ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY
Approved by AICTE & Affliated to anna university
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DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

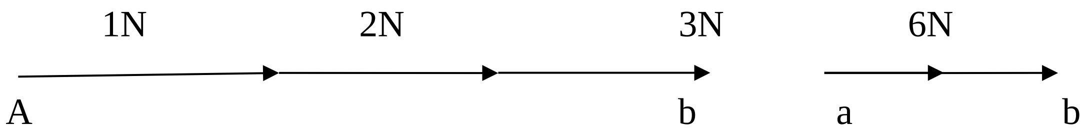
REGULATION 2021

UNIT I: BASIC & STATICS OF PARTICLES

Statics of Particles in Two Dimensions- Resultant Force<u>Resultant</u>

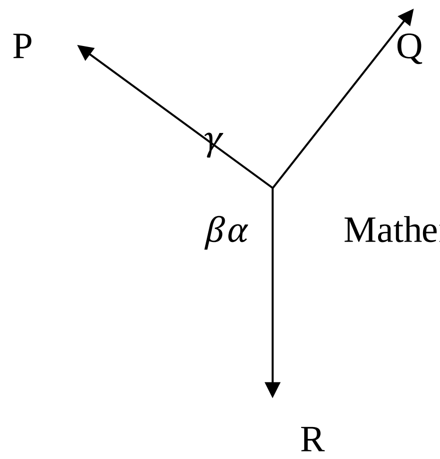
<u>Force:</u>

If a number of forces acting on a particle simultaneously are replaced by a single force which could produce the same effort as produced by the given forces, that single force is called resultant force.



<u>Lami's Theorem:</u>

It state that "if three coplanar forces acting at a point be in equilibrium, than each force is propositional to the sin of the angle between the other two forces.

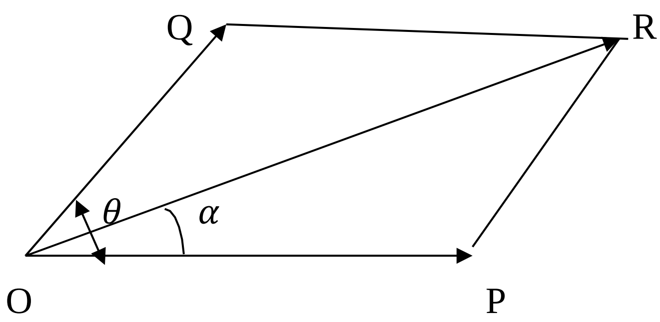


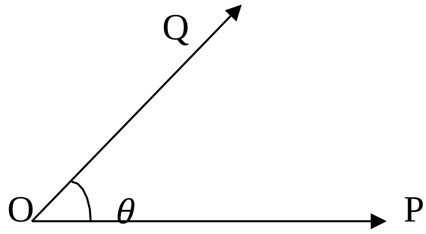
Mathematically

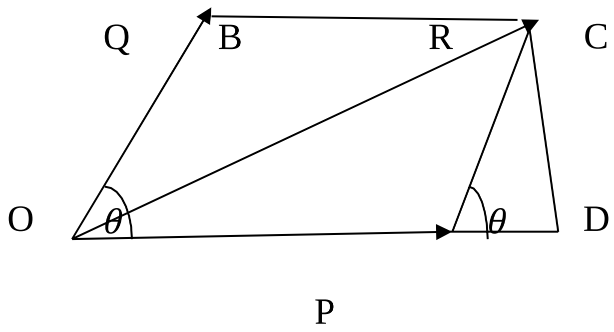
$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

<u>**Parallelogram Law of forces:**</u>

It states that "if the two forces acting simultaneously at a point represented in magnitude and direction by the two adjacent sides of the parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of the parallelogram originating from that point.







Let pand Q are two concurrent force acting on a point O at an angle of θ.

The forces P and Q are graphically represented by the lines OA and OB respectively.

The parallelogram θACB is completed by drawing the lines BC and AC parallel to OA and OB respectively.

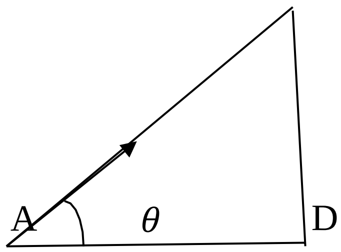
In parallelogram OACB, the diagonal OC represents the resultant force of P and q. by II Law of forces.

