1.4 ABSOLUTE MAXIMUM BENDING MOMENT

Absolute maximum bending moment

When a given load system moves from one end to the other end of a girder, depending upon the position of the load, there will be a maximum bending moment for every section. The maximum of these bending moments will usually occur near or at the mid span. The maximum of maximum bending moments is called the absolute maximum bending moment.

Absolute maximum bending moment in a simply supported beam

When a series of wheel loads crosses a simply supported beam, the absolute maximum bending moment will occur near mid span under the load Wcr, nearest to mid span (or the heaviest load). If Wcr is placed to one side of mid span C, the resultant of the load system R shall be on the other side of C; and Wcr and R shall be equidistant from C. Now the absolute maximum bending moment will occur under Wcr. If Wcr and R coincide, the absolute maximum bending moment will occur at mid span.

Example:

A girder having a span of 18m is SS at the ends. It is traversed by a train of loads as shown in figure. The 50KN load leading. Find the maximum bending moment which can occurs i)under the 200KN load ii)under 50KN load using influence line diagram.
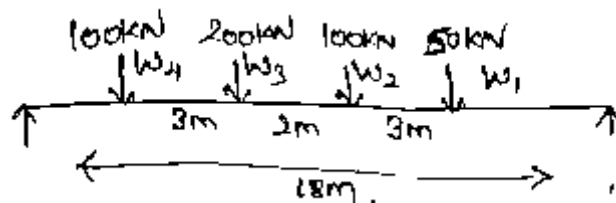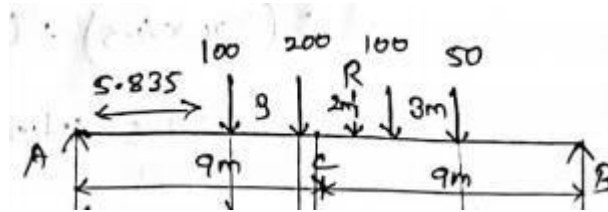

Fig. 1.4.1

Solution:

Fig. 1.4.2

Maximum bending moment

i) under 200KN load

      Resultant loads   =100+200+100+50

          =450 KN

Taking moment about W4

$(200 \times 3)+(100 \times 5)+(50 \times 8)$

           =R x

     1500 =450x

    $\bar{x}$ =3.33m

Ordinate max   =x(l-x)/l

          =9(9)/18

          =4.5

Distance between C and 200KN

        = dist b/w C and R

        =0.33/2

=0.165

Ordinate under 100KN   =4.5/8.835× 5.385

                                       =2.97

Ordinate under 100KN   =4.5/9.165× 7.165

                                       =3.52

Ordinate under 100KN   =4.5/9.165× 4.165

                                       =2.05

BM under 200KN load

                    =(200×4.5)+(100×2.97)+(100×3.51)+(50×2.05)

                    =1650.5 KNm
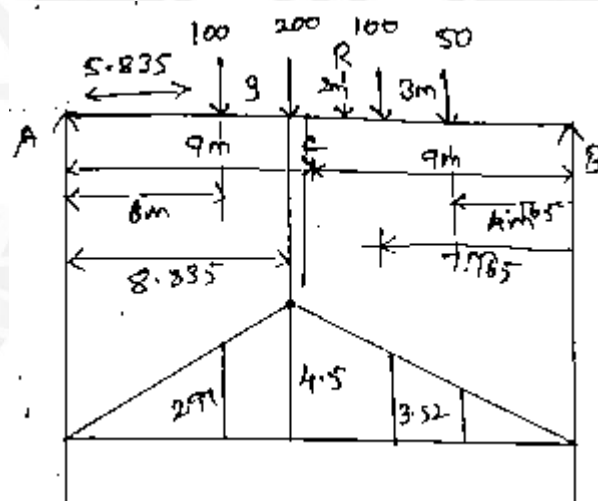
Fig. 1.4.3 Maximum Bending Moment

