RC DC Circuits:

The resistor and capacitor may be the equivalent resistance and equivalent capacitance of combinations of resistors and capacitors.) Our objective is to determine the circuit response, which, for pedagogic reasons, we assume to be the voltage *v(t)* across the capacitor. Since the capacitor is initially charged, we can assume that at time $t = 0$, the initial voltage is

$$v(0) = V_0$$

with the corresponding value of the energy stored as

$$w(0) = \frac{1}{2} C V_0^2$$

Applying KCL at the top node of the circuit

$$i_C + i_R = 0$$

By definition, $i_C = C\, dv/dt$ and $i_R = v/R$. Thus,

$$C \frac{dv}{dt} + \frac{v}{R} = 0$$

or

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

This is a *first-order differential equation*, since only the first derivative of v is involved. To solve it, we rearrange the terms as

$$\frac{dv}{v} = -\frac{1}{RC} dt$$

Integrating both sides, we get

$$\ln v = -\frac{t}{RC} + \ln A$$

where $\ln A$ is the integration constant. Thus,

$$\ln \frac{v}{A} = -\frac{t}{RC}$$

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3} e^{-t} \text{ A}, \qquad t > 0$$

Taking powers of e produces

$$v(t) = Ae^{-t/RC}$$

But from the initial conditions, $v(0) = A = V_0$. Hence,

$$v(t) = V_0 e^{-t/RC}$$

This shows that the voltage response of the RC circuit is an exponential decay of the initial voltage. Since the response is due to the initial energy stored and the physical characteristics of the circuit and not due to some external voltage or current source, it is called the *natural response* of the circuit.

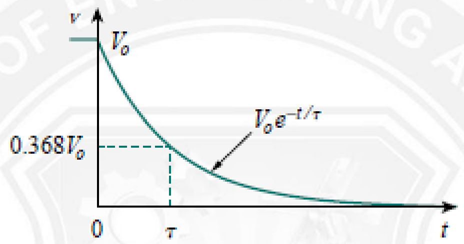

Fig. 3.3.1 plot.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 240]

This implies that at $t = \tau$,

$$V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368 V_0$$

or

$$\boxed{\tau = RC}$$

In terms of the time constant, Eq. (7.7) can be written as

$$\boxed{v(t) = V_0 e^{-t/\tau}}$$

Problem1:

The switch in the circuit in Fig. 7.8 has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.

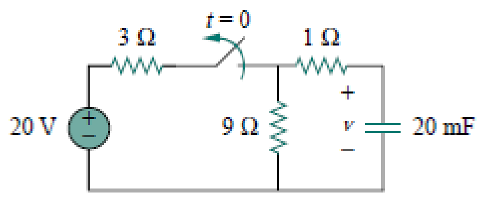

Fig. 3.3.2 For problem1.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 242]

Solution:

For $t < 0$, the switch is closed; the capacitor is an open circuit to dc, as represented in Fig. 3.3.3(a). Using voltage division

$$v_C(t) = \frac{9}{9+3}(20) = 15 \text{ V}, \qquad t < 0$$

Since the voltage across a capacitor cannot change instantaneously, the voltage across the capacitor at $t = 0^-$ is the same at $t = 0$, or

$$v_C(0) = V_0 = 15 \text{ V}$$

For $t > 0$, the switch is opened, and we have the RC circuit shown the independent source is needed to provide V_0 or the initial

The 1-Ω and 9-Ω resistors in series give

$$R_{eq} = 1 + 9 = 10\ \Omega$$

The time constant is

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2 \text{ s}$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2} \text{ V}$$

or

$$v(t) = 15e^{-5t} \text{ V}$$

The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25 \text{ J}$$

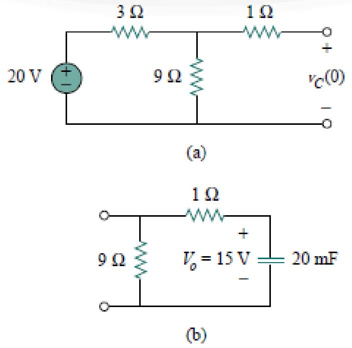

Fig. 3.3.3 For problem1.

[Source: "Fundamentals of Electric Circuits" by charles K. Alexander, page: 243]