network is

$$Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.07286 & 0.03857 & 0.05571 \\ 0.03857 & 0.05571 & 0.04714 \\ 0.05571 & 0.04714 & 0.10143 \end{bmatrix} \end{array}$$

Faulted system is shown in Fig. 3.28

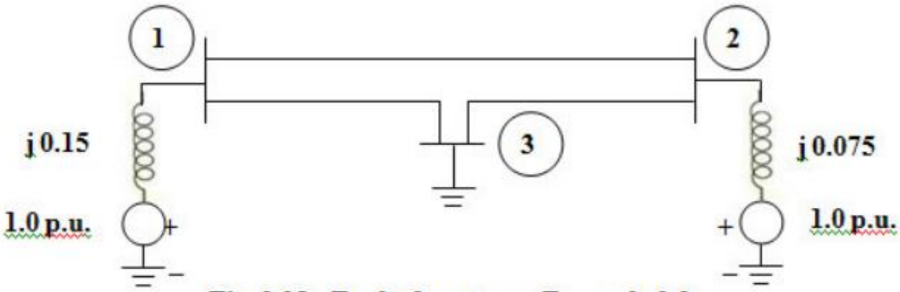


Fig. 3.28 Faulted system – Example 3.3

Faulted bus current $I_{3(F)} = -\dfrac{1}{j0.10143} = j9.8590$ p.u.

Fault current $I_F = -I_{3(F)} = -j9.8590$ p.u.

$$\text{Base current} = \frac{50 \times 1000}{\sqrt{3} \times 12} = 2405.6 \text{ amp.}$$

Fault current $I_F = -j\,9.8590 \times 2405.6 = -j\,23717$ amp.

Since fault impedance is zero, $V_{3(F)} = V_F = 0$

Taking the pre-fault voltage $V_0 = 1.0$ p.u.

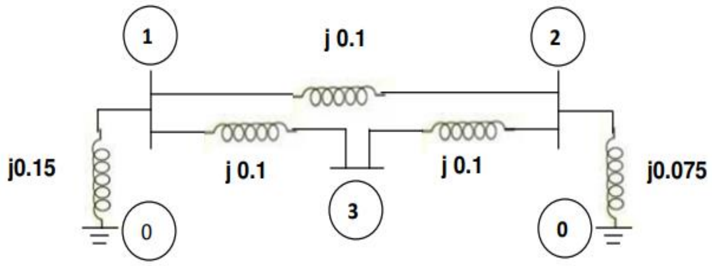
$$V_{1(F)} = 1.0 - \frac{Z_{13}}{Z_{33}} = 1.0 - \frac{0.05571}{0.10143} = 0.45075 \text{ p.u.} = \frac{12}{\sqrt{3}} \times 0.45075 = 3.1229 \text{kV}$$

$$V_{2(F)} = 1.0 - \frac{Z_{23}}{Z_{33}} = 1.0 - \frac{0.04714}{0.10143} = 0.53525 \text{ p.u.} = \frac{12}{\sqrt{3}} \times 0.53525 = 3.7083 \text{kV}$$

$$\text{Current supplied by gen. 1, } I_{G1} = \frac{1.0 - 0.45075}{j0.15} = -j3.6617 \text{ p.u.} = -j8808.6 \text{ amp.}$$

$$\text{Current supplied by gen. 2, } I_{G2} = \frac{1.0 - 0.53525}{j0.075} = -j6.1967 \text{ p.u.} = -j14906.8 \text{ amp.}$$

5.For the transmission-generator system shown in Fig.3.29, the bus impedance matrix is obtained as



$$Z_{bus} = j \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0.072857 & 0.038571 & 0.055714 \\ 2 & 0.038571 & 0.055714 & 0.047143 \\ 3 & 0.055714 & 0.047143 & 0.101429 \end{array}$$

Symmetrical three phase fault with fault impedance j 0.052143 p.u. occurs at bus 1. Find the p.u. currents in all the elements and mark them on the single line diagram.

Solution:

Fault occurs at bus 1 and we need the first column of ZBus, which is

$$j \begin{array}{c|c} & 1 \\ \hline 1 & 0.072857 \\ 2 & 0.038571 \\ 3 & 0.055714 \end{array} \quad \text{and} \quad Z_F = j\,0.052143$$

$$\text{Faulted bus current } I_{1(F)} = -\frac{1}{j0.072857 + j0.052143} = -\frac{1}{j0.125} = j\,8$$

$$\text{Fault current } I_F = -I_{1(F)} = -j\,8 \text{ p.u.}$$

$$V_1\,(F) = V_F = Z_F\, I_F = (j\,0.052143)(-j\,8) = 0.41714 \text{ p.u.}$$

$$V_2(F) = 1 - \frac{0.038571}{0.125} = 0.69143 \text{ p..u.}\ V_3(F) = 1 - \frac{0.055714}{0.125} = 0.55429 \text{ p..u.}$$

