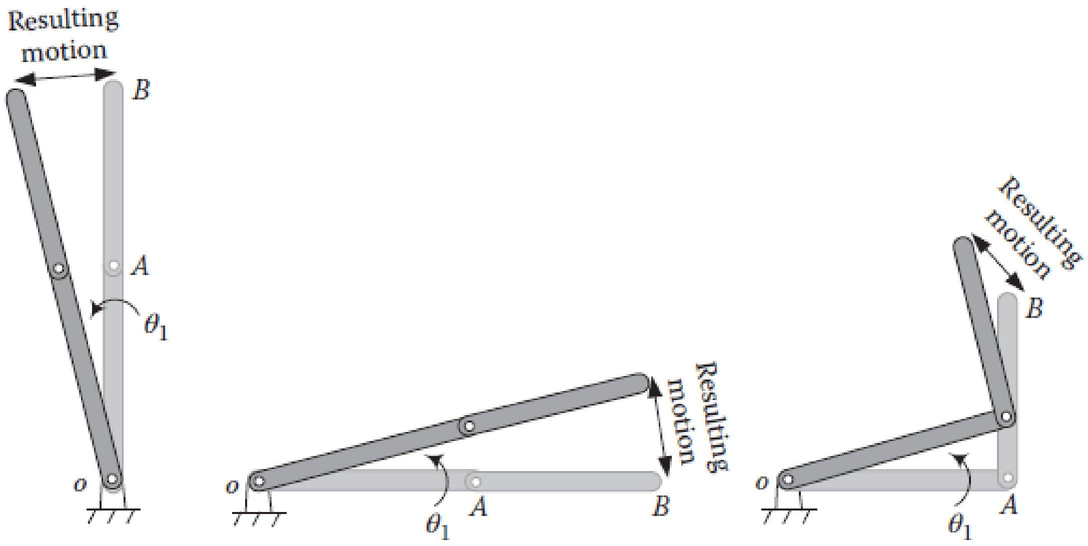
THE JACOBIAN:

The Jacobian is a representation of the geometry of the elements of a mechanism in time. In a robot, it allows the conversion of differential motions or velocities of individual joints to differential motions or velocities of points of interest (e.g. the end effector). It also relates the individual joint motions to overall mechanism motions. The Jacobian is time-related; since the values of joint angles vary in time, the magnitude of the elements of the Jacobian vary in time as well. As shown in Figure for a simple 2-DOF mechanism, if joint-1 of the robot moves an angle of θ, depending on the starting location and configuration of the mechanism, the magnitude and direction of the resulting motion of point B at its end will be very different. This dependence on the geometry of the mechanism is expressed by the Jacobian. Therefore, the Jacobian is a representation of the geometry and the interrelationship between different parts of the mechanism and where they are at any given time. Clearly, as time goes on and the relative po



ׅ.2 Resulting motions of the robot are dependent on the geometry of the robot.

The Jacobian was formed from the position equations, which were differentiated with respect to $\theta 1$ and $\theta 2$. Therefore, the Jacobian can be calculated by taking the derivatives of each position equation with respect to all variables. Suppose we have a set of equations yi in terms of a set of variables xj as:

$$y_i = f_i(x_1, x_2, x_3, \ldots, x_j)$$

The differential change in y_i as a result of a differential change in x_j will be:

$$\begin{cases} \delta y_1 = \dfrac{\partial f_1}{\partial x_1}\delta x_1 + \dfrac{\partial f_1}{\partial x_2}\delta x_2 + \cdots + \dfrac{\partial f_1}{\partial x_j}\delta x_j \\ \delta y_2 = \dfrac{\partial f_2}{\partial x_1}\delta x_1 + \dfrac{\partial f_2}{\partial x_2}\delta x_2 + \cdots + \dfrac{\partial f_2}{\partial x_j}\delta x_j \\ \vdots \\ \delta y_i = \dfrac{\partial f_i}{\partial x_1}\delta x_1 + \dfrac{\partial f_i}{\partial x_2}\delta x_2 + \cdots + \dfrac{\partial f_i}{\partial x_j}\delta x_j \end{cases}$$

Equation can be written in matrix form, representing the differential relationship between individual variables and the functions. The matrix encompassing this relationship is the Jacobian. Therefore, the Jacobian can be calculated by taking the derivative of each equation with respect to all variables. We apply the same principle for the calculation of the Jacobian of a robot.

$$\begin{bmatrix} \delta y_1 \\ \delta y_2 \\ \vdots \\ \delta y_i \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & & \frac{\partial f_1}{\partial x_j} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \cdots & \cdots \\ \vdots & & & \\ \frac{\partial f_i}{\partial x_1} & & & \frac{\partial f_i}{\partial x_j} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \vdots \\ \delta x_j \end{bmatrix} \quad \text{or} \quad [\delta y_i] = \left[\frac{\partial f_i}{\partial x_j}\right][\delta x_j]$$

Differentiating the position equations of a robot relates its joint differential motions to the differential motion of the hand frame:

$$\begin{bmatrix} dx \\ dy \\ dz \\ \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} \\ Robot \\ Jacobian \\ \\ \end{bmatrix} \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \\ d\theta_5 \\ d\theta_6 \end{bmatrix}$$

where dx, dy, dz in [D] represent the differential motions of the hand along the x-, y-, and z-axes; δx, δy, δz in [D] represent the differential rotations of the hand around the x-, y-, and z-axes; and [Dθ] represents the differential motions of the joints. As mentioned earlier, if these two matrices are divided by dt, they represent velocities instead of differential motions.