

3.3. ANALYSIS OF PLANE RIGID FRAMES WITH AND WITHOUT SWAY IN MOMENT DISTRIBUTION METHOD.

3.3.1. INTRODUCTION

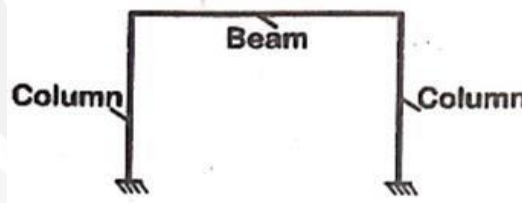
PLANE RIGID FRAMES

It is an indeterminate structure consisting of horizontal and inclined beams resting over columns. The nodes (or) joints of beams and columns behave like rigid joints.

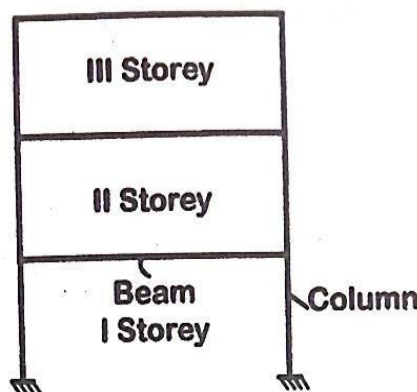
Depending on the number of bays and number of storeys these are classified as;

- Single bay single storey rigid frames
- Single bay multistorey rigid frames
- Multi bay single storey rigid frames and
- Multistorey rigid frames

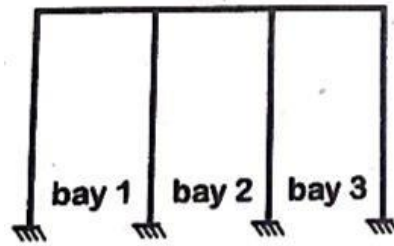
SKETCHES OF RIGID FRAMES



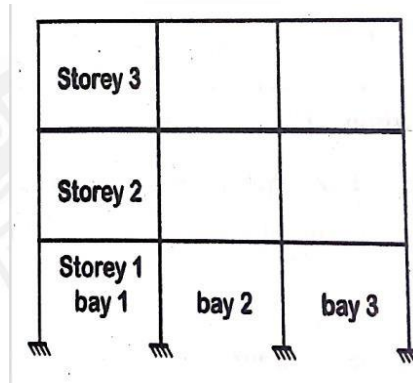
(a) Single bay single storey rigid frames



(b) Single bay multistorey rigid frames



(c) Multi bay single storey rigid frames

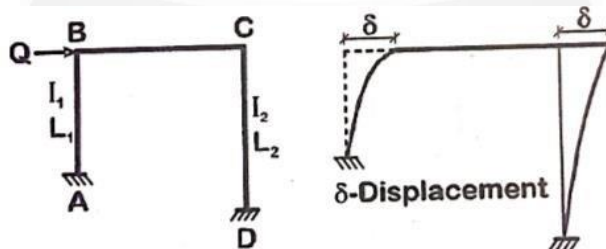


(d) Multistorey rigid frames

3.3.2. SWAY AND NON-SWAY FRAMES

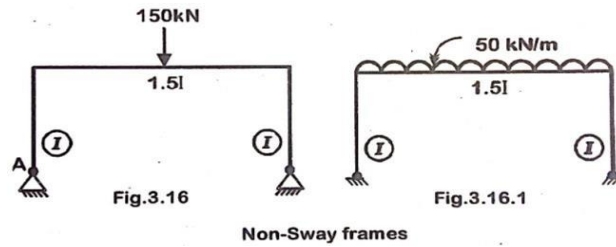
With Sway Frames:

A portal frame is said to be a sway frame when it is subjected to unbalanced and resultant horizontal forces.



Without Sway Frames:

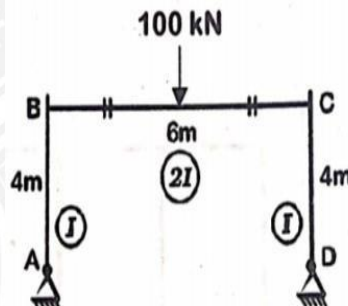
A portal frame is said to be a non-sway frame or without sway when it is subjected to vertical forces.