In order to prove the, parallelogram law of forces, extend the lines of action of force P, till its meet the perpendicular drawn from point C.

Let the point of intersection of these two lines be D. from the geometry of the parallelogram.

OB=AC

OA=BC



In triangle ACD

$\cos\theta = \frac{AD}{Q}$ $\sin\theta = \frac{CD}{Q}$

AD = Q $\cos\theta$ --------- (1) CD = Q $\sin\theta$-------------- (2)

Also = $AD^2 + CD^2 = AC^2$

$= AD^2 + CD^2 = Q^2$ ---------------(3)

In triangle OCD

$OC^2 = OD^2 + CD^2$

$= (OA + AD)^2 + CD^2$

$= OA^2 + AD^2 + 2 \times OA \times AD + CD^2$

$= OA^2 + (AD^2 + CD^2) + 2OA\ AD$

$= OA^2 + AC^2 + 2\ OA\ AD$

$R^2 = P^2 + Q^2 + 2 \times PQ\cos\theta$

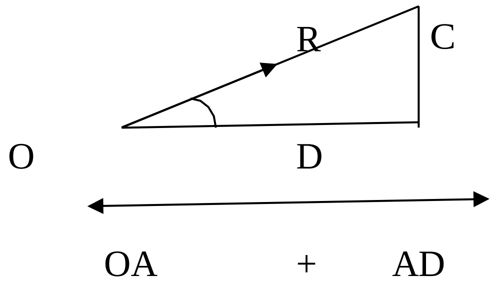
$R = \sqrt{P^2 + Q^2 + 2 \times PQ\cos\theta}$

Inclination of the resultant force with the force P

Let the angle of inclinator of R with the line of action of the force P be a

In triangle OCD

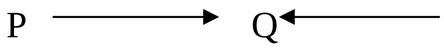
$\tan a = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}$



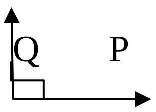
$\therefore \tan a = \frac{Q\sin\theta}{P+Q\cos\theta}$

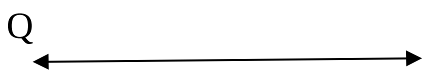
Important results:

1. If $\theta = 0°$ then the resultant forces pand Q will be like collinear, then, R=P +Q



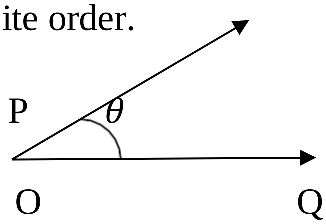
2. If $\theta = 90°$, the forces P and Q are at right angles then R= $\sqrt{P^2 + Q^2}$
3. If $\theta = 180°$, then the forces P and Q will be unlike collinear forces, then R=P-Q.

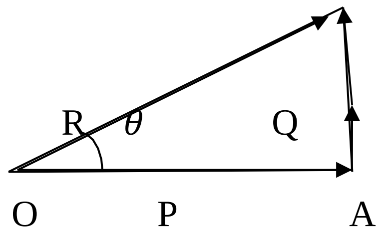




Triangle law of Forces:

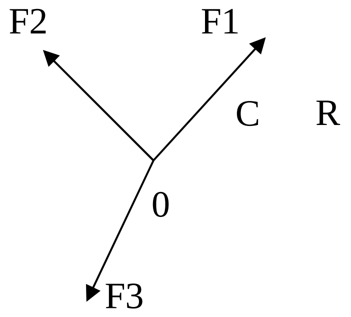
If two forces acting at a point are represented by two sides of a triangle taken in order, then their resultant force is represented by the third side taken in opposite order.

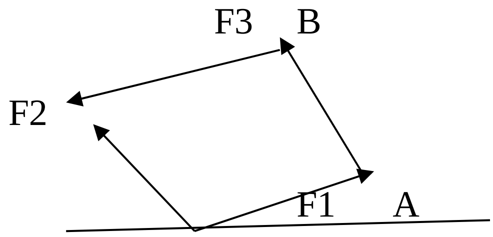




Polygon Law of forces:

Polygon Law of forces states that, ‘if a number of coplanar concurrent forces are represented in magnitude and direction by the sides of a polygon taken in an order then their resultant force is represented by the closing side of the polygon taken in the opposite order.

