Problems:

1. The single line diagram of an unloaded power system is shown in Fig 1. The generator transformer ratings are as follows.

   G1=20 MVA, 11 kV, X''=25%

   G2=30 MVA, 18 kV, X''=25%

   G3=30 MVA, 20 kV, X''=21%

   T1=25 MVA, 220/13.8 kV (Δ/Y), X=15%

   T2=3 single phase units each rated 10 MVA, 127/18 kV(Y/Δ), X=15%

   T3=15 MVA, 220/20 kV(Y/Δ), X=15%

   Draw the reactance diagram using a base of 50 MVA and 11 kV on the generator1.

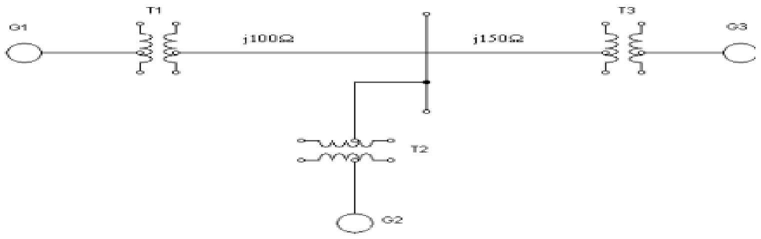


Fig 1

Solution:

Base megavoltampere, MVAb,new=50 MVA

Base kilovolt kVb,new=11 kV (generator side)

Formula:

$$\text{The new p.u. reactance } X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

Reactance of Generator G1

kVb,old=11 kV kVb,new=11 kV

MVAb,old= 20 MVA MVAb,new=50 MVA

Xp.u,old=0.25p.u

$$\text{The new p.u. reactance of Generator } G = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.25 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{20}\right) = j0.625p.u$$

Reactance of Transformer T1

kVb,old=11 kV kVb,new=11 kV

MVAb,old= 25 MVA MVAb,new=50 MVA

Xp.u,old=0.15p.u

$$\text{The new p.u. reactance of Transformer T1} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{11}{11}\right)^2 \times \left(\frac{50}{25}\right) = j0.3\ p.u$$

Reactance of Transmission Line

It is connected to the HT side of the Transformer T1

$$\text{Base kV on HT side of transformer T 1} = \text{Base kV on LT side} \times \frac{\text{HT voltage rating}}{\text{LT voltage rating}}$$

$$= 11 \times \frac{220}{11} = 220\ kV$$

$$\text{Actual Impedance } X_{actual} = 100ohm$$

$$\text{Base impedance } X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{220^2}{50} = 968\ ohm$$

$$\text{p.u reactance of 100 } \Omega \text{ transmission line} = \frac{\text{Actual Reactance ,ohm}}{\text{Base Reactance ,ohm}} = \frac{100}{968} = j0.103\ p.u$$

$$\text{p.u reactance of 150 } \Omega \text{ transmission line} = \frac{\text{Actual Reactance ,ohm}}{\text{Base Reactance ,ohm}} = \frac{150}{968} = j0.154\ p.u$$

Reactance of Transformer T2

kVb,old=127 * √3 kV =220 kV kVb,new=220 kV

MVAb,old= 10 * 3=30 MVA MVAb,new=50 MVA

Xp.u,old=0.15p.u

$$\text{The new p.u. reactance of Transformer T2} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{220}{220}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

Reactance of Generator G2

It is connected to the LT side of the Transformer T2

$$\text{Base kV on LT side of transformer T 2} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{18}{220} = 18\ kV$$

kVb,old=18 kV	kVb,new=18 kV
MVAb,old= 30 MVA	MVAb,new=50 MVA
Xp.u,old=0.25 p.u	

$$\text{The new p.u. reactance of Generator G 2} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.25 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{30}\right) = j0.4167\ p.u$$

Reactance of Transformer T3

kVb,old=20 kV	kVb,new=20 kV
MVAb,old= 20 MVA	MVAb,new=50 MVA
Xp.u,old=0.15p.u	

$$\text{The new p.u. reactance of Transformer T3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.25\ p.u$$

