UNIT-I

TESTING THE HYPOTHESIS

5.3 Small sample Tests (t - Test)

PROCEDURE FOR TESTING OF HYPOTHESIS

- State the null hypothesis H_0
- Decide the alternative hypothesis H_1 (i.e., one tailed or two tailed)
- Choose the level of significance α at 5% (or) 1%.
- Compute the test statistic $Z = \frac{t-E(t)}{S.E\ of\ (t)}$
- Compare the computed value of with the table value of $|Z|$ with the table value of Z and decide the acceptance or the rejection of H_0.
- If $|Z| < 1.96, H_0$ is accepted at 5% level of significance.
- If $|Z| > 1.96, H_0$ is rejected at 5% level of significance.
- If $|Z| < 2.58, H_0$ is accepted at 1% level of significance.
- If $|Z| > 2.58, H_0$is rejected at 1% level of significance.
- For a single tail test (right tail or left tail) we compare the computed value of $|Z|$ with 1.645 (at 5% level of significance) and 2.33 (at 1% level of significance) and accept or reject H_0**accordingly.**

TEST OF SIGNIFICANCE OF SMALL SAMPLES

When the size of the sample (n) is less than 30, then that sample is called a small sample.

The following are some important tests for small samples.

(i) Student's t – test

(ii) F – test

(iii) χ^2 test

Test of significance of the difference between sample mean and population mean

The student's "t" is defined by the statistic

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$Where $\bar{x}$ = sample mean

μ = population mean

S = s.d of sample

n = sample size

Note:

If s. d of a sample is not given directly then, the static is given by $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

Where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, $S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$

Working Rule:

Set the null hypothesis H_0: μ = a specified value

Set the alternative hypothesis H_1: $\mu \neq$ a specified value

we choose α = 0.05(5%) (or) 0.01(1%) as the Level of significance

The test statistic is $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$ with $v = n - 1$ degrees of freedom.

If $|t| < t_{0.05}$ H_0 is accepted at 5% level of significance.

If $|t| > t_{0.05}$, H_0 is rejected at 5% level of significance

1. **The mean lifetime of a sample of 25 bulbs is found as 1550hours, with an S.D of 120 hours. The company ,manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% level of significance?**

Solution:

Given $n = 25, \bar{x} = 1550, s = 120, \mu = 1600$

Set the null hypothesis H_0: μ= 1600

Set the alternative hypothesis H_1: $\mu \neq$ 1600

Level of significance at 5%

he test statistic is $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$ with $v = n - 1$ degrees of freedom.

$$t = \frac{1550 - 1600}{120/\sqrt{24}} = -2.0412$$

$$|t| = 2.0412$$

Critical value: At 5% level, the tabulated value of t_α is 2.064 for

$$v = n - 1 = 24$$

Conclusion: Since $|t|$ = 2.0412 < 2.064

Hence Null Hypothesis H_0 is accepted at 5% level of significance.

i.e.,The claim is acceptable.

2. **Tests made on the breaking strength of 10 pieces of a metal gave the following results: 578, 572, 570, 568, 572, 570, 570, 572, 596, 584 kg. Test if the mean breaking strength of the wire can be assumed as 577kg.**
Solution:

Let us first compute sample mean $\bar{x}$ and sample S. D and then if $\bar{x}$differs significantly from the population mean $\mu = 577$

Where $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{5752}{10} = 575.2$

x	$x - \bar{x}$	$(x - \bar{x})^2$
578	2.8	7.84
572	- 3.2	10.24
570	- 5.2	27.04
568	- 7.2	51.84
572	- 3.2	10.24
570	- 5.2	27.04
570	- 5.2	27.04
572	- 3.2	10.24
596	20.8	432.64
584	8.8	77.44
5752	0	681.6

$$S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{681.6}{10-1} = 75.733$$

Set the null hypothesis H_0: μ = 577

Set the alternative hypothesis H_1: $\mu \neq$ 577

Level of significance at 5%

The test statistic is $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ with $v = n - 1$ degrees of freedom.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{572.2 - 577}{\sqrt{75.733}/\sqrt{10}} = -0.654$$

$$|t| = 0.654$$

Critical value: At 5% level, the tabulated value of t_α is 2.262 for

$$v = n - 1 = 9$$

Conclusion: Since $|t|$= 0.654 < 2.262

Hence Null Hypothesis H_0 is accepted at 5% level of significance.