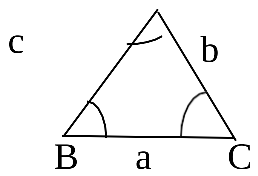




Sine Law:

The law of sines can be used when two angles and a side are known a technique known as triangulation.

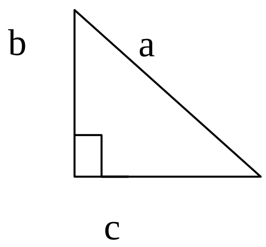
$$= \frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$



Cosine Law:

It two side and the angle between the sides are known,

Then the third is given by



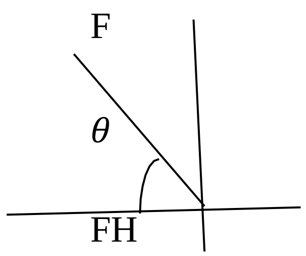
$$a^2 = b^2 + c^2 - 2bccos\alpha$$

$$b^2 = c^2 + a^2 - 2cacos\beta$$

$$c^2 = a^2 + b^2 - 2abcos\gamma$$

Resolution of a force in to its horizontal and vertical part.

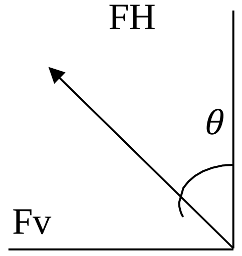
Method I



$$sin\theta = \frac{Fv}{F} => FV = Fsin\theta$$

$$cos\theta = \frac{FH}{F} => FH = Fcos\theta$$

Method II

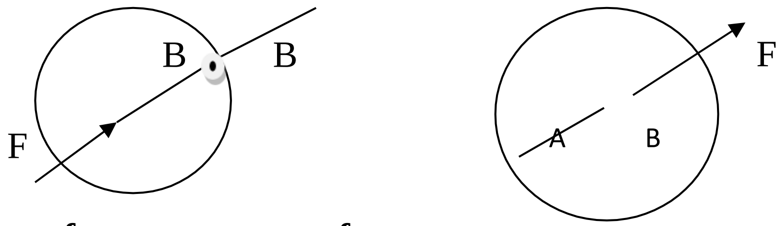


$$sin = \frac{FH}{F} => FH = Fsin\theta$$

$$cos\theta = \frac{Fv}{F} => Fv = Fcos\theta$$

Principle of transmissibility of Forces:

If a force act at any point of on a rigid body it may also be considered to act at any other point on its line of action.

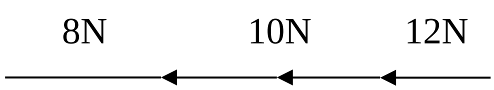


Resultant force of two concurrent forces:

1. Resultant force of two concurrent force
2. Resultant force of more than two concurrent force

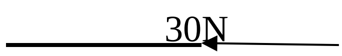
Problem based on parallelogram & Resultant forces:

1. Find the resultant force of the collinear forces shown in fig.

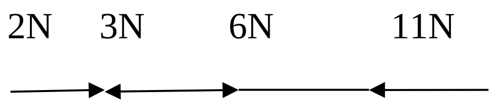


Soln:

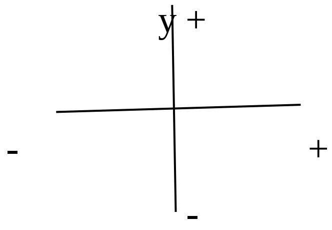
Resultant force R= 8+10+12=30N



2. Find the resultant force of the collinear forces, shown in fig



<u>Soln:</u>



Magnitude of resultant force

= 2-3+6-11

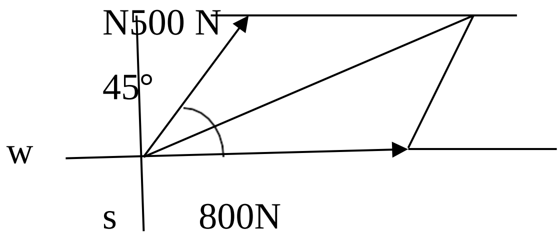
R = -6N

3. Find the resultant force an 800 N force acting towards eastern direction and a 500 n force acting towards north eastern direction.