ii)Bending moment under 50KN load

Centre of span to BM equal distance

$$=(5-0.33)$$

Centre $=4.67/2$

$$=2.335m$$
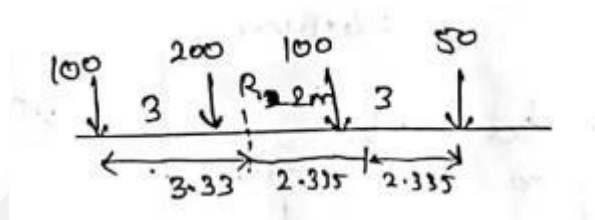
Fig. 1.4.4

Ordinate under 50KN

$$=x(l-x)/l$$

$$=(11.335\times6.665)/18$$

$$=4.2$$

Ordinate under 100KN

$$=4.2/11.335 \times8.335$$

$$=3.08$$

Ordinate under 200KN

$$=4.2/11.335 \times6.335$$

$$=2.35$$

Ordinate under 100KN

$$=4.2/11.335 \times3.335$$

$$=1.23$$

Maximum bending moment

$$=(50\times4.2)+(100\times3.09)+(200\times2.35)+(100\times1.24)$$
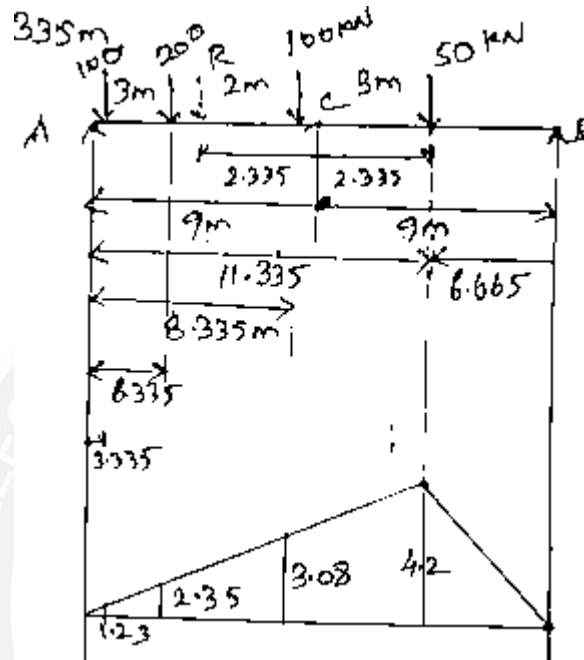
$$=1113 \text{ KNm}$$

Fig. 1.4.5 Maximum Bending Moment

Example :

Draw the influence line diagram for shear force and bending moment for a section at 5m from the left hand support of a simply supported beam, 20m long. Hence calculate the max bending moment and shear force at the section, due to an uniformly distributed rolling load of length 8m and intensity 10kN/m run
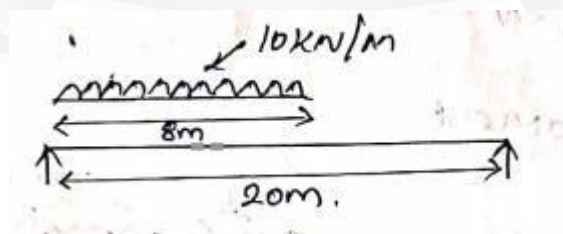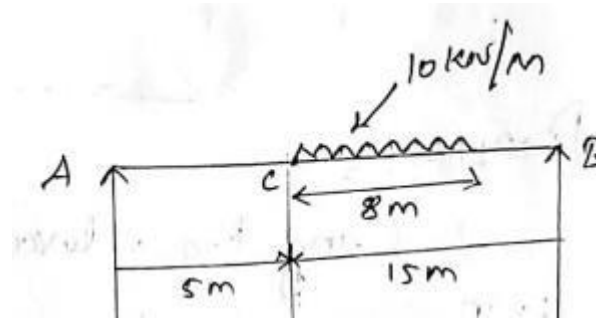

Fig. 1.4.6

Solution :

Fig. 1.4.7

a) Maximum shear force

I) Positive shear force

$$=l-x/l$$

$$=15/20$$

$$=0.75$$

Ordinate under C

$$=0.75/7$$

$$=0.35$$

Maximum positive shear force

$$=10\times[h/2(a+b)]$$

$$=10\times[0.75+0.35]8 / 2$$
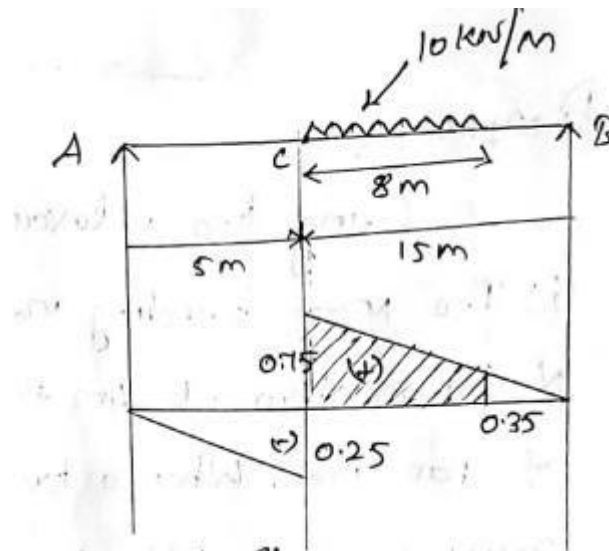
$$=44KN$$

Fig. 1.4.8 ILD For Positive Shear Force
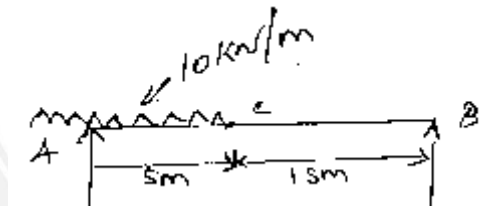
ii)negative shear force

