# Centroids and Centers of Gravity

A ***centroid*** is the geometric center of a geometric object: a one-dimensional curve, a two-dimensional area or a three-dimensional volume. Centroids are useful for many situations in Statics and subsequent courses, including the analysis of distributed forces, beam bending, and shaft torsion.

Two related concepts are the ***center of gravity***, which is the average location of an object's *weight*, and the ***center of mass*** which is the average location of an object's *mass*. In many engineering situations, the centroid, center of mass, and center of gravity are all coincident. Because of this, these three terms are often used interchangeably without regard to their precise meanings.
We consciously and subconsciously use centroids for many things in life and engineering, including:

- *Keeping your body's balance:* Try standing up with your feet together and leaning your head and hips in front of your feet. You have just moved your body's center of gravity out of line with the support of your feet.
- *Computing the stability of objects in motion like cars, airplanes, and boats:* By understanding how the center of gravity interacts with the accelerations caused by motion, we can compute safe speeds for sharp curves on a highway.
- *Designing the structural support to balance the structure's own weight and applied loadings on buildings, bridges, and dams:* We design most large infrastructure not to move. To keep it from moving, we must understand how the structure's weight, people, vehicles, wind, earth pressure, and water pressure balance with the structural supports.

The center of gravity of a body is fixed with respect to the body, but the coordinates depend on the choice of coordinate system. For example, in figure the center of gravity of the block is at its geometric center meaning that $\bar{x}$ and $\bar{y}$ are positive, but if the block is moved to the left of the $y$ axis, or the coordinate system is translated to the right of the block, $\bar{x}$ would then become negative.

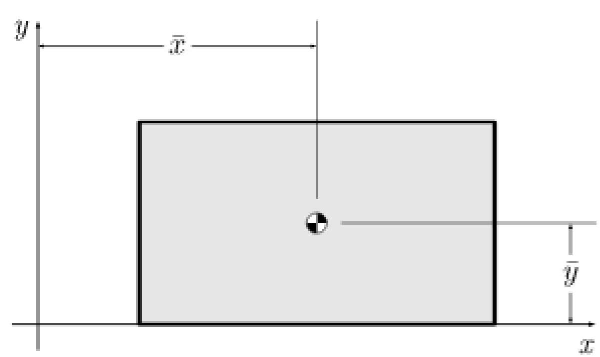



**Figure 3.1.** Location of the centroid, measured from the origin.

A centroid is a weighted average like the center of gravity, but weighted with a geometric property like area or volume, and not a physical property like weight or mass. This means that centroids are properties of pure shapes, not physical objects. They represent the coordinates of the "middle" of the shape.

The defining equations for centroids are similar to the equations for Centers of Gravity but with *volume* used as the weighting factor for three-dimensional shapes

$$\bar{x} = \frac{\sum \bar{x}_i V_i}{\sum V_i} \quad \bar{y} = \frac{\sum \bar{y}_i V_i}{\sum V_i} \quad \bar{z} = \frac{\sum \bar{z}_i V_i}{\sum V_i},$$

and *area* for two-dimensional shapes

$$\bar{x} = \frac{\sum \bar{x}_i A_i}{\sum A_i} \quad \bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}.$$

If the shape has an axis of symmetry, every point on one side of the axis is mirrored by another point equidistant on the other side. One has a positive distance from the axis, and the other is the same distance away in the negative direction. These two points will add to zero the numerator, as will every other point making up the shape, and the first moment will be zero. This means that the centroid must lie along the line of symmetry if there is one. If a shape has multiple symmetry lines, then the centroid must exist at their intersection.

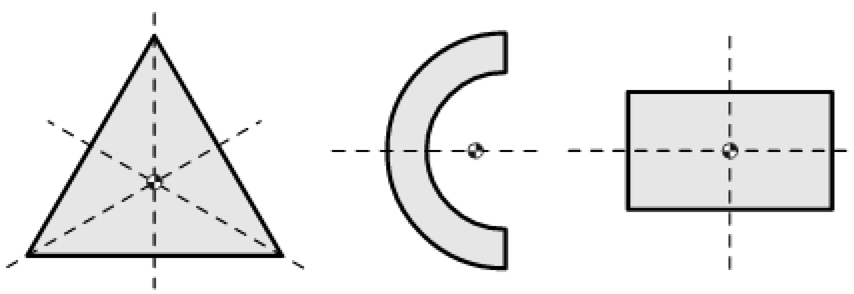
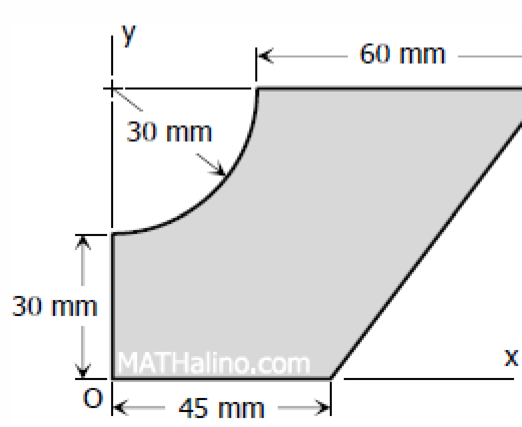


Figure 3.1.2 Centroids lie upon axes of symmetry.

### Problem

Locate the centroid of the shaded area in Fig.

