

3.5 Stationary wave or standing wave:

When two progressive waves of same amplitude and wavelength travelling along a straight line in opposite directions superimpose on each other, standing waves are formed.

Analytical method:

Let us consider a progressive waves of amplitude 'a' and wavelength λ travelling in the direction of X-axis

$$y_1 = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

This wave is reflected from a free end and it travels in the negative direction of X – axis then

$$y_2 = a \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

According to principle of superposition, the resultant displacement is

$$y = y_1 + y_2$$

$$a\left[\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

[using trigonometry identity

$$\sin(A-B) + \sin(A+B) = 2 \sin A \cos B]$$

$$= a\left[\sin\left(2\pi\frac{t}{T} - 2\pi\frac{x}{\lambda}\right) + \sin\left(2\pi\frac{t}{T} + 2\pi\frac{x}{\lambda}\right)\right]$$

$$= a\left[2\sin 2\pi\frac{t}{T} \cos 2\pi\frac{x}{\lambda}\right]$$

$$y = 2a \sin 2\pi\frac{t}{T} \cos 2\pi\frac{x}{\lambda}$$

This is the equation of a stationary wave.