Fig. 1.4.9

Negative shear force

$$=x/l$$

$$=5/20$$

$$=0.25$$

Max Negative shear force
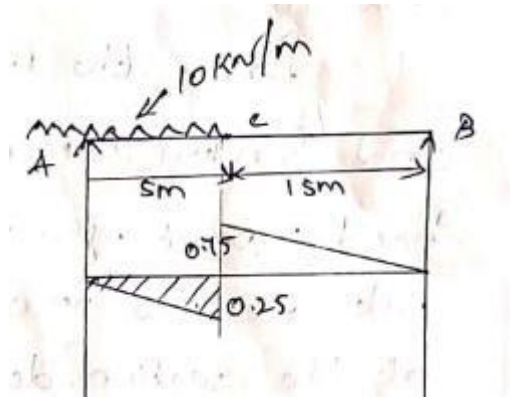
$$=10\times[1/2 \times5\times0.5]$$

$$=6.25 \text{ KN}$$

Fig. 1.4.10 ILD For Negative Shear Force

b) Max bending moment
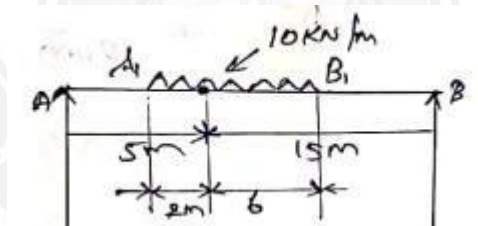
Equal ratio

$$20/4 = 5$$

$$8/4 = 2$$

Fig. 1.4.11

Max ordinate

$$=x(l-x)/l$$

$$=5(15)/20$$

$$=3.75m$$

Ordinate under A1

$$=3.75/5 \times 3$$

$$=2.25m$$

Ordinate under B1

$$=3.75/15 \times 9$$

$$=2.25 \text{ m}$$

Max bending moment

$$=10[(2.25+3.75)2/2 +(2.25+3.75)6/2]$$
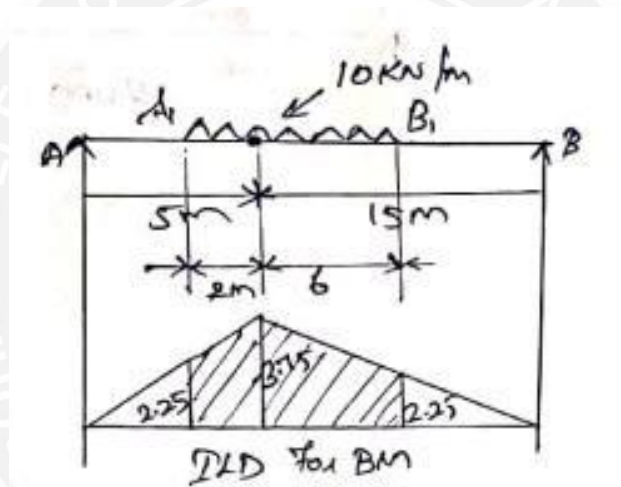
$$=10(6+18)$$

$$=240KNm$$

Fig. 1.4.12 ILD For Max Bending Moment

Example:

Four equal loads of 150KN each equally spaced at 2m apart. Followed by a UDL of 60KN/m at a distance of 1.5m from the last 150KN load across a girder of 20m from right to left. Using influence lines. Calculate the shear force and bending moment at a distance of 8m from the left hand support. When the leading 150KN load is at 5m from the left hand support.
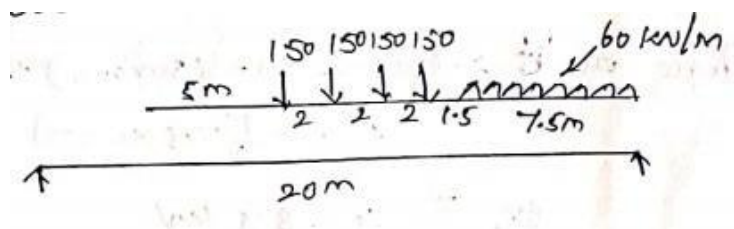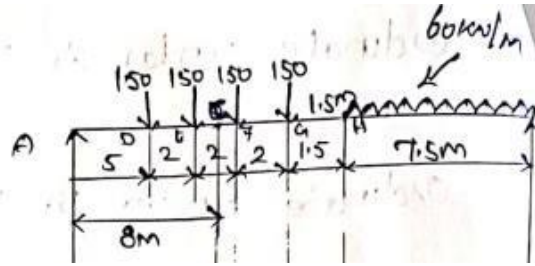