The mean breaking strength of the wire can be assumed as 577 kg at 5% level of significance.

3. **A machinist is making engine parts with axle diameter of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D of 0.040. Test whether the work is meeting the specification at 5% Los**
   Solution:

   Given $n = 10,\ \bar{x} = 0.742,\ s = 0.040,\ \mu = 0.700$

   Set the null hypothesis H_0: $\mu = 0.700$

   Set the alternative hypothesis H_1: $\mu \neq 0.700$

   Level of significance at 5%

   The test statistic is $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$ with $v = n - 1$degrees of freedom.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}} = \frac{0.742 - 0.700}{0.040/\sqrt{9}} = 3.15$$

Critical value: At 5% level, the tabulated value of t_α is 2.26 for

$$v = n - 1 = 9$$

Conclusion: Since $|t|$ = 3.15 > 2.26

Hence Null Hypothesis H_0 is rejected at 5% level of significance.

Test of significance of the difference between means of two small samples

- **To test the significance of the difference between the mean $\overline{x_1}$ and $\overline{x_2}$ of samples of size n_1 and n_2, use the statistic $t = \frac{\overline{x_1}-\overline{x_2}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$**

- **Where** $S = \sqrt{\frac{n_1 {s_1}^2 + n_2 {s_2}^2}{n_1+n_2-2}}$ **with** $n_1 + n_2 - 2$ **degrees of freedom**
- **(OR)** $\boldsymbol{S^2} = \frac{\sum(x_1-\overline{x_1})^2+\sum(x_2-\overline{x_2})^2}{n_1+n_2-2}$ **if** s_1 **and** s_2 **are not given directly**

1.Two independent samples from normal pop's with equal variances gave the following results

Sample	Size	Mean	S.D
1	16	23.4	2.5
2	12	24.9	2.8

Test for the equations of means.

Solution:

Given $n_1 = 16, n_2 = 12,\ s_1 = 2.5, s_2 = 2.8, \overline{x_1} = 23.4\ , \overline{x_2} = 24.9$

Set the null hypothesis $H_0\colon\ \mu_1 = \mu_2$

Set the alternative hypothesis $H_1\colon\ \mu_1 \neq \mu_2$

Level of significance at 5%

The test statistic is $t = \frac{\overline{x_1}-\overline{x_2}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

$$S = \sqrt{\frac{n_1 {s_1}^2 + n_2 {s_2}^2}{n_1+n_2-2}} = \sqrt{\frac{16*(2.5)^2+12*(2.8)^2}{16+12-2}} = \mathbf{2.732}$$

$$\Rightarrow\ t = \frac{23.4-24.9}{2.732\sqrt{\frac{1}{16}+\ \frac{1}{12}}} = -\mathbf{1.432}$$

$|t|$ **= 1.432**

Critical value: At 5% level, the tabulated value of t_α **is 2.056 for**

$$v = n_1 + n_2 - 2 = 16 + 12 - 2 = 26$$

Conclusion: Since$|t|$ **= 1.432 < 2.056**

Hence Null Hypothesis H_0 **is accepted at 5% level of significance.**

i.e., There is no significant difference between their means

2. Two independent samples of 8 and 7 items respectively had the following values

Sample I : 9 13 11 11 15 9 12 14

Sample II : 10 12 10 14 9 8 10

Is the difference between the means of the samples significant?