3.3.3. NUMERICAL EXAMPLES ON(PLANE RIGID FRAMES):

PROBLEM NO:01

For the portal rigid frame compute the bending moments and draw the BMD



Solution:

- Fixed End Moments:**

$$MF_{AB} = MF_{BA} = 0$$

$$MF_{BC} = Wl/8 = 100 \times 6/8 = -75 \text{ kNm}$$

$$MF_{CB} = -Wl/8 = 100 \times 6/8 = 75 \text{ kNm}$$

$$MF_{CD} = MF_{DC} = 0$$

- Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor ($k/\Sigma k$)
B	BA	$3EI/l = 3EI/4$	$7EI/4$	0.43
	BC	$3E(2I)/l = 6EI/6$		0.57
C	CB	$3E(2I)/l = 6EI/6$	$7EI/4$	0.57
	CD	$3EI/l = 3EI/4$		0.43

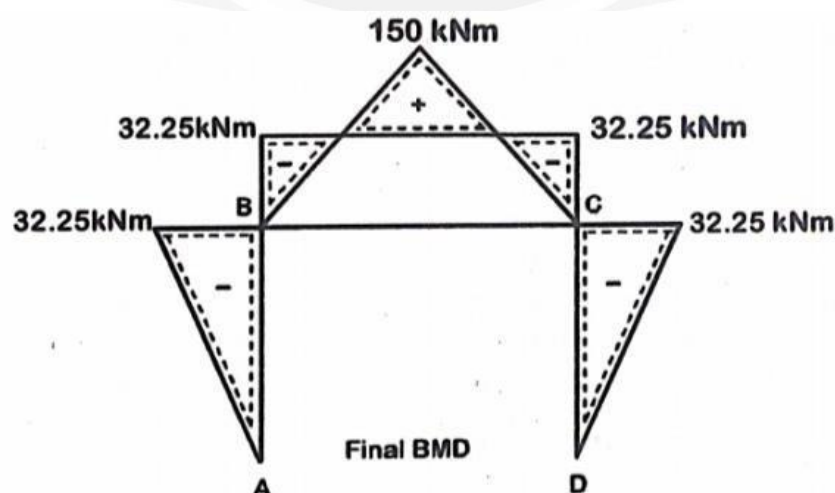
- **Free BMD:**

$$MBC = Wl/4 = 100 \times 6/4 = 150\text{kNm}$$

- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.43	0.57	0.57	0.43	-
F.E.M	0	0	-75	75	0	0
Distribute		32.25	42.75	-42.75	-32.25	
Final Moments	0	32.25	-32.25	32.25	-32.25	0
Conventional Moments	0	-32.25	-32.25	-32.25	-32.25	0

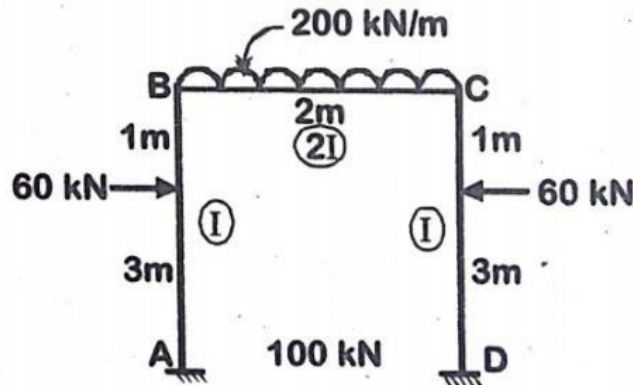
- **Moment Diagram:**



PROBLEM NO:02

Analyse the portal frames shown in fig by moment distribution method and draw the BMD.

Solution:



- Fixed End Moments:**

$$MF_{AB} = -Wab^2/l^2 = -60 \times 3 \times 1^2/4^2 = -11.25 \text{ kNm};$$

$$MF_{BA} = Wa^2b/l^2 = 60 \times 3^2 \times 1/4^2 = 33.76 \text{ kNm};$$

$$MF_{BC} = -Wl^2/12 = -200 \times 2^2/12 = -66.67 \text{ kNm};$$

$$MF_{CB} = Wl^2/12 = 200 \times 2^2/12 = 66.67 \text{ kNm};$$

$$MF_{CD} = -Wab^2/l^2 = -60 \times 3 \times 1^2/6^2 = -33.75 \text{ kNm};$$

$$MF_{DC} = Wa^2b/l^2 = 60 \times 3^2 \times 1/4^2 = 11.25 \text{ kNm}.$$

- Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor ($k/\Sigma k$)
B	BA	$4EI/l = 4EI/4$	$4EI$	0.25
	BC	$3E(2I)/l = 6EI/2$		0.75
C	CB	$3E(2I)/l = 6EI/2$	$4EI$	0.75
	CD	$4EI/l = 4EI/4$		0.25