Reactance of Generator G3

It is connected to the LT side of the Transformer T3

$$\text{Base kV on LT side of transformer T 3} = \text{Base kV on HT side} \times \frac{\text{LT voltage rating}}{\text{HT voltage rating}}$$

$$= 220 \times \frac{20}{220} = 20\ kV$$

kVb,old=20 kV kVb,new=20 kV

MVAb,old= 30 MVA MVAb,new=50 MVA

Xp.u,old=0.21 p.u

$$\textit{The new p.u. reactance of Generator G 3} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.21 \times \left(\frac{20}{20}\right)^2 \times \left(\frac{50}{30}\right) = j0.35\ p.u$$

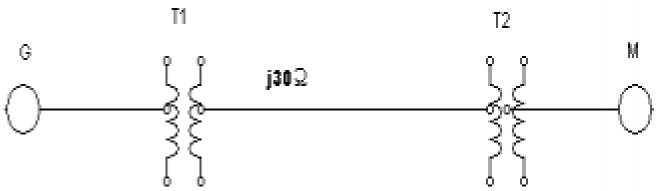
2). Draw the reactance diagram for the power system shown in fig. Use a base of 50MVA 230 kV in 30 Ω line. The ratings of the generator, motor and transformers are

Generator = 20 MVA, 20 kV, X=20%

Motor = 35 MVA, 13.2 kV, X=25%

T1 = 25 MVA, 18/230 kV (Y/Y), X=10%

T2 = 45 MVA, 230/13.8 kV (Y/Δ), X=15%



Solution:

Base megavoltampere,MVAb,new=50 MVA

Base kilovolt kVb,new=230 kV (Transmission line side)

Formula

$$\textit{The new p.u. reactance } X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

Reactance of Generator G

It is connected to the LT side of the T1 transformer

$$Base\ kV\ on\ LT\ side\ of\ transformer\ T\ 1 = Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$$

$$= 230 \times \frac{18}{230} = 18\ kV$$

kVb,old=20 kV kVb,new=18 kV

MVAb,old= 20 MVA MVAb,new=50 MVA

Xp.u,old=0.2p.u

$$The\ new\ p.u.\ reactance\ of\ Generator\ G = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.2 \times \left(\frac{20}{18}\right)^2 \times \left(\frac{50}{20}\right) = j0.617\ p.u$$

Reactance of Transformer T1

kVb,old=18 kV kVb,new=18 kV

MVAb,old= 25 MVA MVAb,new=50 MVA

Xp.u,old=0.1p.u

$$The\ new\ p.u.\ reactance\ of\ Transformer\ T1 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.1 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{50}{25}\right) = j0.2\ p.u$$

Reactance of Transmission Line

It is connected to the HT side of the Transformer T1

Actual Impedance X actual= j30 ohm

$$Base\ impedance\ X_{base} = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{230^2}{50} = 1058\ ohm$$

$$p.u\ reactance\ of\ j30\ \Omega\ transmission\ line = \frac{Actual\ Reactance\ ,ohm}{Base\ Reactance\ ,ohm} = \frac{j30}{1058} = j0.028\ p.u$$

Reactance of Transformer T2

kVb,old=230 kV kVb,new=230 kV

MVAb,old= 45 MVA MVAb,new=50 MVA

Xp.u,old=0.15p.u

$$The\ new\ p.u.\ reactance\ of\ Transformer\ T2 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.15 \times \left(\frac{230}{230}\right)^2 \times \left(\frac{50}{45}\right) = j0.166\ p.u$$

Reactance of Motor M2

It is connected to the LT side of the Transformer T2

$$Base\ kV\ on\ LT\ side\ of\ transformer\ T\ 2 = Base\ kV\ on\ HT\ side \times \frac{LT\ voltage\ rating}{HT\ voltage\ rating}$$

$$= 230 \times \frac{13.8}{230} = 13.8\ kV$$

kVb,old=13.2 kV kVb,new=13.8 kV

MVAb,old= 35 MVA MVAb,new=50 MVA

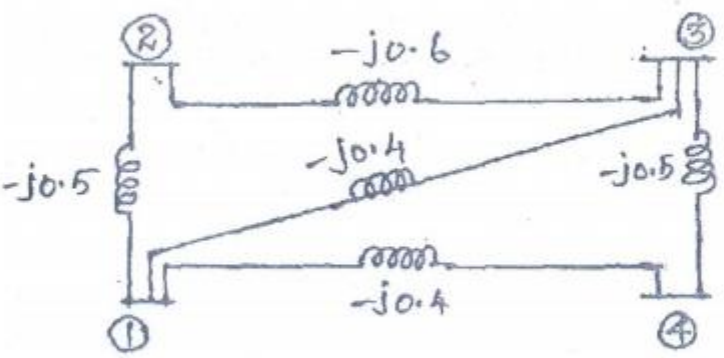
Xp.u,old=0.25 p.u

$$The\ new\ p.u.\ reactance\ of\ Generator\ G\ 2 = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}}\right)$$