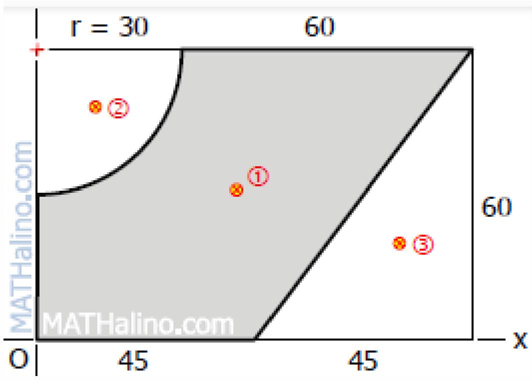


For rectangular area

$A_1 = 90(60) = 5400 \ \text{mm}^2$

$x_1 = \frac{1}{2}(90) = 45 \ \text{mm}$

$y_1 = \frac{1}{2}(60) = 30 \ \text{mm}$




For quarter circle

$A_2 = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(30^2) = 706.86 \ \text{mm}^2$

$$x_2 = \frac{4r}{3\pi} = \frac{4(30)r}{3\pi} = 12.73 \ \text{mm}$$

$$y_2 = 60 - \frac{4r}{3\pi} = 60 - \frac{4(30)}{3\pi} = 47.27 \ \text{mm}$$

For the shaded region

$A = A_1 - A_2 - A_3 = 5400 - 706.86 - 1350$

$A = 3343.14 \ \text{mm}^2$

$A\bar{x} = \Sigma ax$

$3343.14\bar{x} = 5400(45) - 706.86(12.73) - 1350(75)$

$\bar{x} = 39.71 \ \text{mm} \qquad \textit{answer}$

$A\bar{y} = \Sigma ay$

$3343.14\bar{y} = 5400(30) - 706.86(47.27) - 1350(20)$

$\bar{y} = 30.39 \ \text{mm} \qquad \textit{answer}$

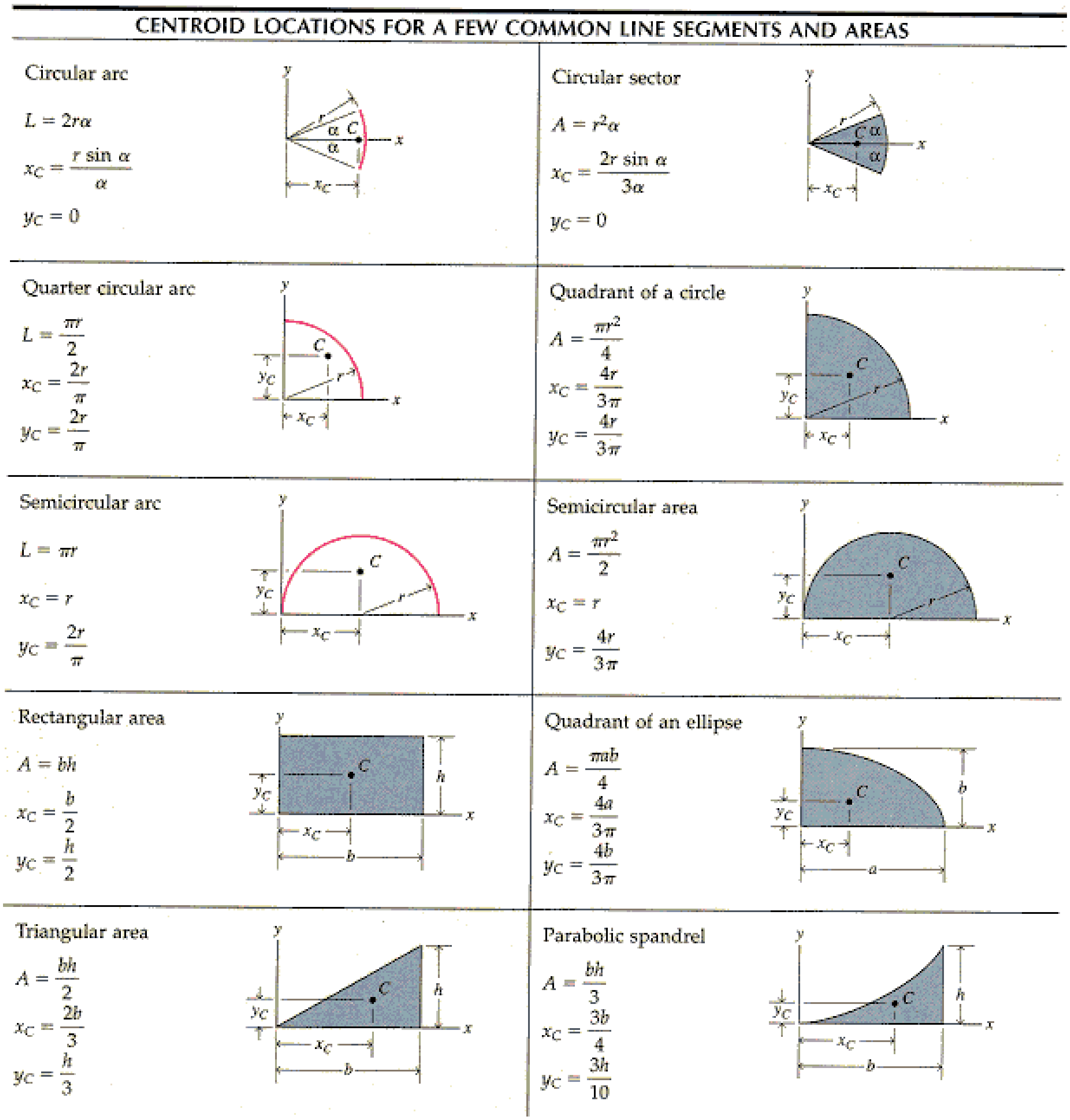



**Figure 3.2.** Location of the centroid, measured from the origin for different shapes