$$i_{2\text{-}1} = (0.69143 - 0.41714) / (j\,0.1) = -j\,2.7429 \text{ p.u.}$$

$$i_{3\text{-}1} = (0.55429 - 0.41714) / (j\,0.1) = -j\,1.3715 \text{ p.u.}$$

$$i_{2\text{-}3} = (0.69143 - 0.55429) / (j\,0.1) = -j\,1.3714 \text{ p.u.}$$

$$i_{G1} = (1 - 0.41714) / (j\,0.15) = -j\,3.8857 \text{ p.u.}$$

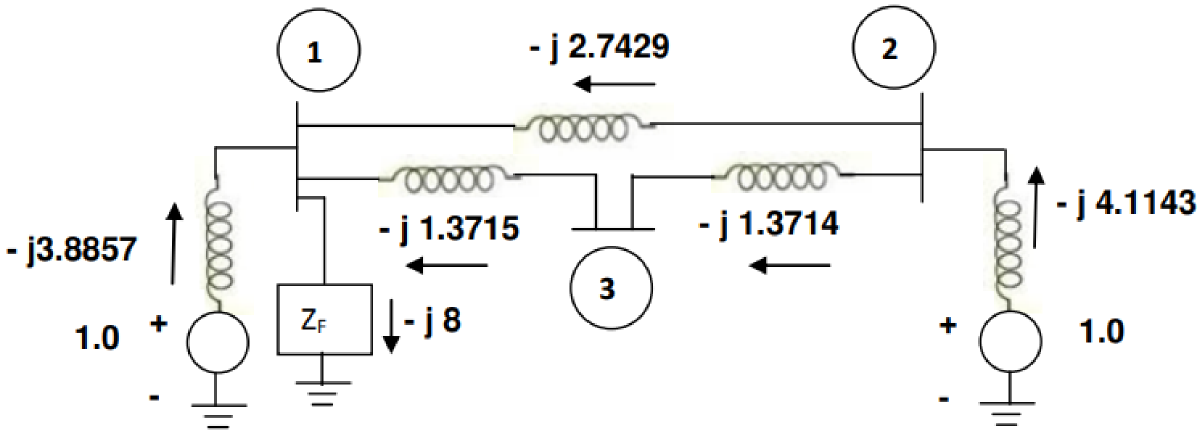
$$i_{G2} = (1 - 0.69143) / (j\,0.075) = -j\,4.1143 \text{ p.u.}$$

$V_1(F) = 0.41714$ p.u.

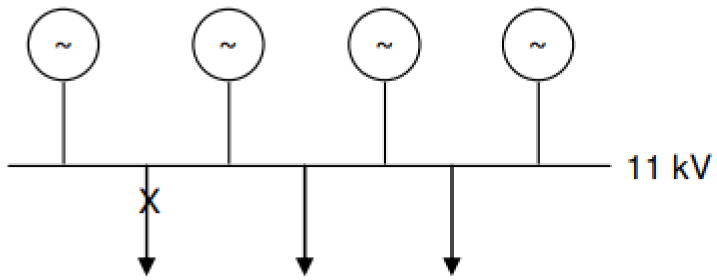
$V_2(F) = 0.69143$ p.u.

$V_3(F) = 0.55429$ p.u.

Currents are marked in Fig.

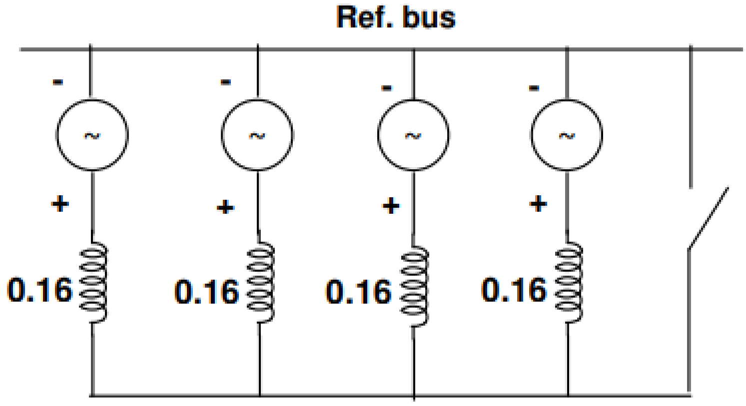


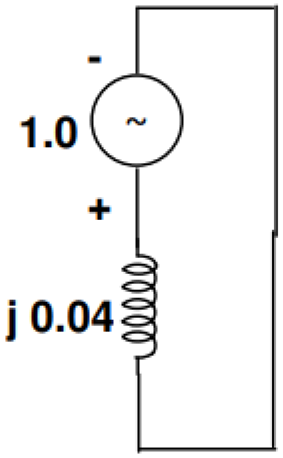
6.Fig. shows four identical alternators in parallel. Each machine is rated for 25 MVA, 11 kV and has a subtransient reactance of 16 % on its rating. Compute the short circuit MVA when a three phase fault occurs at one of the outgoing feeders.



Solution

Fault is simulated by closing the switch shown in the p.u. reactance diagram shown in fig and Its Thevenin's equivalent is also shown in next Fig.





Fault current $|I_F| = \dfrac{1}{0.04} = 25$ p.u.

Short circuit MVA = prefault voltage in p.u. x fault current in p.u. x Base MVA

= 1.0 x 25 x 25

= 625