by 1. Parallelogram Law

2. Triangle Law

Also find the direction



<u>Given</u>

P=800 N Q=500N θ=45°

To find

Resultant force & direction

<u>Soln</u>

1. Parallelogram Law

Resultant Force R= $\sqrt{P^2} + Q^2 + 2PQcos\theta$

R = $\sqrt{800^2} + 500^2 + 2 \times 800 \times 500 \times cos45$

R= 1206.52 N

Direction of magnitude

Q $= tan^{-1}[\frac{Qsin\theta}{P+Q\ cos\theta}]$

Q $= tan^{-1}[\frac{500sin45}{800+500\ cos45}]$

Q = 17°04’

Summing of components:

R $= \sqrt{FH^2 + \sum FV^2}$

$\sum FH = 800+500\sin 45=1153.55N$

$\sum FH = 500 \sin 45 = 353.55$ N

R = $\sqrt{1153.55^2 + 353.55^2}$

R = 1206.52 N

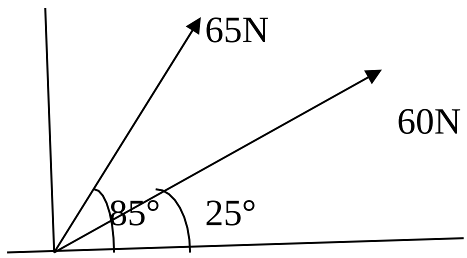
$\alpha = tan^{-1} \frac{[\sum FV]}{\sum FH}$

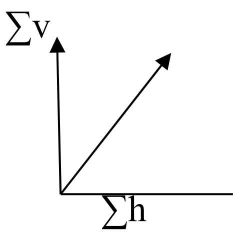
$= tan^{-1} \frac{[1153.55]}{353.55}$

$\alpha = 17°04'$

4. Two forces 60 N and 65 N act on a screw at an angle of 25° and 85° from the base. Determine the magnitude and direction of their resultant.

Given:





P_1=60 N1,$_1$=25°

P_2=65 N, θ_2=85°

To find:

Magnitude & direction of their resultant

Soln:

1. Magnitude of resultant force

R $= \sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ = 60 cosθ_1 +65 cosθ_2

= 60 cos25 +65 cos85

$\sum FH$ =60 N

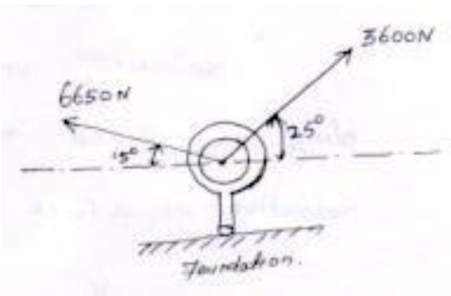
$\sum FV$ = 60 sinθ_1 +65 sinθ_2 = 60 sin25 +65 sin85

$\sum FV$ = 90 N

R = $\sqrt{60^2 + 90^2}$

R= 108.17 N

5. Two wires are attached to a bolt in a foundation as shown in fig. below. Determine the pull exerted by the bolt on the foundation.



Soln:

Resultant force R = $\sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ = 3600 cos 25-6650 cos 15

