Bus Impedance matrix building algorithm

Bus impedance matrix Zbus of a power network can be obtained by inverting the bus admittance matrix Ybus, which is easy to construct. However, when the order of matrix is large, direct inversion requires more core storage and enormous computer time. Therefore, inversion of Ybus is prohibited for large size network. Bus impedance matrix can be constructed by adding the one after the other. Using impedance parameters, performance equations in bus frame of reference can be written as

Ebus = Zbus Ibus

In the expanded form the above becomes

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

From this we can write

Ep = Zp1 I1 + Zp2 I2 + ... + ZpqIq +.... + ZpN

From the above, it can be noted that with Iq = 1 p.u. other bus currents set to zero, Ep = Zpq. Thus Zpq can be obtained by measuring Ep when 1 p.u. current is injected at bus q and leaving the other bus currents as zero. In fact, p and q can be varied from 1 to N. While making measurements all the buses except one, are open circuited. Hence, the bus impedance parameters are called open circuit impedances. The diagonal elements in Zbus are known as driving point impedances, while the off-diagonal elements are called transfer impedances.

Symmetrical fault analysis through bus impedance matrix. Once the bus impedance matrix is constructed, symmetrical fault analysis can be carried out with a very few calculations. Bus voltages and currents in various elements can be computed quickly. When faults are to be simulated at different buses, this method is proved to be good. Symmetrical short circuit analysis essentially consists of determining the steady state solution of linear network with balanced sources. Since the short circuit currents are much larger compared to prefault currents the following assumptions are made while conducting short circuit study.

1. All the shunt parameters like loads, line charging admittances etc. are neglected.

2. All the transformer taps are at nominal position.

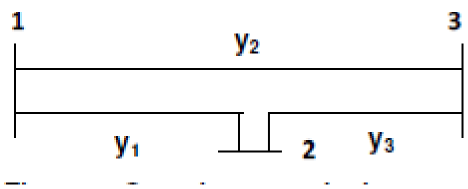
3. Prior to the fault, all the generators are assumed to operate at rated voltage of 1.0 p.u. with their emf's in phase. With these assumptions, in the prefault condition, there will not be any current flow in the network and all the bus voltages will be equal to1.0 p.u.

The linear network that has to be solved comprises of

i) Transmission network ii) Generation system and iii) Fault

By properly combining the representations of the above three components, we can solve the short circuit problem.

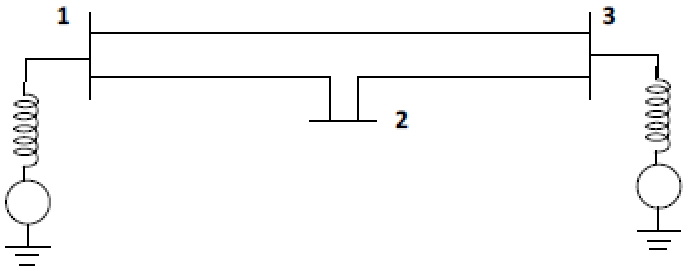
Consider the transmission network shown in Fig



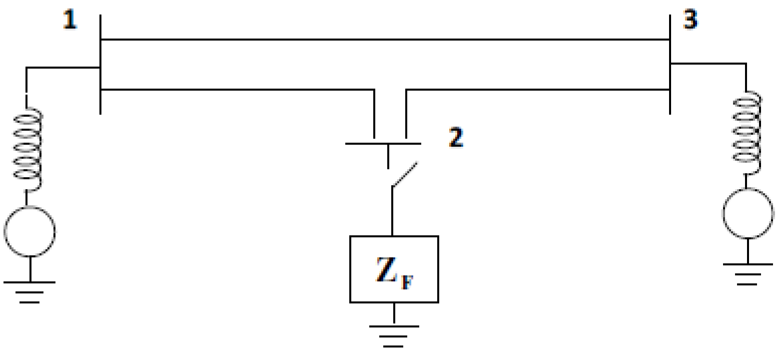
Taking the ground as the reference bus, the bus admittance matrix is obtained as

$$Y_{bus} = \begin{matrix} & 1 & 2 & 3 \\ 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} y_1 + y_2 & -y_1 & -y_2 \\ -y_1 & y_1 + y_3 & -y_3 \\ -y_2 & -y_3 & y_2 + y_3 \end{bmatrix}$$

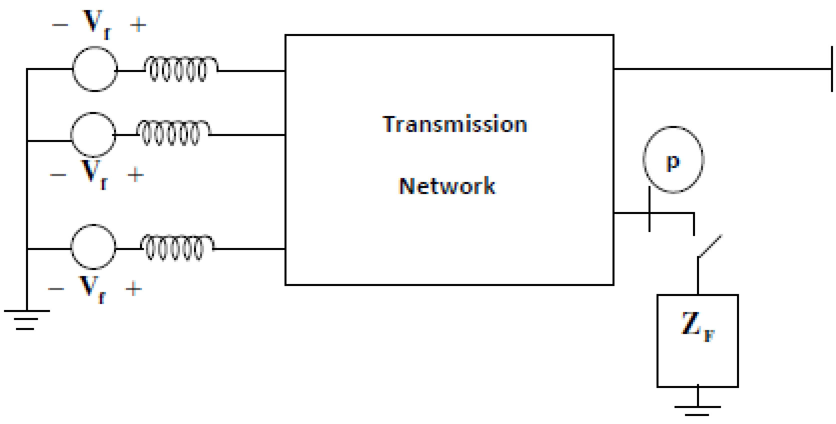
If we add all the columns (or rows) we get a column (or row) of all zero elements. Hence this Ybus matrix is singular and hence corresponding Zbus matrix of this transmission network does not exist. Thus, when all the shunt parameters are neglected, Zbus matrix will not exist for the transmission network. However, connection to ground is established at the generator buses, representing the generator as a constant voltage source behind appropriate reactance as shown in Fig.