Solution:

Given $n_1 = 8, n_2 = 7$

x_1	$d_1 = (x_1 - \overline{x_1}) = x_1 - 11.75$	${d_1}^2 = (x_1 - \overline{x_1})^2$	x_2	$d_2 = (x_2 - \overline{x_2}) = x_2 - 10.43$	${d_2}^2 = (x_2 - \overline{x_2})^2$
9	-2.75	7.5625	10	-0.43	0.1849
13	1.25	1.5625	2	1.57	2.4649
11	-0.75	0.5625	10	-0.43	0.1849
11	-0.75	1.5625	14	3.57	12.7449
15	3.25	10.5625	9	-1.43	2.0449
9	-2.75	7.5625	8	-2.43	5.9049
12	0.25	0.0625	10	-0.43	0.1849
14	2.25	5.0625			
	$\sum d_1$ =3.5	$\sum {d_1}^2$ =33.5		$\sum d_2$ =-0.01	$\sum {d_2}^2$ =23.7143

Set the null hypothesis $H_0\colon\ \mu_1 = \mu_2$

Set the alternative hypothesis $H_1\colon\ \mu_1 \neq \mu_2$

Level of significance at 5%

$$\overline{x_1} = \frac{\sum x_1}{n} = \frac{94}{8} = 11.75$$

$$\overline{x_2} = \frac{\sum x_2}{n} = \frac{73}{7} = 10.43$$

$$\boldsymbol{S^2} = \frac{\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2}{n_1 + n_2 - 2} = \frac{33.5 + 23.71}{8 + 7 - 2}$$

$$\boldsymbol{S = 2.097}$$

The test statistic is $t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$\Rightarrow t = \frac{11.75 - 10.43}{2.097\sqrt{\frac{1}{8} + \frac{1}{7}}} = \mathbf{1.218}$$

Critical value:At 5% level, the tabulated value of t_α **is 2.16 for**

$$v = n_1 + n_2 - 2 = \mathbf{8 + 7 - 2 = 13}$$

Conclusion: Since $|t|$ **= 1.432 < 2.16**

Hence Null Hypothesis H_0 **is accepted at 5% level of significance.**

i.e., There is no significant difference between their means

3.Two independent samples of 8 and 7 items respectively had the following values

Sample I : 19 17 15 21 16 18 16 14

Sample II : 15 14 15 19 15 18 16

Is the difference between the means of the samples significant?

Solution:

Given $n_1 = 8, n_2 = 7$

x_1	$d_1 = (x_1 - \overline{x_1}) = x_1 - 11.75$	${d_1}^2 = (x_1 - \overline{x_1})^2$	x_2	$d_2 = (x_2 - \overline{x_2}) = x_2 - 10.43$	${d_2}^2 = (x_2 - \overline{x_2})^2$
19	**2**	**4**	**15**	**-1**	**1**
17	**0**	**0**	**14**	**-2**	**4**
15	**-2**	**4**	**15**	**-1**	**1**
21	**4**	**16**	**19**	**3**	**9**
16	**-1**	**1**	**15**	**-1**	**1**
18	**1**	**1**	**18**	**2**	**4**

16	**-1**	**1**	**16**	**0**	**0**
14	**-3**	**9**			
136	**0**	**36**	**112**	**0**	**20**

Set the null hypothesis $H_0: \mu_1 = \mu_2$

Set the alternative hypothesis $H_1: \mu_1 \neq \mu_2$

Level of significance at 5%

$$\overline{x_1} = \frac{\sum x_1}{\boldsymbol{n}} = \frac{\mathbf{136}}{\mathbf{8}} = \mathbf{17}$$

$$\overline{x_2} = \frac{\sum x_2}{\boldsymbol{n}} = \frac{\mathbf{112}}{\mathbf{7}} = \mathbf{16}$$

$$\boldsymbol{S}^2 = \frac{\sum(x_1 - \overline{x_1})^2 + \sum(x_2 - \overline{x_2})^2}{n_1 + n_2 - 2} = \frac{36 + 20}{8 + 7 - 2} = \mathbf{4.3076}$$

$$\boldsymbol{S} = \mathbf{2.0754}$$

The test statistic is $t = \frac{\overline{x_1} - \overline{x_2}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$\Rightarrow t = \frac{17 - 16}{2.0754\sqrt{\frac{1}{8} + \frac{1}{7}}} = 0.9309$$

Critical value:At 5% level, the tabulated value of t_α **is 2.16 for**

$$v = n_1 + n_2 - 2 = \mathbf{8 + 7 - 2 = 13}$$

Conclusion: Since $|t|$**= 0.9309 < 2.16**

Hence Null Hypothesis H_0 **is accepted at 5% level of significance.**

i.e., There is no significant difference between their means.