$\sum FH$ = -3160 N

$\sum FV$ = 3600 sin25+ 6650 sin 15

$\sum FV$ = 3242N

R = $\sqrt{-3160^2 + 3242^2}$

R = 4527 N

$$\alpha = tan^{-1[\frac{\sum FV}{\sum FH}]}$$

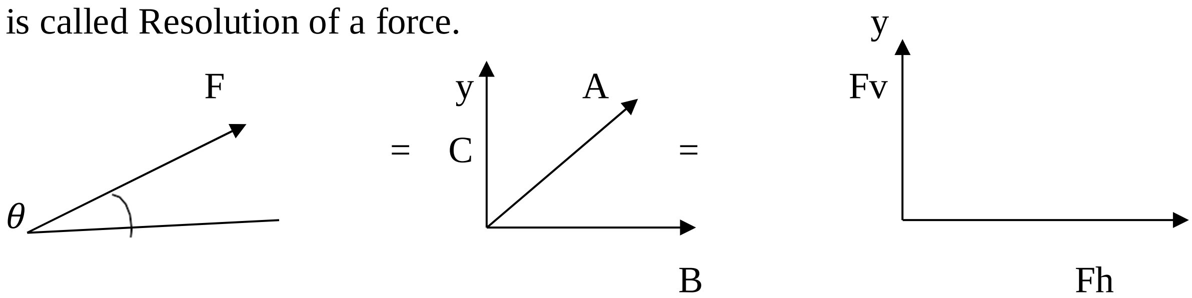
$$\alpha = tan^{-1[\frac{\sum 3242}{\sum 3160}]}$$

$$\alpha = 45°73'$$

⇒ **<u>Resultant force of more than Two concurrent Forces:</u>**

Resolution of Forces:

Splitting up a force into components along the fixed reference axis is called Resolution of a force.



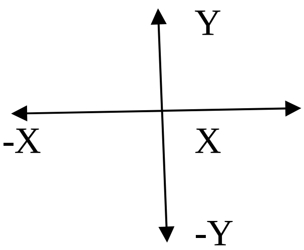
F_h = horizontal component = +F cosθ

F_v = vertical component = +F sinθ

Sign conversion:

Horizontalcomponent - ⟵ + ⟶

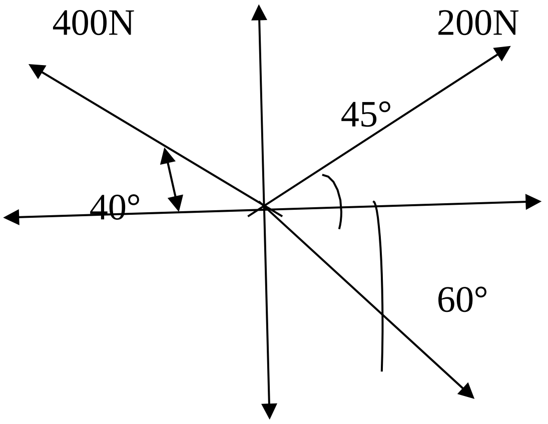
Vertical component - ↓ ↑ + sine



$$R = \sqrt{\sum FH^2 + \sum FV^2}$$

$$\alpha = tan^{-1[\frac{\sum FV}{\sum FH}]}$$

1. Three coplanar concurrent forces are acting at a point as shown in fig. Determine the resultant in magnitude of direction.



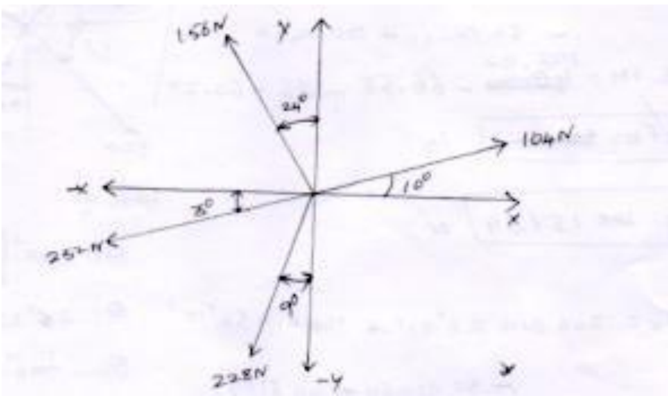
Soln:

Force & magnitude	θ	F$\cos\theta$	$Fsin\theta$
F_1=200	45°	200 cos45°=141.42	200 sin45°=141.42N
F_2=400	150°	400 cos150°=-326.41	400 sin150°=200
F_3=600	300°	600 cos300°=300 N	600 cos300°=-519.61
		$\sum FH$ = 95.01 N	$\sum FV$ = -178.19N