Fig. 1.4.13

Solution :

Fig. 1.4.14

a) shear force at this section

Positive shear force

$$=l-x/l$$

$$=20-8/20$$

$$=0.6$$

Ordinate under C left

$$=x/l$$

$$=8/20$$

$$=0.4$$

Ordinate under F

$$=0.6/12 \times 11$$

$$=0.55$$

Ordinate under G

$$=0.6/12 \times 9$$

$$=0.45$$

Ordinate under H

$$=0.6/12 \times 7.5$$

$$=0.375$$

Ordinate under E

$$=-0.4/8 \times 7$$

$$=-0.35$$

Ordinate under D

$$=-0.4/8 \times 7$$

$$=-0.25$$

Shear force at C

$$=-(150 \times 0.25)-(50 \times 0.35)+(150 \times 0.55)+(150 \times 0.45)+(60 \times (1/2 \times 7.5) \times 0.35)$$
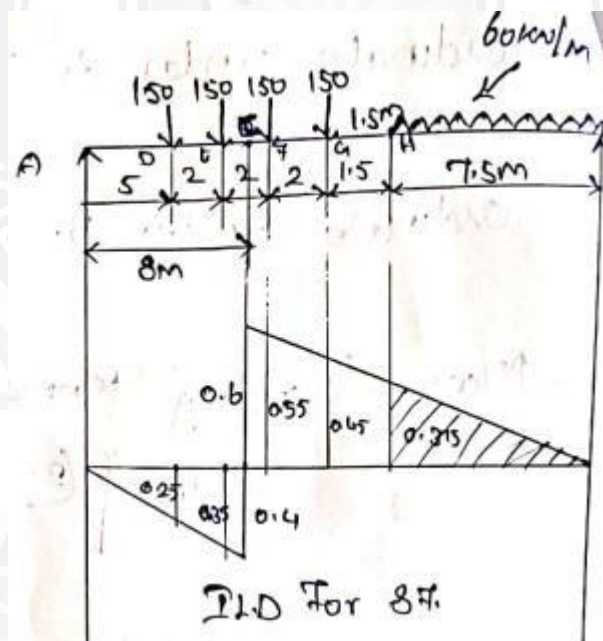
SF@c $=144.375$ KN

Fig. 1.4.15 ILD For Shear Force

b) Bending moment for given load position

Max Ordinate at C

$$=x(l-x)/l$$

=8(12)/20

=4.8

Ordinate under E

=4.8/8 ×7

=4.2m

Ordinate under D

=4.8/8×5

=3m

Ordinate under F

=4.8/12 ×11

=4.4m

Ordinate under G

=4.8/12 ×9

=3.6 m

Ordinate under H

=4.8/12 ×7.5

=3m

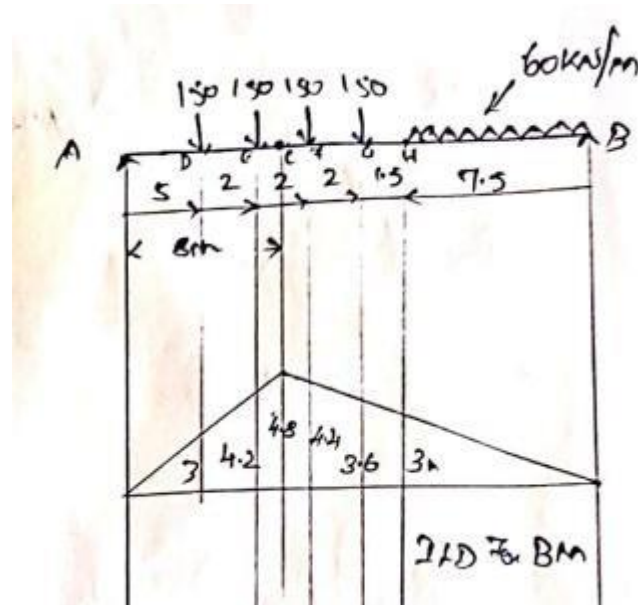Bending moment at C

=(150×3)+(150×4.2)+(150×4.4)+(50×3.6)+(60×(1/2)×7.5×3)

=2955 KNm

Fig. 1.4.16 ILD For Max Bending Moment