$$= 0.25 \times \left(\frac{13.2}{13.8}\right)^2 \times \left(\frac{50}{35}\right) = j0.326\ p.u$$

Bus admittance matrix

1. Find the bus admittance matrix for the given network in Fig. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u.



Solution:

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

$Y11 = y12 + y13 + y14 = -j0.5 - j0.4 - j0.4 = -j1.3$

$Y22 = y12 + y23 = -j0.5 - j0.6 = -j1.1$

$Y33 = y32 + y31 + y34 = -j0.6 - j0.4 - j0.5 = -j1.5$

$Y44 = y41 + y43 = -j0.4 - j0.5 = -j0.9$

$Y12 = -y12 = j0.5$ $Y13 = -y13 = j0.4$

$Y14 = -y14 = j0.4$ $Y21 = Y12 = j0.5$

$Y23 = -y23 = j0.6$

$Y24 = -y24 = 0$

$Y31 = Y13 = j0.4$

$Y32 = Y23 = j0.6$

$Y34 = -y34 = j0.5$

$Y41 = Y14 = j0.4$

$Y42 = Y24 = 0$

$Y43 = Y34 = j0.5$

$$Y_{BUS} = \begin{bmatrix} -j1.3 & j0.5 & j0.4 & j0.4 \\ j0.5 & -j1.1 & j0.6 & 0 \\ j0.4 & j0.6 & -j1.5 & j0.5 \\ j0.4 & 0 & j0.5 & -j0.9 \end{bmatrix}$$

Elements of new bus admittance matrix after eliminating 4th row and 4th column

$$Y_{jk,new} = Y_{jk} - \frac{Y_{jn} Y_{nk}}{Y_{nn}}$$

N=4, j=1,2,3 k=1,2,3

$$Y_{11,new} = Y_{11} - \frac{Y_{14} Y_{41}}{Y_{44}} = -j1.3 - \frac{(j0.4)(j0.4)}{-j0.9} = -j1.12$$

$$Y_{12,new} = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}} = j0.5 - \frac{(j0.4)(j0)}{-j0.9} = j0.5$$

$$Y_{13,new} = Y_{13} - \frac{Y_{14}Y_{43}}{Y_{44}} = j0.4 - \frac{(j0.4)(j0.5)}{-j0.9} = j0.622$$

$$Y_{21,new} = Y_{12,new} = j0.5$$

$$Y_{22,new} = Y_{22} - \frac{Y_{24}Y_{42}}{Y_{44}} = -j1.1 - \frac{(j0)(j0)}{-j0.9} = -j1.1$$

$$Y_{23,new} = Y_{23} - \frac{Y_{24}Y_{43}}{Y_{44}} = j0.6 - \frac{(j0)(j0.5)}{-j0.9} = j0.6$$

$$Y_{31,new} = Y_{13,new} = j0.622$$

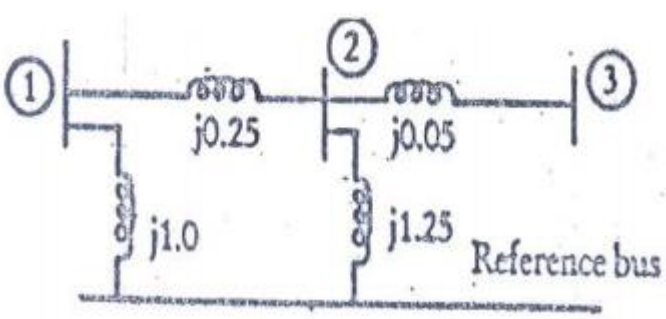
$$Y_{32,new} = Y_{23,new} = j0.6$$

$$Y_{33,new} = Y_{33} - \frac{Y_{34}Y_{43}}{Y_{44}} = -j1.5 - \frac{(j0.5)(j0.5)}{-j0.9} = -j1.22$$

Reduced admittance matrix after eliminating 4 th row and 4th column

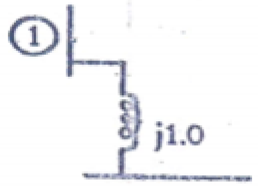
$$Y_{BUS} = \begin{bmatrix} -j1.12 & j0.5 & j0.622 \\ j0.5 & -j1.1 & j0.6 \\ j0.622 & j0.6 & -j1.222 \end{bmatrix}$$

2) Find the bus impedance matrix for the system whose reactance diagram is shown in fig 3. All the impedances are in p,u.