R $= \sqrt{\sum FH^2 + \sum FV^2} = \sqrt{95.01^2 + -178.19^2}$

R=201.95 N

2. The four coplanar forces acting at a point a as shown in fig. Determine the resultant in magnitude and direction.



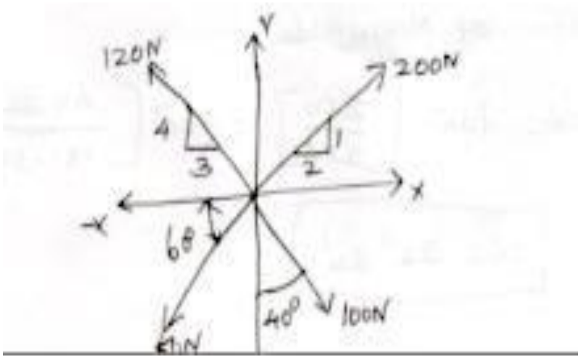
<u>Soln:2 nd method</u>

Note:$_1$=10° θ_2=90-24=66° θ_3=3° θ_4=90-9=81°

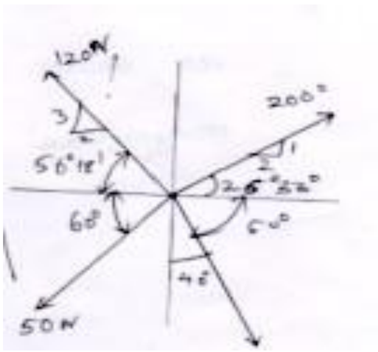
$\sum FH$ =-248.36 $\sum FV$ = -77.82

R=260.26N θ= 17.39°

3. A system of four forces acting on a body is shown in fig below. Determine the resultant force and direction.



Soln:



Resultant force R= $\sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ =200 cos26°33$' - 120 \cos 56°18' - 50 \cos 60 + 100cos50$

$\sum FH$ =178.90-66.58-25+64.27

$\sum FH$ =151.59

$\sum FV \ = 200sin26°33' + 120sin56°18' - 50sin60 - 100sin50$

$\sum FV \ = 89.39 + 99.83 - 43.30 - 76.60$

$\sum FV$ =69.32N

R $=\sqrt{(151.59)^2 + (69.32)^2}$

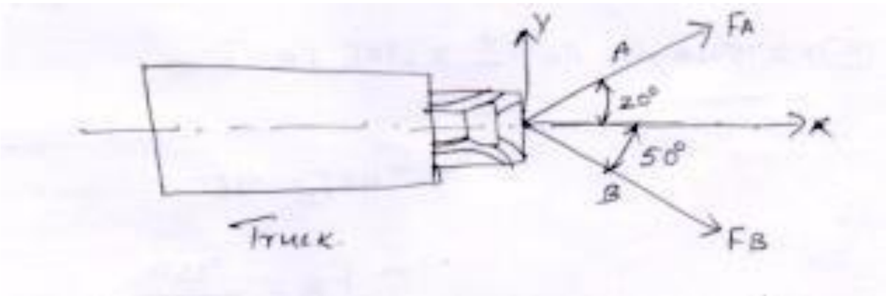
R = 166.68N

Direction of magnitude

$\alpha = tan^{-1}[\frac{\sum FV}{\sum FH}] = tan^{-1}[\frac{69.32}{151.59}]$

$\alpha = 24°34'$

4. The truck shown is to be toward using two ropes. Determine the magnitude of forces F_A& F_B acting on each rope in order to develop a resultant force of 950N directed along the positive X axis.



Resultant force R= 950 N in positive $\times\, direction$

∴Hence $\sum FH$ = 950

$\sum FH$ = 0

Resolving forces horizontally

$\sum FH$=FAcos20°+FB cos 50°

$FAcos20 + FBcos50 = 950$--------- (1)

Resolving forces vertically

$\sum FV = FAsin20 - FBsin50 = 0$

$FAsin20 - FBsin50 = 0$------------------- (2)

Solving eq(1) & (2)

$FAcos20 + FBcos50 = 950$

$FAsin20 - FBsin50 = 0$

$0.939FA + 0.642FB = 950$--------------- (1)

$0.342FA + 0.766FB = 0$--------------- (2)

$0.939FA + 0.642FB = 950$--------------- (1)

(2)$\times$ 2.75 $\quad 0.342FA \pm 0.766FB = \quad 0$

2.748FB = 950

FB =$\frac{950}{2.748}$

FB = 345.64 N

FB value sub in eqn(1)