If the generator reactances are included with the transmission network, Zbus matrix of the combined network can be obtained. As stated earlier, there is no current flow in the network in the pre-fault condition and all the bus voltages will be 1.0 p.u. Consider the network shown in Fig. Symmetrical fault occurring at bus 2can be simulated by closing the switch shown in Fig. Here Zf is the fault impedance.



When the fault is simulated, there will be currents in different elements and the bus voltages will be different from 1.0 p.u. These changes have occurred because of i) Generator voltages and ii) Fault current. Any general power system with a number of generators and N number of buses subjected to symmetrical fault at pth bus will be represented as shown in Fig.



In the faulted system there are two types of sources:

1. Current injection at the faulted bus
2. Generated voltage sources.

The bus voltages in the faulted system can be obtained using Superposition Theorem.

Bus voltages due to current injection:

Make all the generator voltages to zero. Then we have Generator-Transmission system without voltage sources. Such network has transmission parameters and generator reactances between generator buses and the ground. Let Zbus be the bus impedance matrix of such Generator-Transmission network. Then the bus voltages due to the current injection will be given by

Vbus = Zbus Ibus (F)

where Ibus (F) is the bus current vector having only one non-zero element.

Thus when the fault is at the pth bus

$$I_{bus\,(F)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_{p\,(F)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Here Ip (F) is the faulted bus current

Thus, bus voltages due to current injection will be

$$V_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Np} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_{p\,(F)} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_{p\,(F)}$$

Bus voltages due to generator voltages

Make the fault current to be zero. Since there is no shunt element, there will be no current flow and all the bus voltages are equal to V0 , the pre-fault voltage which will be normally equal to 1.0 p.u. Thus, bus voltages due to generator voltages will be

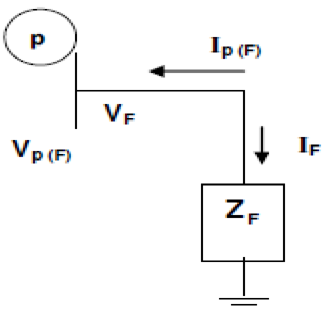
$$V_{bus} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0$$

Thus for the faulted system, wherein both the current injection and generator sources are simultaneously present, the bus voltages can be obtained by adding the voltages. Therefore, for the faulted system the bus voltages are

$$V_{bus\,(F)} = \begin{bmatrix} V_{1(F)} \\ V_{2(F)} \\ \vdots \\ V_{p(F)} \\ \vdots \\ V_{N(F)} \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_{p\,(F)} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0$$

To calculate Vbus (F) we need the faulted bus current Ip (F) which can be, determined as discussed below.

The fault can be described as shown in Fig.



It is clear that VF = ZF IF ,Vp (F) = VF and Ip (F) = - IF (3.40)

Therefore Vp (F) = - ZF Ip (F) .

The pth equation extracted from eqn gives Vp (F) = ZppIp (F) + V0

Vp (F) = - ZF Ip (F)

The pth equation extracted from eqn. (3.39) gives

Vp (F) = ZppIp (F) + V0

Substituting eqn. in the above, we get

- ZF Ip (F) = ZppIp (F) + V0

Thus the faulted bus current Ip (F) is given by

$$I_{p\,(F)} = - \frac{V_0}{Z_{pp} + Z_F}$$

Substituting the above in eqn. (3.41), the faulted bus voltage Vp (F) is

$$V_{p\,(F)} = \frac{Z_F}{Z_{pp} + Z_F} V_0$$

Finally, voltages at other buses at faulted condition are to be obtained. The ith equation extracted from eqn. (3.39) gives

Vi (F) = Zip Ip (F) + V0

Substituting eqn. (3.43) in the above, we get

$$V_{i\,(F)} = V_0 - \frac{Z_{ip}}{Z_{pp} + Z_F} V_0 \qquad \begin{matrix} i = 1,2,\ldots\ldots\ldots,N \\ i \neq p \end{matrix}$$

Knowing all the bus voltages, current flowing through the various network elements can be computed as ikm (F) = (Vk (F) – Vm (F)) ykm where ykm is the admittance of element k-m.

When the fault is direct, ZF = 0 and hence

$$I_{p(F)} = -\frac{V_0}{Z_{pp}}$$

$$V_{p(F)} = 0 \text{ and}$$

$$V_{i(F)} = V_0 - \frac{Z_{ip}}{Z_{pp}} V_0 \qquad i = 1,2,\ldots\ldots,N \quad i \neq p$$

It is to be noted that when the fault occurs at the pth bus, only the pth column of Zbus matrix (and not the entire Zbus matrix) is required for further calculations.

The following are the various steps for conducting symmetrical short circuit analysis.

Step 1 Read
i) Transmission line data
ii) Generator reactances data
iii) Faulted bus number p and
iv) Fault impedance Z_F.

Step 2 Construct the bus impedance matrix of the transmission network including the generator reactances.

Step 3 Compute $I_{p(F)} = -\dfrac{V_0}{Z_{pp} + Z_F}$

Step 4 Compute $V_{p(F)} = \dfrac{Z_F}{Z_{pp} + Z_F} V_0$

Step 5 Compute $V_{i(F)} = V_0 - \dfrac{Z_{ip}}{Z_{pp} + Z_F} V_0 \qquad i = 1,2,\ldots\ldots,N \quad i \neq p$

Step 6 Calculate the element currents from $i_{km(F)} = (V_{k(F)} - V_{m(F)})\, y_{km}$