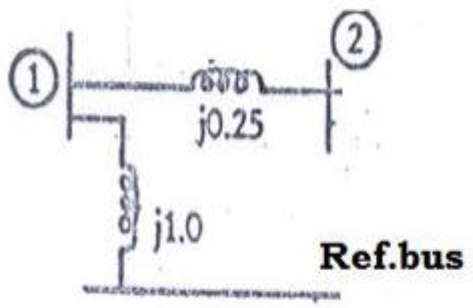
Solution:

Step 1: connect bus 1 to ref bus through impedance j1.0



$$Z_{bus} = [j1.0]$$

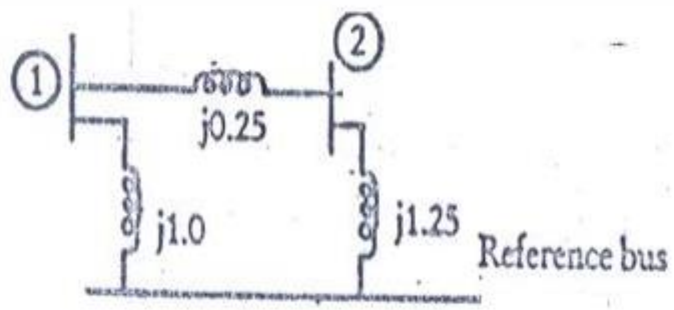
Step 2: connect bus 2 to the bus 1 through impedance j0.25



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0 + j0.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

Step 3: connect bus 2 to ref bus through impedance j1.25



$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j1.25 + j1.25 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

Number of buses is only 2. But matrix size is 3*3. The matrix size is reduced by eliminating 3rd row and 3rd column

$$Z_{jk,ack} = Z_{jk} - \frac{Z_{j(n+1)}Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

Where n=2 j=1,2 k=1,2

n=2 j=1 k=1

$$Z_{11,ack} = Z_{11} - \frac{Z_{13}Z_{31}}{Z_{33}}$$

$$Z_{11,ack} = j1.0 - \frac{j1.0 * j1.0}{j2.5} = j0.6$$

n=2 j=1 k=2

$$Z_{12,ack} = Z_{12} - \frac{Z_{13}Z_{32}}{Z_{33}}$$

$$Z_{12,ack} = j1.0 - \frac{j1.0 * j1.25}{j2.5} = j0.5$$

n=2 j=2 k=1

$$Z_{21,ack} = Z_{12,ack} = j0.5$$

n=2 j=2 k=2

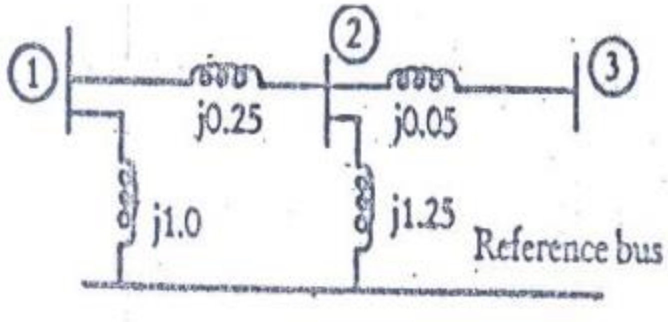
$$Z_{22,ack} = Z_{22} - \frac{Z_{23} Z_{32}}{Z_{33}}$$

$$Z_{22,ack} = j1.25 - \frac{j1.25 * j1.25}{j2.5} = j0.625$$

The reduced matrix

$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j.625 \end{bmatrix}$$

Step 4: connect bus 3 to bus 2 through impedance j0.05



$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j.625 & j.675 \end{bmatrix}$$