Power Input and Power Developed Equations

Net input to the synchronous motor is the three phase input to the stator.

∴ $P_{in} = \sqrt{3}\ V_L\ I_L\ \cos\Phi$ W where

V_L = Applied Line Voltage

I_L = Line current drawn by the motor

$\cos\Phi$ = operating p.f. of synchronous motor or P_{in}

= 3 ([er phase power)

$= 3 \times V_{ph}\ I_{aph} \cos\Phi$ W

Now in stator, due to its resistance R_a per phase there are stator copper losses.

Total stator copper losses $= 3 \times (I_{aph})^2 \times R_a$ W

∴ The remaining power is converted to the mechanical power, called gross mechanical power developed by the motor denoted as P_m.

∴ $P_m = P_{in}$ - Stator copper losses

Now $P = T \times \omega$

∴ $P_m = T_g \times (2\pi N_s/60)$ as speed is always N_s

$$\therefore \quad \boxed{T_g = \frac{P_m \times 60}{2\pi N_s}\ N_m}$$

This is the gross mechanical torque developed. In d.c. motor, electrical equivalent of gross mechanical power developed is $E_b \times I_a$, similar in synchronous motor the electrical equivalent of gross mechanical power developed is given by,

$$P_m = 3\ E_{bph} \times I_{aph} \times \cos(E_{bph} \wedge I_{aph})$$

i) For lagging p.f.,

$E_{bph} \wedge I_{aph} = \Phi - \delta$

ii) For leading p.f.,

$E_{bph} \wedge I_{aph} = \Phi + \delta$

iii) For unity p.f.,

$E_{bph} \wedge I_{aph} = \delta$

Note : While calculating angle between E_{bph} and I_{aph} from phasor diagram, it is necessary to reverse E_{bph} phasor. After reversing E_{bph}, as it is in opposition to V_{ph}, angle between E_{bph} and I_{aph} must be determined.

In general,

$$\boxed{P_m = 3\ E_{bph}\ I_{aph} \cos(\phi \pm \delta)}$$

Positive sign for leading p.f.

Neglecting sign for lagging p.f.

Net output of the motor then can be obtained by subtracting friction and windage i.e. mechanical losses from gross mechanical power developed.

∴ $P_{out} = P_m$- mechanical losses.

$$\therefore \quad T_{shaft} = \frac{P_{out} \times 60}{2\pi N_s} \ N_m$$

where T_{shaft} = Shaft torque available to load.

P_{out} = Power available to load

∴ Overall efficiency = P_{out}/P_{in}

$$\therefore \quad \boxed{\%\eta = \frac{P_{out}}{P_{in}} \times 100}$$

Condition for Maximum Power Developed

The value of δ for which the mechanical power developed is maximum can be obtained as,

$$\frac{dP_m}{d\delta} = 0$$

$$\therefore \quad \frac{d}{d\delta}\left[\frac{E_b V_{ph}}{Z_s}\cos(\theta-\delta) - \frac{E_b^2}{Z_s}\cos\theta\right] = 0$$

$$\therefore \quad \frac{E_b V_{ph}}{Z_s} \cdot \sin(\theta-\delta)(-1) = 0$$

$$\therefore \quad \sin(\theta-\delta) = 0$$

$$\therefore \quad \theta = \delta \qquad \ldots(6)$$

Note : Thus when R_a is negligible, $\theta = 90^o$ for maximum power developed. The corresponding torque is called pull out torque.

The Value of Maximum Power Developed

The value of maximum power developed can be obtained by substituting θ =δ in the equation of P_m.

$$(P_m)_{max} = \frac{E_b V_{ph}}{Z_s}\cos(0) - \frac{E_b^2}{Z_s}\cos(\delta)$$

$$\therefore \quad (P_m)_{max} = \frac{E_b V_{ph}}{Z_s} - \frac{E_b^2}{Z_s}\cos\delta \qquad \ldots(6a)$$

$$\therefore \quad (P_m)_{max} = \frac{E_b V_{ph}}{Z_s} - \frac{E_b^2}{Z_s}\cos\theta \qquad \ldots(6b)$$

When R_a is negligible, $\theta = 90^o$ and $\cos(\theta) = 0$ hence,

$$\therefore \quad \boxed{(P_m)_{max} = \frac{E_b V_{ph}}{Z_s}} \qquad \ldots \text{ when } R_a \text{ is negligible}$$

We know that $Z_s = R_a + j X_s = |Z_s| \angle \theta$

∴ $R_a = Z_s \cos\theta$ and $X_s = Z_s \sin\theta$

Substituting $\cos\theta = R_a/Z_s$ in equation (6b) we get,

$$\therefore \quad \boxed{(P_m)_{max} = \frac{E_b V_{ph}}{Z_s} - \frac{E_b^2 R_a}{Z_s^2}} \quad \dots (7)$$

Solving the above quadratic in E_b we get,

$$\boxed{E_b = \frac{Z_s}{2R_a}\left[V \pm \sqrt{V^2 - 4R_a (P_m)_{max}}\right]} \quad \dots (8)$$

As E_b is completely dependent on excitation, the equation (8) gives the excitation limits for any load for a synchronous motor. If the excitation exceeds this limit, the motor falls out of step.

Condition for Excitation When Motor Develop$_m$s (P_{max}) R

Let us find excitation condition for maximum power developed. The excitation controls E_b. Hence the condition of excitation can be obtained as,

$$\frac{dP_m}{dE_b} = 0$$

$$\therefore \quad \frac{d}{dE_b}\left[\frac{E_b V_{ph}}{Z_s}\cos(\theta - \delta) - \frac{E_b^2}{Z_s}\cos\theta\right] = 0$$

Assume load constant hence δ constant.

$$\therefore \quad \frac{V_{ph}}{Z_s}\cos(\theta - \delta) - \frac{2E_b}{Z_s}\cos\theta = 0$$

but $\theta = \delta$ for $P_m = (P_m)_{max}$

$$\therefore \quad \frac{V_{ph}}{Z_s} - \frac{2E_b}{Z_s}\cos\theta = 0$$

Substitute $\cos\theta = R_a/Zs$

$$\therefore \quad \frac{V_{ph}}{Z_s} - \frac{2E_b}{Z_s}\cdot\frac{R_a}{Z_s} = 0$$

$$\therefore \quad \boxed{E_b = \frac{V_{ph} Z_s}{2R_a}} \quad \dots (9)$$

This is the required condition of excitation.

Note : Note that this is not maximum value of but this is the value of foe which power developed is maximum.

The corresponding value of maximum power is,

$$(P_m)_{max} = \frac{V_{ph}^2}{2R_a} - \frac{V_{ph}^2}{4R_a} \quad \dots (10)$$