5.2 CONVERSION OF STATE VARIABLE MODELS TO TRANSFER FUNCTIONS

The state model of a system consists of state equation and output equation. (or) the state equation and output equation together called as state model of the system.

$X(t) = A X (t) + B U (t)$ --------------------s tate equa ti on

( ) ( ) ( )

Where

X (t) = state vector of order (n x 1)

U (t) = Input vector of order (m x 1)

A = System matrix of order (n x n)

B = Input matrix of order (n x m)

Y (t) = Output vector of order (p x 1)

C = Output matrix of order (p x n)

D = Transmission matrix of order (p x m)

Taking Laplace transform (with zero initial condition) in state equation and output equation

$$sX (s') = AX (s') + B U(s)$$

$$Y (s) = CX (s) + D U(s)$$

The state equation can be placed in the form

$$(s I-A) X (s) = B U (s)$$

Premultiply both sides by $(sI-A)^{-1}$

$$X (s') = (si - A)~*B U(s)$$

Subtitling X(s) in the output equation

$$Y (S') = [C (si - A) ^rB + D] U(s)$$

Hence transfer function Matrix $T (s) = [C(s I — A)~ ^rB + D]$

EXAMPLE:

State space model is given by

Xt | ■ 0 1 ■ | ■*r | | * | O |
_X2_ | * + | * + | | | -

X [ ] .Find the transfer functions of the system.

x2

Let compare given state space model equation with standard state space model equation,

X(t) = A X(t) + B U(t)

( ) ( ) ( ) *-state equation*

Hence *output equation*

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W.K.T, $3]; B = [{}^{0}_{1}]; C = [1 \quad 0]; D = [0];$

$$T(s) = [C(sI - A)^{-1}B + D]$$

$T(s) = [1 \quad 0] \Big($ Is 0 *0 s. | [0 11)-1 1. - 2 - 3 1) | O' -1 | $+ [0]$

$$T(s) = \frac{1}{s^2 + 3s + 2}$$