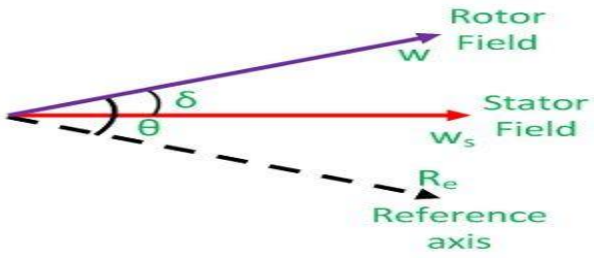
SWING EQUATION

The Swing Equation of generator describes the relative motion between the rotor axis and the synchronously rotating stator filed axis with respect to time. This equation is very helpful in analyzing the stability of connected machines (machine here means generator).

The transient stability of the system can be determined by the help of the swing equation. Let θ be the angular position of the rotor at any instant t. θ is continuously changing with time, and it is convenient to measure it with respect to the reference axis shown in the figure below. The angular position of the rotor is given by the equation



$$\theta = w_s t + \delta \ldots\ldots\ldots\ldots\ldots\ldots equ(1)$$

Where,θ – angle between rotor field and a reference axis, w_s – synchronous speed, δ – angular displacement

Differentiation of equation (1) gives

$$\frac{d\theta}{dt} = w_s + \frac{d\delta}{dt} \ldots\ldots\ldots\ldots\ldots\ldots equ(2)$$

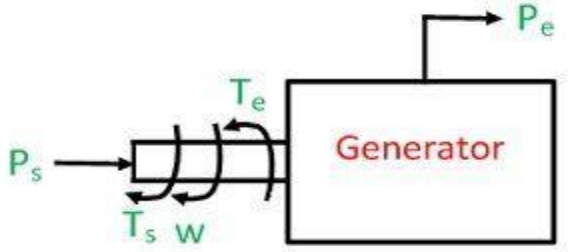
Differentiation of equation (2) gives

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \ldots\ldots\ldots\ldots\ldots\ldots equ(3)$$

Angular acceleration of rotor

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}\ elect.rad/s^2$$

Power flow in the synchronous generator is shown in the diagram below. If the damping is neglected the accelerating torques, T_a in a synchronous generator is equal to the difference of mechanical input shaft and the electromagnetic output torque, i.e.,



$$T_a = T_s - T_e \ldots\ldots\ldots\ldots\ldots\ldots equ(5)$$

Where,

T_a – accelerating torque, T_s – shaft torque,T_e – electromagnetic torque

Angular momentum of the rotor is expressed by the equation

$$M = Jw \ldots\ldots\ldots\ldots\ldots\ldots equ(6)$$

Where,w- the synchronous speed of the rotor, J – moment of inertia of the rotor M – angular momentum of the rotor

Multiplying both the sides of equation (5) by **w** we get

$$wT_a = wT_s - wT_e$$

$$P_a = P_s - P_e$$

Where,P_s – mechanical power input, P_e – electrical power output, P_a – accelerating power

But,

$$J\frac{d^2\delta}{dt^2} = T_a$$

$$J\frac{d^2\theta}{dt^2} = T_a$$

$$wJ\frac{d^2\delta}{dt^2} = wT_a \quad M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots equ(7)$$

Equation (7) gives the relation between the accelerating power and angular acceleration. It is called the swing equation. Swing equation describes the rotor dynamics of the synchronous machines and it helps in stabilizing the system.