4.4 CONCEPTS OF CONTROLLABILITY AND OBSERVABILITY

CONCEPT OF CONTROLLABILITY

A system is said to be completely controllable, if it is possible to transfer the system state from any initial state x(t0) to any desired state x(t) in specified finite time by a control vector u(t).

If any of the state variable is independent of the control u(t), there would be no way of driving this particular state variable to desired state in finite time by means of control effort. Therefore, this particular state is said to be uncontrollable. As long as there is at least one uncontrollable state, the system is said to be not completely controllable or 'uncontrollable'.

Consider a single input, linear time invariant system:

$$\dot{X}(t) = AX(t) + BU(t)$$

Let the initial system state be x(0) and the final state be $x(t_f)$. The system is controllable if it is possible to construct a control signal, which in finite time interval $0 < t <= t_f$, will transfer the system state from x(0) to $x(t_f)$. The above equation is completely controllable if and only if the rank of the composite matrix is n.

$$Q_C = [B \vdots AB \vdots \cdots \vdots A^{n-1}B]$$

Since only matrices A and B are involved, we may say that the pair (A;B) is controllable if rank of Q_C is n.

CONCEPT OF OBSERVABILITY

A system is said to be completely observable, if every state $x(t_0)$ can be completely identified by measurement of outputs y(t) over a finite time interval. Given a LTI system that is described by the dynamic equations, the state $x(t_0)$ is said to be observable if given any input u(t), there exists a finite time $t_f >= t_0$ such that knowledge of u(t) for $t_0 <= t < t_f$, matrices A,B,C, & D and the output y(t); for $t_0 <= t < t_f$ are sufficient to determine $x(t_0)$. The necessary and sufficient condition for the system to be completely observable it is necessary and sufficient that the following n x n_p observability matrix has rank of n.

$$Q_O = [C^T \quad A^TC^T \quad (A^2)^TC^T \quad \cdots \quad (A^{n-1})^TC^T]$$