Formulation of power flow problem in polar coordinates

The Newton-Raphson method can also be applied to the solution of power flow problem when the bus voltages are expressed in polar form. In fact, only polar form is used in practice because the use of polar form results in a smaller number of equations than the total number of equations involved in rectangular form.

For any i^{th} bus, we have

$$\mathbf{V}_i = V_i e^{j\delta i}, \text{ then } \mathbf{V}_i^* = V_i e^{-j\delta i},$$
$$\text{and } \mathbf{V}_k = V_k e^{j\delta k} \qquad \text{...(6.114)}$$
$$\text{and } \mathbf{Y}_{ik} = Y_{ik} e^{-j\theta ik}$$

When δ is the phase angle of the bus voltages and θ_{ik} is and admittance angle. Then according to Eq. 6.57(b) for any i^{th} bus –

$$\mathbf{S}_i^* = P_i - jQ_i = \mathbf{V}_i^* \sum_{k=1}^{n} \mathbf{Y}_{ik}\mathbf{V}_k \,;\, i = 1, 2, \ldots n \qquad \text{...(6.115)}$$

Substituting the values of V_i, V_k and Y_{ik} from Eq. (6.114) in Eq. (6.115) we have –

$$P_i - jQ_i = \sum_{k=1}^{n} V_i V_k Y_{ik}\, e^{-j(\theta_{ik} + \delta_i - \delta_k)} \qquad \text{...(6.116)}$$

$$\text{Thus } P_i = \text{Real } \mathbf{V}_i^* \sum_{k=1}^{n} \mathbf{Y}_{ik}\mathbf{V}_k = \sum_{k=1}^{n} V_i V_k Y_{ik} \cos(\theta_{ik} + \delta_i - \delta_k)$$

$$= V_i V_i Y_{ii} \cos\theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} V_i V_k Y_{ik} \cos(\theta_{ik} + \delta_i - \delta_k) \qquad \text{...(6.117)}$$

$$\text{and } Q_i = \text{Imaginary } \mathbf{V}_i^* \sum_{k=1}^{n} \mathbf{Y}_{ik}\mathbf{Y}_k = \sum_{k=1}^{n} V_i V_k Y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k)$$

$$= V_i V_i Y_{ii} \sin\theta_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} V_i V_k Y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k) \qquad \text{...(6.118)}$$

for $i = 2, 3, 4, \ldots n$ because bus 1 is slack bus

Now the linear equation in polar form becomes –

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \end{array}\right] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \qquad \text{...(6.119)}$$

where J_1, J_2, J_3 and J_4 are the elements of Jacobian matrix and can be determined from power Eqs. (6.117) and (6.118) as follows:

The off-diagonal and diagonal elements of J_1 are –

$$\frac{\partial P_i}{\partial \delta_k} = V_i V_k Y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k) \text{ for } k \neq i \qquad \text{...(6.120)}$$

$$\text{and } \frac{\partial P_i}{\partial \delta_i} = -\sum_{\substack{k=1 \\ k \neq i}}^{n} V_i V_k Y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k) \qquad \text{...(6.121)}$$

The off-diagonal and diagonal elements of J_2 are –

$$\frac{\partial P_i}{\partial V_k} = V_i Y_{ik} \cos(\theta_{ik} + \delta_i - \delta_k) \text{ for } k \neq i \qquad ...(6.122)$$

$$\text{and } \frac{\partial P_i}{\partial V_i} = 2V_i Y_{ii} \cos\theta_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} V_k Y_{ik} \cos(\theta_{ik} + \delta_i - \delta_k) \qquad ...(6.123)$$

The off-diagonal and diagonal elements of J_3 are –

$$\frac{\partial Q_i}{\partial \delta_k} = -V_i V_k Y_{ik} \cos(\theta_{ik} + \delta_i - \delta_k) \qquad ...(6.124)$$

$$\text{and } \frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} V_i V_k Y_{ik} \cos(\theta_{ik} + \delta_i - \delta_k) \qquad ...(6.125)$$

The off-diagonal and diagonal elements of J_4 are –

$$\frac{\partial Q_i}{\partial V_k} = V_i Y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k) \text{ for } k \neq i \qquad ...(6.126)$$

$$\text{and } \frac{\partial Q_i}{\partial V_i} = 2V_i Y_{ii} \sin\theta_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} V_k Y_{ik} \sin(\theta_{ik} + \delta_i - \delta_k) \qquad ...(6.127)$$

The elements of Jacobian matrix are computed with the latest voltage estimate and computed power. However, the procedure (i.e., algorithm, here) is the same as that of the rectangular coordinates. The formulation in the polar coordinates takes less computational efforts and also needs less memory space.