1.5 ELECTRICAL ANALOGY OF THERMAL SYSTEMS

There are two fundamental physical elements that make up thermal networks, thermal resistances and thermal capacitance. There are also three sources of heat, a power source, a temperature source, and fluid flow.

Example:

In practice temperature when we discuss temperature, we will use degree Celsius (ºC), while SI unit for temperature is to use Kelvins (0ºK = - 273.15ºC). Generally reference temperature (T_1) is taken and all temperatures are measured relative to this reference. Reference temperature is assumed to be constant.

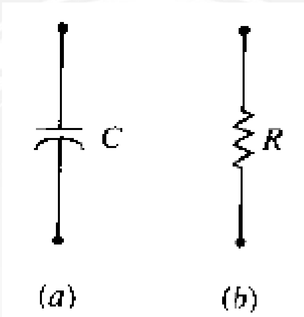

Figure 1.5.1 Network elements of thermal systems

[Source: "Linear Control System Analysis and Design" by John J. D'Azzo, Page: 76]

Thermal resistance

Consider the situation in which there is a wall, one side of which is at a temperature T_1, with the other side at temperature T_2, the wall has a thermal resistance of R_{12}.

Thermal capacitance

In addition to thermal resistance, objects can also have thermal capacitance (also called thermal mass). The thermal capacitance of an object is a measure of how much heat it can store. If an object has thermal capacitance its temperature will rise as heat flows into the object, and the temperature will lower as heat flows out. To understand this, envision a rock in the sun. During the day heat goes in to the rock from the sunlight, and the temperature of the rock increases as energy is stored in the rock as an increased temperature. At night energy is released, and the rock cools down. We represent a thermal capacitance in isolation in diagrams (and equations) as shown in Figure (in the drawing at the left the coil represents a power source and the stippled object is the thermal capacitance). In the thermal analogy, one end of the capacitor is always connected to the constant ambient temperature. The electrical model will always have one side of the capacitance connected to ground, or reference. Also, we could write the equation as = q but since T_1 is constant, it can be removed from the derivative. The thermal capacitance of an object is determined by its mass and specific heat.

$$C = mc_p$$

Where C is the thermal capacitance, m is the mass in kilograms, and c_p is the specific heat in J/(kg-°K). it is always assumed that the capacitor is at a single uniform temperature, though this is obviously a simplification in many cases.

$$C\frac{dT_2}{dt} = q$$

Power source (or heat source)

A common part of a thermal model is a controlled power source that generates a predetermined amount of power, or heat, in a system. This power can either be constant or a function of time. In the electrical analogy, the power source is represented by a current source. An example of a power source is the quantity *q* in the diagrams for the thermal capacitance, above. In practice a power source is often an electrical heating element comprised of a coil of wire that is heated by a current flowing through it. Therefore, we use a diagram of a coil of wire to represent the power source. An ideal power source generates power that is independent of temperature.

Temperature source

An ideal temperature source maintains a given temperature independent of the amount of power required. Ambient temperature is considered to be reference temperature).

Mass Transfer (Fluid Flow)

If fluid with specific heat c_p J/kg-°K) flows into a system with a flow rate of G kg/sec and a temperature of T_m°C above reference, and flows out at a temperature of T_{out}°C below reference then the rate of heat flow into the system is given by

$$q_{in} = G\left\{\frac{kg}{sec}\right\} \cdot c_p \left\{\frac{J}{kg\ K}\right\} \cdot (T_{in} - T_{out})\{°C\} = Gc_p(T_{in} - T_{out})\{W\}$$

We can cancel the K and °C since a temperature difference $(T_{in} - T_{out})$ is the same in Kelvin r Celsius. If you carefully observe this equation, it makes sense intuitively. Heat into s system goes up with mass flow rate into the system (increased mass flow, yields increased heat flow). Heat into a system also goes up with the specific heat of the mass (Higher specific heat indicates increased capacity to store heat). Finally, heat into system increases with an increased inflow temperature, or a decreased outflow temperature (if the temperature difference between inflow and outflow increases, more heat is being taken from the fluid). Note, the mass flow rate at the input and output must be equal to the mass (and thermal capacitance) of the system would be changing. This is not allowed for the systems being studied (time-invariant systems).

Energy balance

To develop a mathematical model of a thermal system we use the concept of an energy balance. The energy balance equation simply states that at any given location, or node, in a system, the heat into that node is equal to the heat out of the node plus any heat that is stored (heat is stored as increased temperature in thermal capacitances). The terms used in the equations is mentioned below:

Symbol	Quantity	U.S. customary units	Metric units
q	Rate of heat flow	Btu/minute	Joules/second
M	Mass	Pounds	Kilograms
S	Specific heat	Btu/(pounds)(°F)	Joules/(kilogram)(°C)
C	Thermal capacitance $C = MS$	Btu/°F	Joules/°C
K	Thermal conductance	Btu/(minute)(°F)	Joules/(second)(°C)
R	Thermal resistance	Degrees/(Btu/minute)	Degrees/(joule/second)
θ	Temperature	°F	°C
h	Heat energy	Btu	Joules

Additional heat stored in a body whose temperature is raised from θ_1 to θ_2 is given by

$$h = \frac{q}{D} = C(\theta_2 - \theta_1)$$

$$q = CD(\theta_2 - \theta_1)$$

Rate of heat flow through a body in terms of the two boundary temperatures θ_3 to θ_4

$$q = \frac{\theta_3 - \theta_4}{R}$$

The thermal resistance determines the rate of heat flow through the body. This is analogous to the resistance of a resistor in an electric circuit, which determines the current flow.

SIMPLE MERCURY THERMOMETER

Consider a thin glass-walled thermometer filled with mercury that has stabilized at a temperature θ_1. It is plunged into a bath of temperature θ_0 at t=0. In its simplest form, the thermometer can be considered to have a capacitance C that stores heat and a resistance R that limits the heat flow. The temperature at the center of the mercury is θ_m.The flow of heat into the thermometer is

$$q = \frac{\theta_0 - \theta_m}{R}$$

The heat entering the thermometer is stored in the thermal capacitance and is given by

$$h = \frac{q}{D} = C(\theta_m - \theta_1)$$

These equations can be combined to form

$$\frac{\theta_0 - \theta_m}{RD} = C(\theta_m - \theta_1)$$

Differentiating the above equation and rearranging the terms gives,

$$RC\, D\theta_m + \theta_m = \theta_0$$

The thermal network is drawn as in figure 1.5.2. Thus, the state equation is

$$\dot{x}_1 = -\frac{1}{RC}x_1 + \frac{1}{RC}u$$

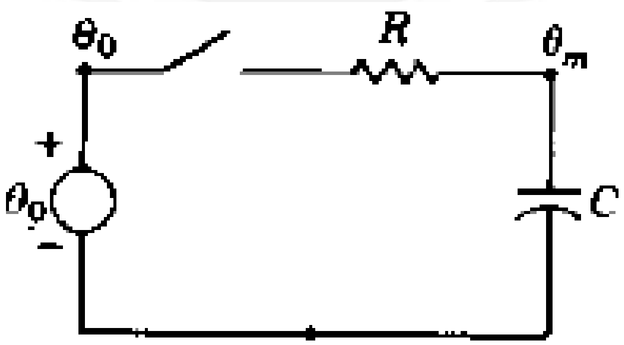

Figure 1.5.2 Network representation of a thermometer

[Source: "Linear Control System Analysis and Design" by John J. D'Azzo, Page: 77]

In general, *Heat in = Heat out + Heat stored*

$$q = \frac{T_r - T_a}{R_{ra}} + C\frac{dT_r}{dt}$$