1.4 ELECTRICAL ANALOGY OF MECHANICAL SYSTEMS

FORCE-VOLTAGE ANALOGY

Consider a simple translational mechanical system as shown in figure 1.4.1.

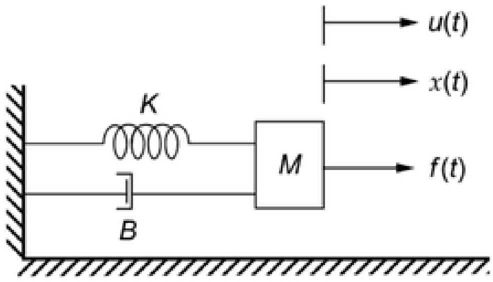

Figure 1.4.1 Translational mechanical system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.51]

Using D' Alembert's principle, we have,

Sum of the applied forces = Sum of the opposing forces

$$f(t) = M\frac{du(t)}{dt} + B\,u(t) + K\int u(t)\,dt$$

Consider a series RLC circuit as shown in figure 1.4.2.

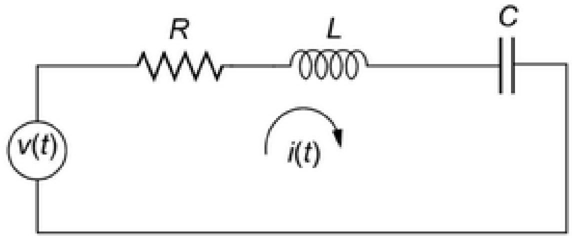

Figure 1.4.2 Series RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.52]

Using KVL, the integro-differential equations can be written as

$$v(t) = L\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

Translational system	Electrical system
Force (f)	Voltage (v)
Velocity (u)	Current (i)
Displacement (x)	Charge (q)
Mass (M)	Inductance (L)
Damping coefficient (B)	Resistance (R)
Spring constant (K)	1/Capacitance (C)

FORCE-CURRENT ANALOGY

Consider a simple parallel RLC circuit as shown in figure 1.4.3.

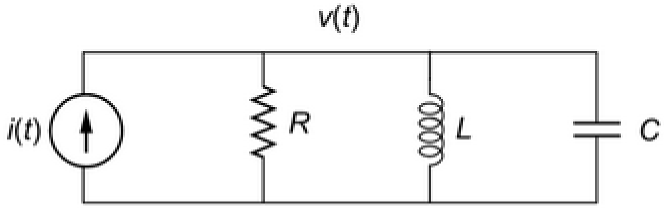

Figure 1.4.3 Parallel RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.52]

Using KCL, the integro-differential equations can be written as follows:

$$i(t) = C\frac{dv(t)}{dt} + G\,v(t) + \frac{1}{L}\int v(t)\,dt$$

where, conductance, G=1/R.

On comparing with the mechanical translational system equation, we get,

Translational System	Electrical System
Force (f)	Current (i)
Velocity (u)	Voltage (v)
Displacement (x)	Flux (Φ)
Mass (M)	Capacitance (C)
Damping coefficient (B)	Conductance (G)
Spring constant (K)	1/Inductance (L)

TORQUE-VOLTAGE ANALOGY

Consider a simple rotational mechanical system as shown in figure 1.4.4.

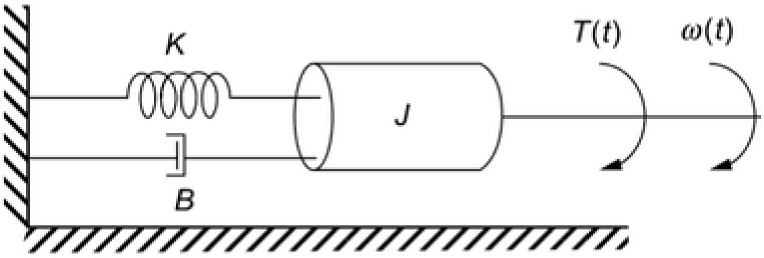

Figure 1.4.4 Rotational mechanical system

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.71]

Using D' Alembert's principle, we have,

Sum of the applied torques = Sum of the opposing torques

$$T(t) = J\frac{d\omega(t)}{dt} + B\omega(t) + K\int \omega(t)\,dt$$

Consider a series RLC circuit as shown in figure 1.4.5.

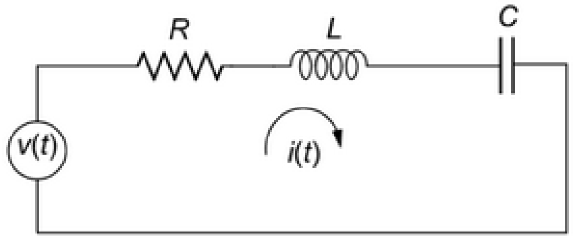

Figure 1.4.5 Series RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.72]

Using KVL, the integro-differential equations can be written as

$$v(t) = L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)\,dt$$

On comparing with the mechanical rotational system equation, we get,

Rotational System	Electrical System
Torque (T)	Voltage (v)
Angular velocity (ω)	Current (i)
Angular displacement (θ)	Charge (q)
Moment of inertia (J)	Inductance (L)
Rotational damping (B)	Resistance (R)
Rotational spring constant (K)	1/Capacitance (C)

TORQUE-CURRENT ANALOGY

Consider a simple parallel RLC circuit as shown in figure 1.4.6.

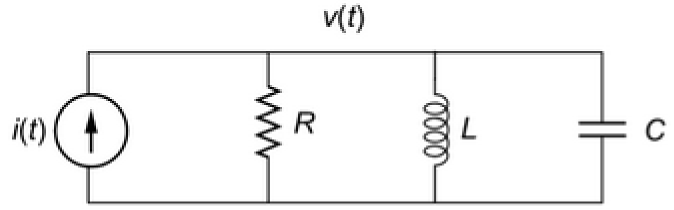

Figure 1.4.6 Parallel RLC circuit

[Source: "Control Systems Engineering" by S.Salivahanan, R.Rengaraj, G.R.Venkatakrishnan, Page: 1.73]

Using KCL, the integro-differential equations can be written as follows:

$$i(t) = C\frac{dv(t)}{dt} + G\,v(t) + \frac{1}{L}\int v(t)\,dt$$

where, conductance, G=1/R.

On comparing with the mechanical rotational system equation, we get,

Rotational Mechanical System	*T-I* Analogous
Torque (T)	Current (i)
Angular velocity (ω)	Voltage (v)
Angular displacement (θ)	Flux (Φ)
Moment of inertia (J)	Capacitance (C)
Rotational spring constant (K)	1/Inductance (L)
Rotational damping (B)	Conductance (G)