- Free BMD:**

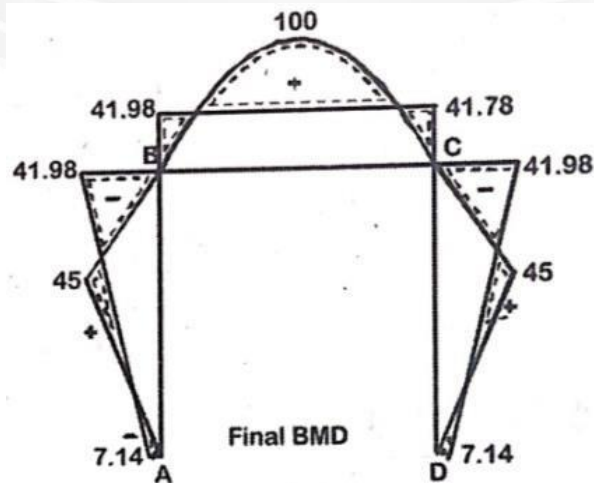
$$M_{AB} = M_{CD} = Wab/l = 60 \times 3 \times 1/4 = 45 \text{ kNm};$$

$$M_{BC} = Wl^2/8 = 200 \times 2^2/8 = 100 \text{ kNm}.$$

• **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.25	0.75	0.75	0.25	-
F.E.M	-11.25	33.75	-66.67	66.67	-33.75	11.25
Distribute		8.23	24.69	-24.69	-8.23	
Carry over	4.11					-4.11
Final Moments	-7.14	41.98	-41.98	41.98	-41.98	7.14
Conventional Moments	-7.14	-41.98	-41.98	-41.98	-41.98	-7.14

• **Moment Diagram:**

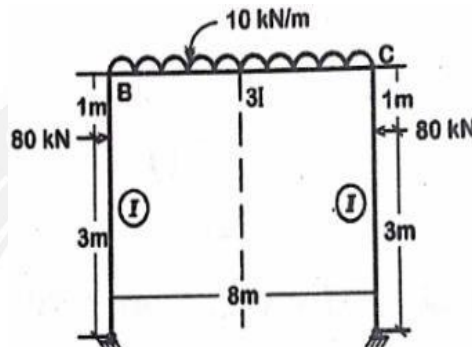


• **Result:**

$$M_A = M_D = -7.14 \text{ kNm}; \quad M_B = M_C = -41.98 \text{ kNm}$$

PROBLEM NO:03

Analyze the symmetrical portal rigid frame shown below by moment distribution method and draw BMD



Solution:

• **Fixed End Moments:**

$$M_{FAB} = -Wab^2/l^2 = -80 \times 3 \times 1^2/4^2 = -15 \text{ kNm};$$

$$M_{FBA} = Wa^2b/l^2 = 80 \times 3^2 \times 1/4^2 = 45 \text{ kNm};$$

$$M_{FBC} = -Wl^2/12 = -10 \times 8^2/12 = -53.33 \text{ kNm};$$

$$M_{FCB} = Wl^2/12 = 10 \times 8^2/12 = 53.33 \text{ kNm};$$

$$M_{FCD} = -Wab^2/l^2 = -80 \times 1 \times 3^2/4^2 = -45 \text{ kNm};$$

$$M_{FDC} = Wa^2b/l^2 = 80 \times 1^2 \times 3/4^2 = 15 \text{ kNm};$$

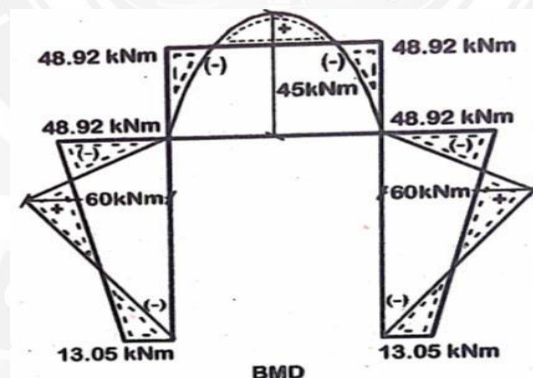
• **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor ($k/\Sigma k$)
B	BA	$4EI/l = 4EI/4$	$17EI/8$	0.47
	BC	$3E(3I)/l = 9EI/8$		0.53
C	CB	$3E(3I)/l = 9EI/8$	$17EI/8$	0.53
	CD	$4EI/l = 4EI/4$		0.47

- Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.47	0.53	0.53	0.47	-
F.E.M	-15	45	-53.33	53.33	-45	15
Distribute		3.92	4.41	-4.41	-3.92	
Carry over	1.95					-1.95
Final Moments	-13.05	48.92	-48.92	48.92	-48.92	13.05
Conventional Moments	-13.05	-48.92	-48.92	-48.92	-48.92	-13.05

- Moment Diagram:**



- Free BMD:**

$$M_{AB} = M_{CD} = W_{ab}/l = 80 \times 3 \times 1/4 = 60 \text{ kNm};$$

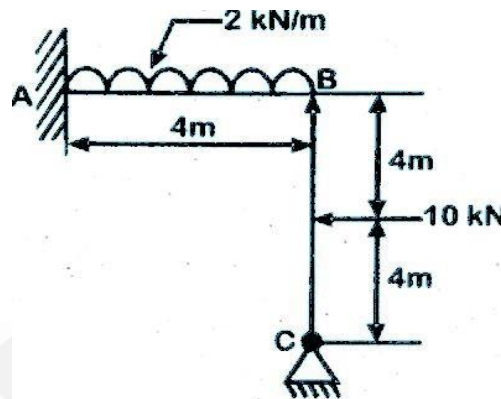
$$M_{BC} = Wl^2/8 = 10 \times 6^2/8 = 45 \text{ kNm}.$$

- Result:**

$$M_A = M_D = -13.05 \text{ kNm}; \quad M_B = M_C = -48.92 \text{ kNm}$$

PROBLEM NO:03

Analyze the symmetrical portal rigid frame shown below by moment distribution method and draw BMD



Solution:

- Fixed End Moments:**

$$MF_{AB} = -Wl^2/12 = -2 \times 4^2/12 = -2.67 \text{ kNm};$$

$$MF_{BA} = Wl^2/12 = 2 \times 4^2/12 = 2.67 \text{ kNm};$$

$$MF_{BC} = -Wl/8 = -10 \times 8/8 = -10 \text{ kNm}$$

$$MF_{CB} = Wl/8 = 10 \times 8/8 = 10 \text{ kNm}$$

- Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor ($k/\Sigma k$)
B	BA	$4EI/l = EI$	$11EI/8$	0.73
	BC	$3EI/l = 3EI/8$		0.27

- Free BMD:**

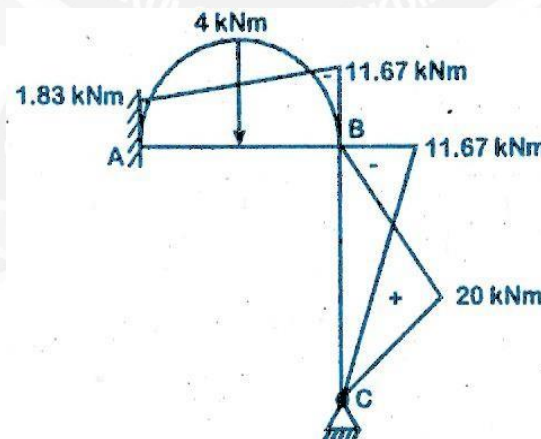
$$M_{AB} = Wl^2/8 = 2 \times 4^2/8 = 4 \text{ kNm.}$$

$$M_{BC} = Wl/4 = 10 \times 8/4 = 20 \text{ kNm};$$

- Moment Distribution Table:**

Joint	A	B		C
Member	AB	BA	BC	CB
D.F	-	0.73	0.27	-
F.E.M	-2.67	2.67	-10	10
Release C and carry over	-	0	-5	-10
Initial Moments	-2.67	2.67	-15	0
Distribute	-90	9.00	3.329	-90
Carry over	4.5			
Final Moments	1.83	11.67	-11.67	0
Conventional Moments	-1.83	-11.67	-11.67	0

- Moment Diagram:**



- Result:**

$M_A = -1.83 \text{ kNm}$; $M_B = -11.67 \text{ kNm}$; $M_C = 0$.