THE LAGRANGIAN

Lagrangian mechanics is based on the differentiation of the system's energy terms with respect to the system's variables and time, as follows. For simple cases, it may take longer to use this technique than Newtonian mechanics. However, as the complexity of the system increases, the Lagrangian method becomes relatively simpler to use. Lagrangian mechanics is based on the following two generalized equations: one for linear motions and one for rotational motions. First, we define a Lagrangian as:

$$L = K - P$$

where L is the Lagrangian, K is the kinetic energy of the system, and P is the potential energy of the system. Then:

$$F_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i}$$

$$T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i}$$

where Fi is the summation of all external forces, Ti is the summation of all external torques, and θi and xi are system variables. As a result, in order to get the equations of motion, we need to derive the system's energy equations and differentiate the Lagrangian. The following five examples demonstrate the application of Lagrangian mechanics in deriving equations of motion. Notice how the complexity of the terms increases as the number of DOF (and variables) increases.

MOMENT OF INERTIA:

Moment of Inertia (MOI) describes how resistant a body is to rotational motion about a given axis—similar to how mass measures resistance to linear motion.

The moment of inertia of a body about an axis is:

I=∑mr2 (discrete masses)

Where:

- m = mass element
- r = perpendicular distance from the axis of rotation
- I = moment of inertia

- Larger I → harder to rotate
- Mass farther from the axis increases I significantly
- Depends on **mass**, **shape**, and **axis of rotation**

APPLICATIONS

- Rotating machinery
- Flywheels
- Gyroscopes
- Sports (spinning skaters, divers)
- Engineering design