3.5 CORRELATION BETWEEN FREQUENCY DOMAIN AND TIME DOMAIN SPECIFICATIONS

For a second order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Put $s=j\omega$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{-\omega^2 + 2\zeta\omega_n j\omega + \omega_n^2}$$

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{-\frac{\omega^2}{\omega_n^2} + 2\zeta j\frac{\omega}{\omega_n} + 1}$$

Let $u = \frac{\omega}{\omega_n}$, then

$$\frac{C(j\omega)}{R(j\omega)} = \frac{1}{(1-u^2) + 2\zeta ju}$$

We know,

$$M(j\omega) = |M(j\omega)|\angle M(j\omega)$$

$$|M(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2 + (2\zeta u)^2}}$$

$$\theta = -\tan^{-1}\left(\frac{2\zeta u}{1-u^2}\right)$$

Now,

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$\omega_b = \omega_n\sqrt{1-2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$PM = -180^o + \phi$$

where,

$$\phi = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{4\zeta^2+1} - 2\zeta^2}}\right)$$