0.94$\times$ FA+0.642$\times$345.64=950

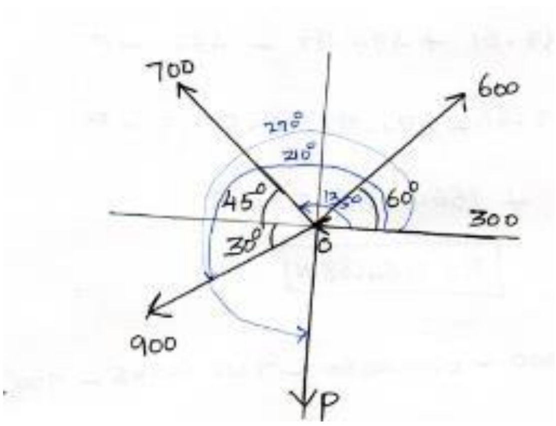
0.94 FA =950-221.9=728

FA= $\frac{728.09}{0.94}$

FA = 774.57 N

5. Five forces are acting on a particle. The magnitude of the forces are 300 N,600N,700N,900n and P and their respective angles with the horizontal are 0°,60°,135°,210°,270°. If the vertical component of all the force is -1000N, Find the value of P. Also calculate the magnitude and the direction of the resultant, assuming that the first force acts towards the point, while all the remaining forces act away from the point.

Given:



θ_1=0° θ_2=60° θ_3=180-135=45°

θ_4=180+[90-60]=210°=30°

θ_5=270=90°

F_1=300, F_2=600 F_3=700 F_4=900 F_5=P

$\sum FV$ = -1000N

Soln

Resultant force R=$\sqrt{(\sum FH)^2 + (\sum \mathrm{FV})^2}$

$\sum$FV= -1000N

To find the value of 'P'

Algebraic sum of vertical components

$\sum$Fv = 600 sin60 +700 sin45-900 sin30-P

-1000=519.61+494.97-450-P

-1000=-519.61-494.97+450=-P

- 1564.58 = - P

P=1564.58N

$\sum$FH=-300+600 cos60-700 cos45-900 cos30

$\sum$FH=-300 +300-494.97-779.42

$\sum$FH=-1274.39 N

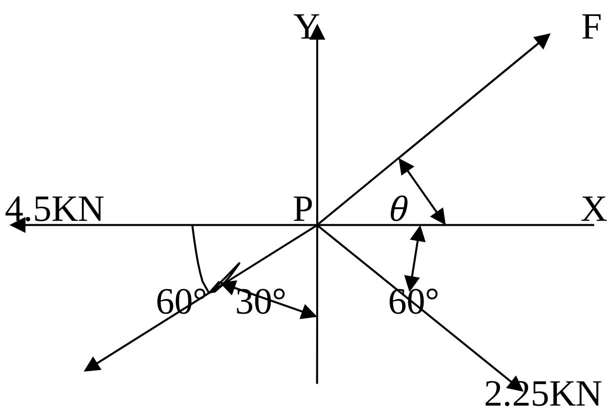
Resultant force R=$\sqrt{(-1274.39)^2 + (-1000)^{\mathbf{2}}}$

R=1619.89 N

Direction $\alpha = tan^{-1}[\frac{\Sigma FV}{\Sigma FH}] = tan^{-1}[\frac{1000}{1274}]$

$\alpha = 38°7'$

6. Determine the magnitude and angle of f so that particle P shown in Fig



Soln:

$\sum$FH=F cosθ-4.5-7.5 cos60+2.25 cos60=0

$F\cos\theta$-4.5-3.75+1.125=0 ----------------- (1)

$\sum$FV= F sinθ-7.5 sin60-2.25 sin 60=0

F sinθ-6.49-1.94=0 --------------- (2)

Eqn (1) Rearrange

F cosθ-7.125=0

F cosθ=7.125 ----------------- (3)

Eqn(2) Rearrange

F sinθ-8.43=0

F sinθ=8.43 ------------------- (4)

$$\frac{(4)}{(3)} = \frac{Fsin\theta}{Fcos\theta} = \frac{8.43}{7.125}$$

$\text{Tan}\theta = 1.183$

$\theta = tan^{-1}[1.183]$

$\theta = 49°47'$

Substitute $\theta = 49°47'$ in eqn(3)

F$\cos\theta$=7.125$\Rightarrow$ F cos $49°47'$=7.125

$F=\frac{7.125}{cos 49°47}$

F= 11.03 N