3.3 Wiener Filtering in Image Processing

Wiener filtering is an advanced image restoration technique used to recover an original image from a blurred and noisy image. It minimizes the mean square error (MSE) between the restored image and the original image. It is an improvement over inverse filtering because it handles noise effectively.

3.3.1. Image Degradation Model

The degradation of an image can be represented as:

$g(x,y)=h(x,y)*f(x,y)+n(x,y)f(x,y)$ → Original image

- $h(x,y)$ → Degradation function / Point Spread Function (PSF)
- $g(x,y)$ → Degraded image
- $n(x,y)$ → Noise
- * → Convolution operation

In the frequency domain:

$G(u,v)=H(u,v)F(u,v)+N(u,v)$

Wiener Filter Formula

The Wiener filter restores the image using:

$F(u,v)=H*(u,v)\mid H(u,v)\mid 2+Sn(u,v)Sf(u,v)\times G(u,v)F(u,v)$ =
\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} \times
$G(u,v)F(u,v)=\mid H(u,v)\mid 2+Sf(u,v)Sn(u,v)H*(u,v)\times G(u,v)$

Where:

- **H(u,v)** → Degradation function
- **H*(u,v)** → Complex conjugate of H(u,v)
- **Sn(u,v)** → Power spectrum of noise
- **Sf(u,v)** → Power spectrum of original image
- **G(u,v)** → Fourier transform of degraded image

. Working Principle

1. Convert the degraded image to the **frequency domain** using Fourier transform.
2. Apply the **Wiener filter formula**.
3. Reduce the effect of noise and blur simultaneously.
4. Convert the result back to the **spatial domain** using inverse Fourier transform.

Advantages

- Reduces **both blur and noise**.
- Produces **better restoration** than inverse filtering.
- Works well even when **noise is present**.

Limitations

- Requires knowledge of **noise statistics**.
- Computationally more complex than inverse filtering

Inverse Filtering in Image Processing

Inverse filtering is a restoration technique used in digital image processing to recover the original image from a blurred or degraded image. It works by reversing the effect of the degradation (blur) that occurred during image acquisition or transmission.

Image Degradation Model

In image processing, a degraded image is usually modeled as:

g(x,y)=h(x,y)∗f(x,y)+n(x,y)g(x,y) = h(x,y) * f(x,y) + n(x,y)g(x,y)=h(x,y)∗f(x,y)+n(x,y)

Where:

- **f(x,y)** → Original image
- **h(x,y)** → Degradation function / Point Spread Function (PSF)
- **g(x,y)** → Degraded image
- **n(x,y)** → Noise
- * → Convolution operation

In the **frequency domain**, this becomes:

$G(u,v)=H(u,v)F(u,v)+N(u,v)$

2. Concept of Inverse Filtering

Inverse filtering attempts to recover the original image by **dividing the degraded image by the degradation function**.

1. Image Degradation Model

In image processing, a degraded image is usually modeled as:

$$g(x,y)=h(x,y)*f(x,y)+n(x,y)$$

$$F(u,v)=G(u,v)H(u,v)F(u,v) = \frac{G(u,v)}{H(u,v)}F(u,v)=H(u,v)G(u,v)$$

Where:

- **G(u,v)** → Fourier transform of degraded image
- **H(u,v)** → Fourier transform of degradation function
- **F(u,v)** → Recovered image in frequency domain

Steps in Inverse Filtering

1. Obtain the degraded image.
2. Determine the **degradation function (PSF)**.
3. Apply **Fourier Transform** to convert the image to frequency domain.
4. Apply the **inverse filter**.
5. Perform **Inverse Fourier Transform** to reconstruct the image.

Advantages

- Simple and easy to implement.
- Effective when **noise is negligible**.
- Useful when the **degradation function is known exactly**.

Limitations

- Very **sensitive to noise**.
- If **H(u,v) is close to zero**, the division amplifies noise.
- Requires accurate knowledge of the **degradation function**.